Selected Topics in Quark-Hadron Physics

-- From Scalar Nonets to Topological Glueballs --

Chihiro Sasaki

Institute of Theoretical Physics

University of Wroclaw, Poland

&

SKCM², Hiroshima University, Japan

Outline

Part I: The lowest scalar nonet

1. A puzzle: non-exotic or exotic?
2. Production of scalar mesons and glueballs in HICs
3. A new classification [Yasui et al., arXiv:2603.13764]

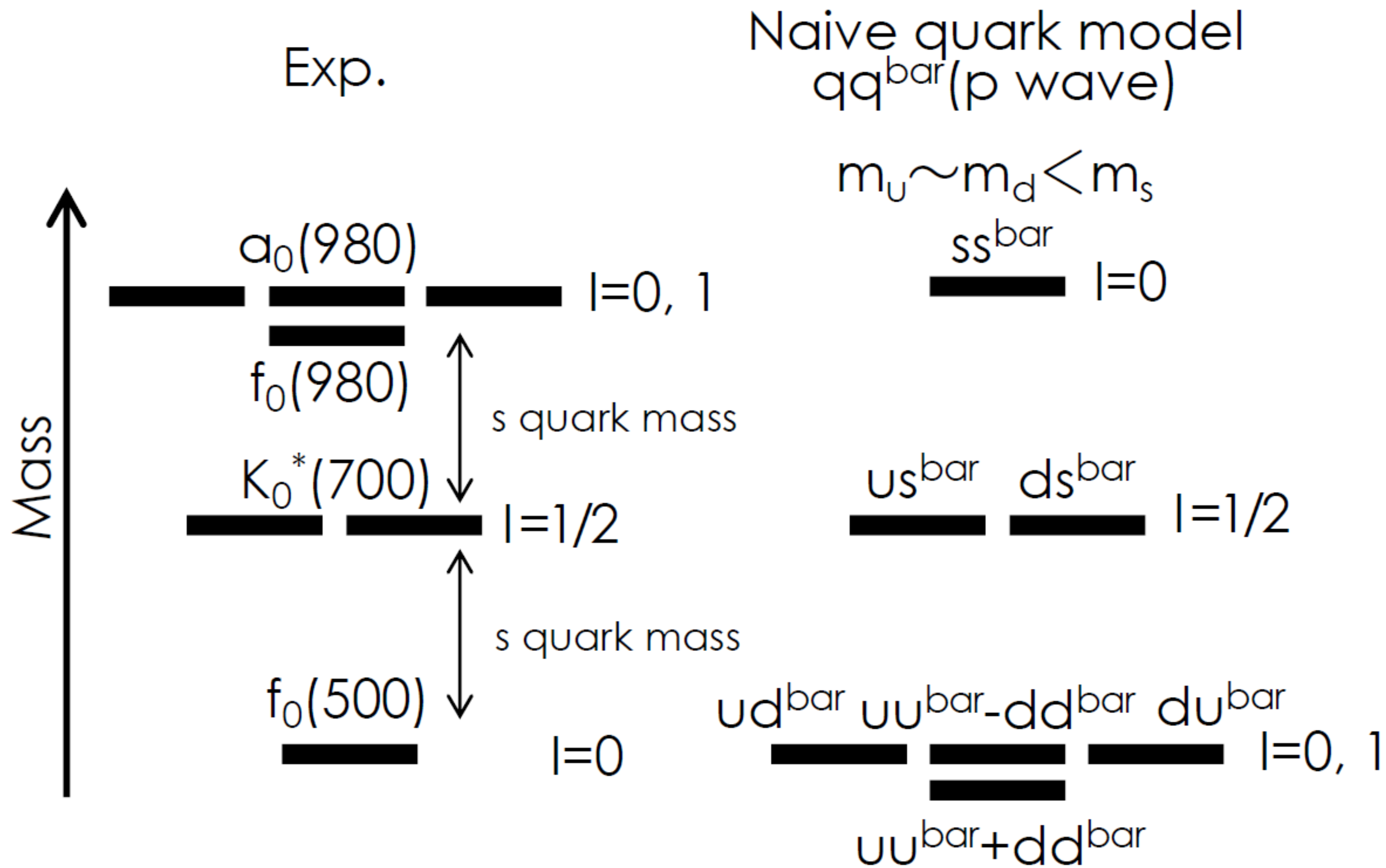
Part II: Glueballs as topological solitons

1. The Skyrme-Faddeev model
2. Glueballonia [Amari et al., Phys. Lett. B (2025)]
3. Gravitational FFs [Hutauruk et al., to appear on arXiv]

Part I:
The lowest scalar nonet

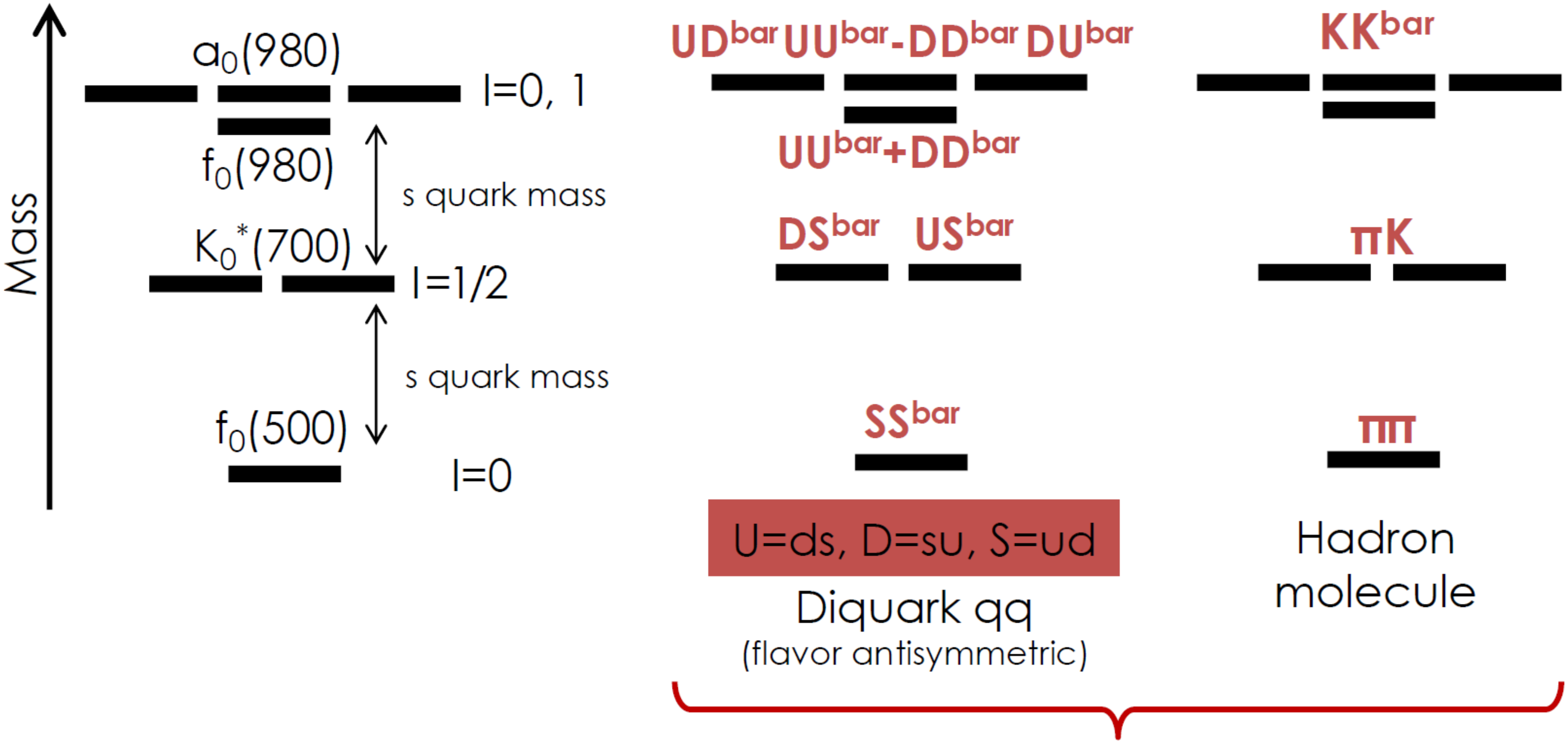
Revisiting *light* exotic mesons

Long-standing puzzle: why is that mass hierarchy?



Bob: “They are compact tetra-quark states.”

Diquarks [Jaffe (1977)] or **Molecules** [Close & Tornqvist (2002)]

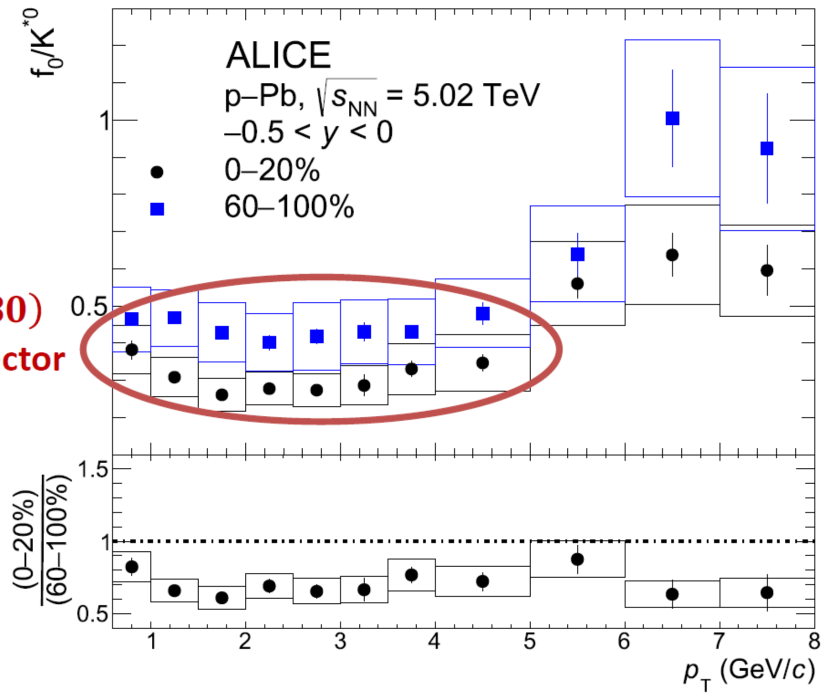


“$f_0(980)$ should include $s\bar{s}$.”

[ALICE Collaboration, PLB (2024)]

ALICE: "No, $f_0(980)$ is not a tetra-quark state."

Suppression of $f_0(980)$ against $K^{*0}(892)$ (vector meson) at low p_T.



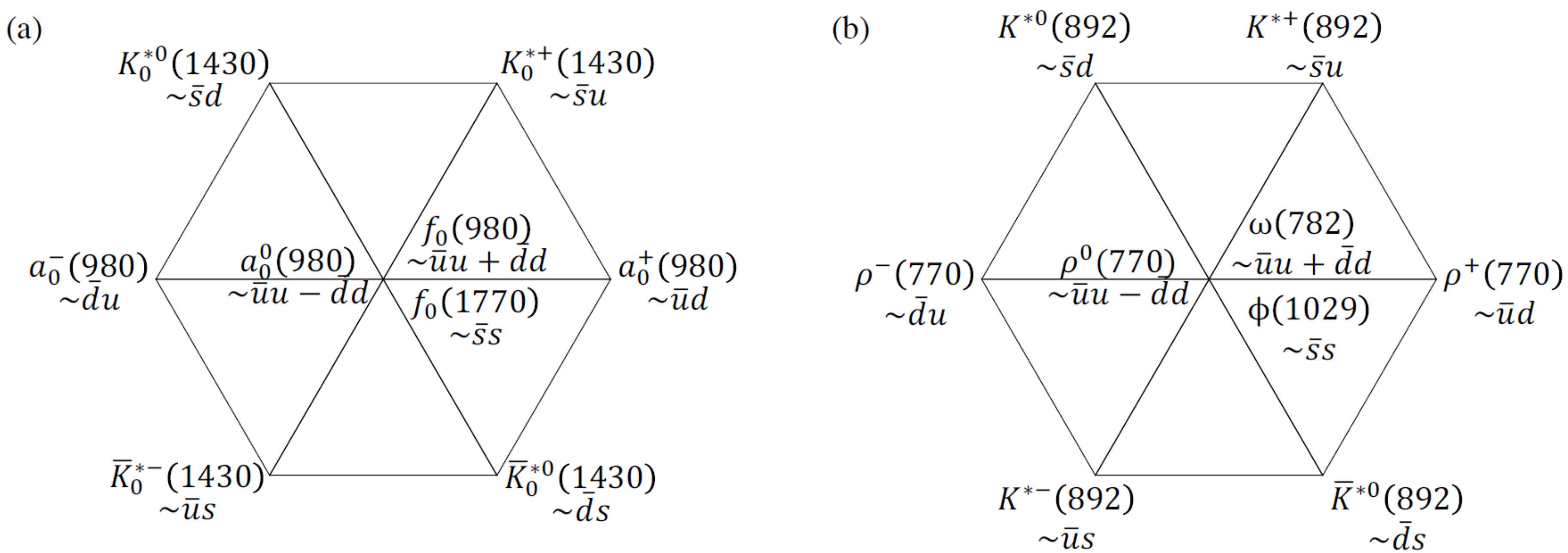
The abnormal suppression in terms of multiplicity and transverse momentum relative to other particles sheds light on the internal structure of $f_0(980)$ suggesting that it is a conventional meson with no hidden strange quarks and provides insight into the properties of the late hadronic phase in p–Pb collisions.

A new classification

✓Abandon $f_0(500)$ and $K_0^*(700)$.

✓$f_0(980)$ and $a_0(980)$ as the lowest states.

➔ Nonet: $f_0(980), a_0(980), K_0^*(1430), f_0(1770)$



Summary of the new nonet

	particle	m [MeV]	Γ [MeV]	$I(J^P)$	confs.(L)
nonet	$f_0(980)$	990 ± 20	10-100	$0(0^+)$	$\bar{n}n(P)$
	$a_0(980)$	980 ± 20	50-100	$1(0^+)$	$\bar{n}n(P)$
	$K_0^*(1430)$	1425 ± 50	270 ± 80	$1/2(0^+)$	$\bar{s}n(P)$
	$f_0(1770)$	1733^{+8}_{-7}	150^{+12}_{-10}	$0(0^+)$	$\bar{s}s(P)$
glueball	$f_0(1370)$	1200-1500	200-500	$0(0^+)$	$\bar{n}n(P)$
					$\bar{n}\bar{n}nn(S)$
					$gg(S)$
	$f_0(1500)$	1522 ± 25	108 ± 33	$0(0^+)$	$\bar{n}n(P)$
					$\bar{n}\bar{n}nn(S)$
					$gg\ (S)$
normal	$\rho(770)$	775	147	$1(1^-)$	$\bar{n}n(S)$
	$\phi(1020)$	1019	4.3	$0(1^-)$	$\bar{s}s(S)$
	$\Omega(1672)$	1672	—	$0(3/2^+)$	$sss(S)$

?

Production in HICs

?

reference particle

Phenomenological approaches

❑Statistical model [Andronic, Braun-Munzinger, Redlich, Stachel (2003)]

$$N_h^{\mathrm{stat}} = V_H \frac{g_h}{2\pi^2} \int_0^\infty \frac{p^2 \mathrm{d}p}{e^{(E_h - \mu_B B - \mu_S S)/T_H} \pm 1}$$

❑Coalescence model [Greco, C. M. Ko, P. Levai (2003)]

$$N_h^{\mathrm{coal}} = g_h \int \left[\prod_{i=1}^{n} \frac{1}{g_i} \frac{p_i \cdot d\sigma_i}{(2\pi)^3} \frac{\mathrm{d}^3\mathbf{p}_i}{E_i} f(x_i, p_i) \right] \times f^W(x_1, \ldots, x_n : p_1, \ldots, p_n).$$

① distribution function $f(x_i, p_i)$ for particle i

$$\int p_i \cdot d\sigma_i \frac{d^3\mathbf{p}_i}{(2\pi)^3 E_i} f(x_i, p_i) = N_i$$

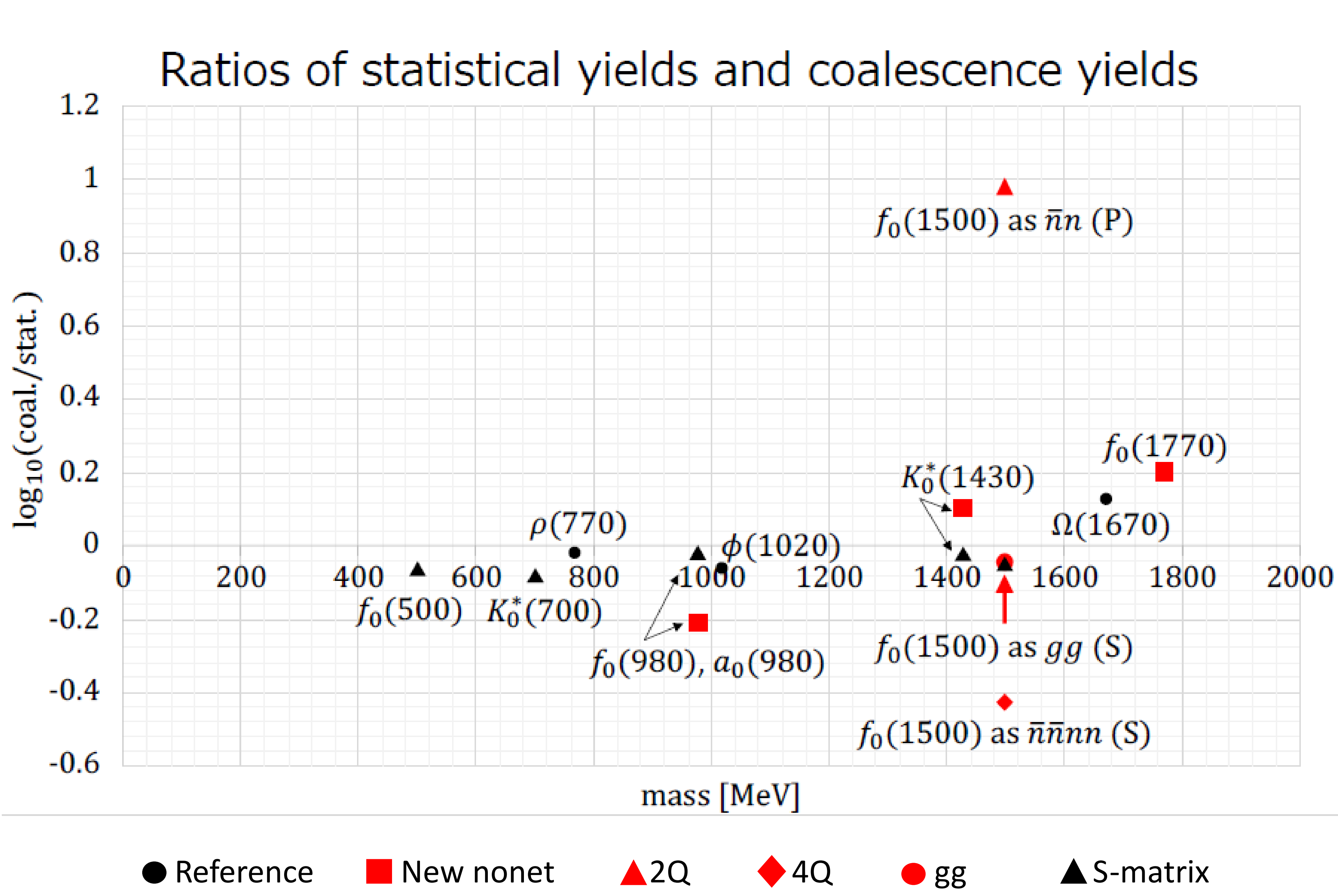
② Wigner function (n # of constituent particles)

$$f^W(x_1, \ldots, x_n : p_1, \ldots, p_n) = \int \prod_{i=1}^{n} dy_i e^{ip_i y_i} \psi^*(x_1 + y_1/2, \ldots, x_n + y_n/2) \times \psi(x_1 - y_1/2, \ldots, x_n - y_n/2),$$

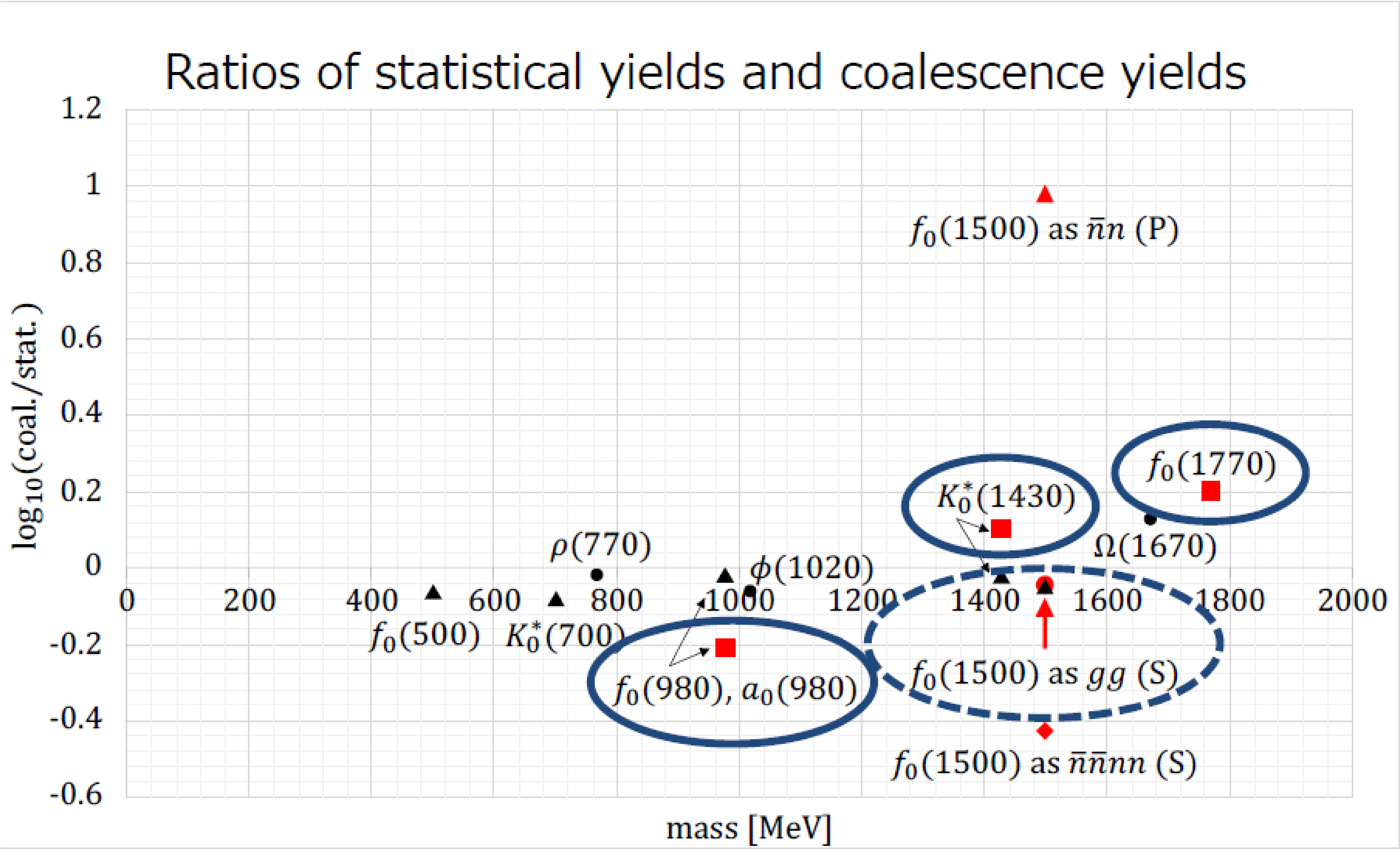
❑S-matrix approach

$$N_h^{\mathrm{S}} = \frac{g_h V_H}{2\pi^2} \int \frac{\mathrm{d}\sqrt{s}}{2\pi} B(s) \int_0^\infty \frac{p^2 \, \mathrm{d}p}{e^{(\varepsilon(p,s) - \mu_B B - \mu_S S)/T_H} \pm 1}$$

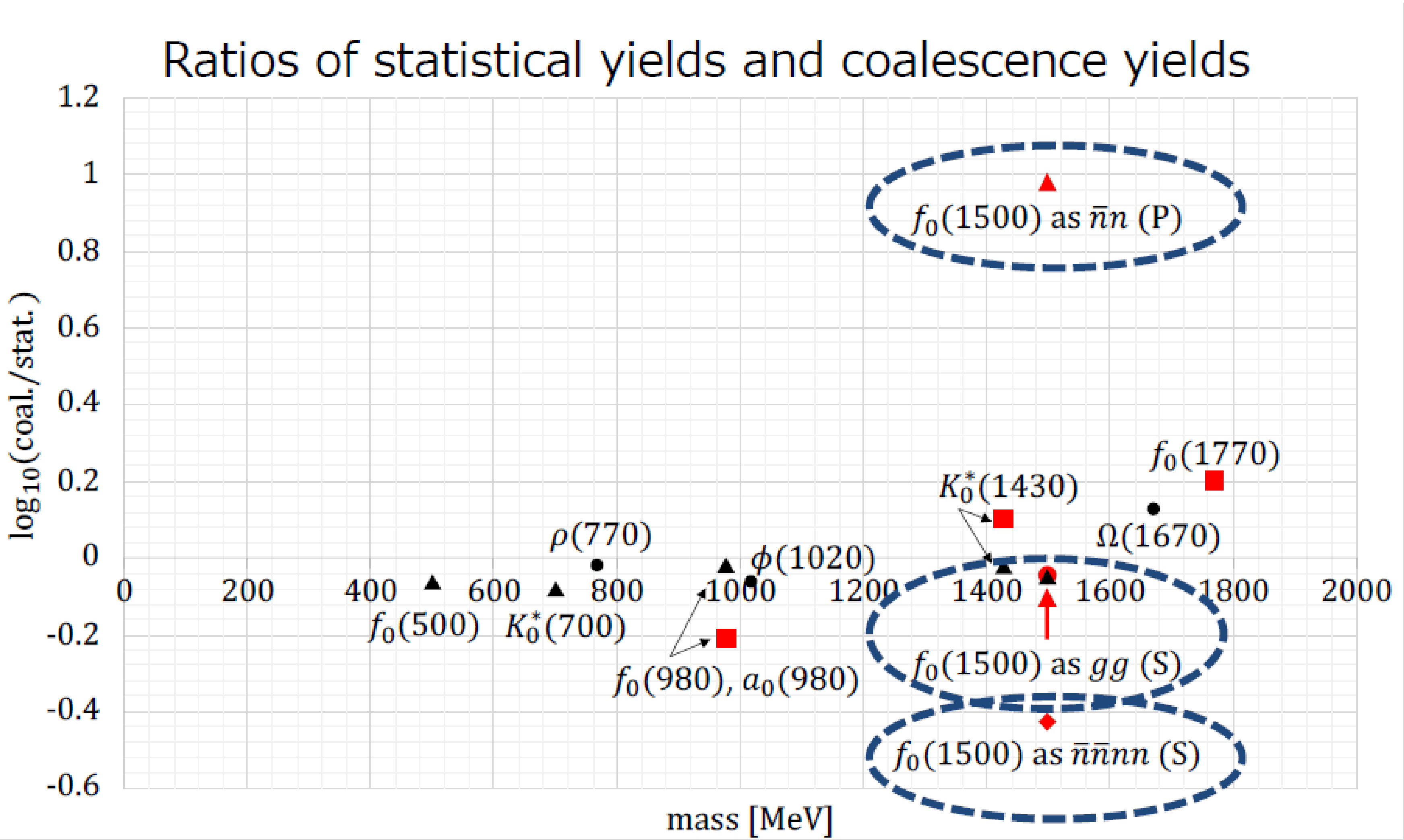
[Dashen, Ma, Bernstein (‘69); Venugopalan, Prakash (‘92); Lo (‘20-)]



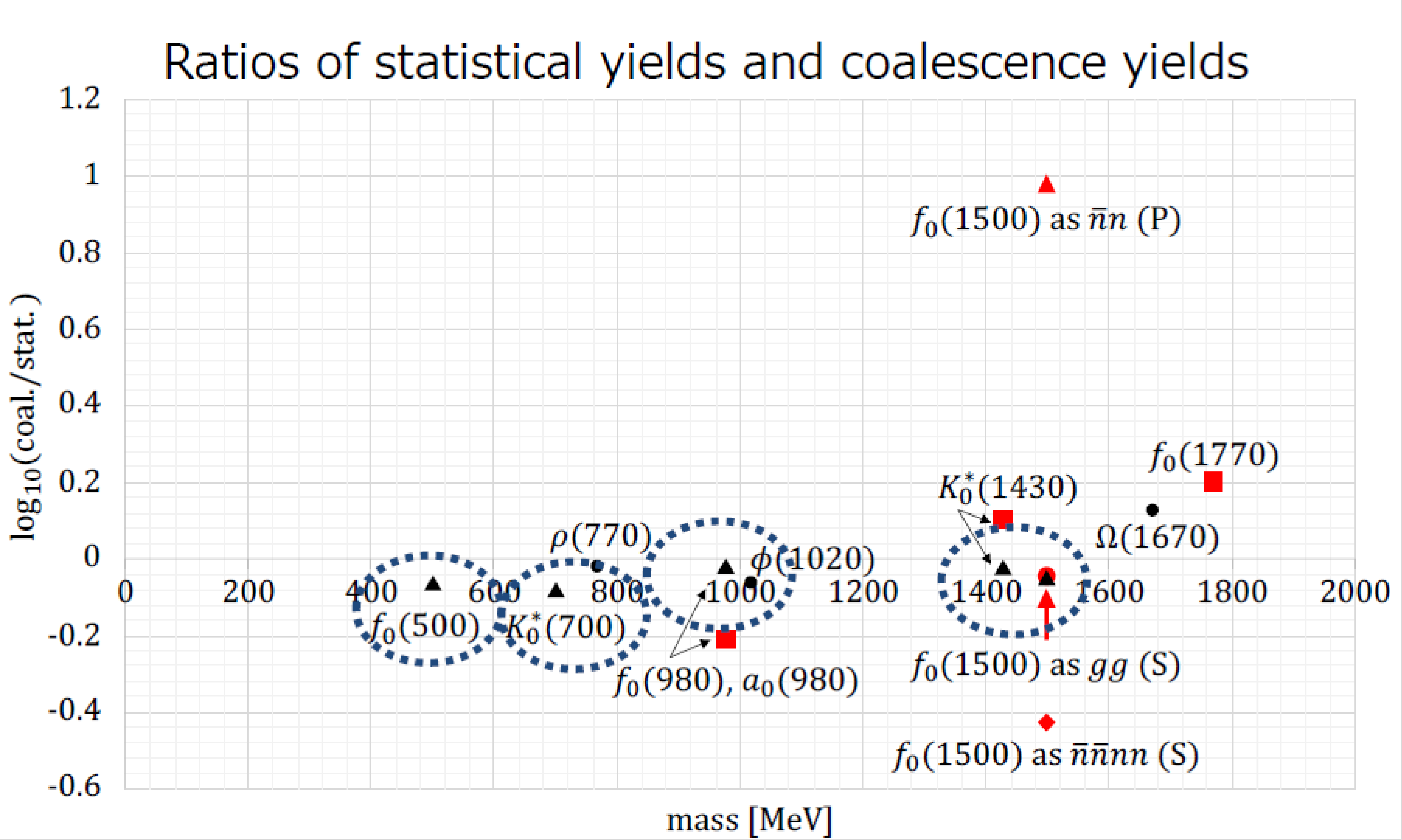
Ratios of statistical yields and coalescence yields
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
200
400
600
800
1000
1200
1400
1600
1800
2000
$\log_{10}$(coal./stat.)
mass [MeV]
$f_0(1500)$ as $\bar{n}n$ (P)
$f_0(1770)$
$K_0^*(1430)$
$\Omega(1670)$
$\rho(770)$
$\phi(1020)$
$f_0(500)$
$K_0^*(700)$
$f_0(980)$, $a_0(980)$
$f_0(1500)$ as gg (S)
$f_0(1500)$ as $\bar{n}\bar{n}nn$ (S)
Reference
New nonet
2Q
4Q
gg
S-matrix



Point 1: Ratios are almost close to *one* like normal hadrons.



nt 2: Glueball (gg) is favored for $f_0(1500)$ for the ratio consistenc



Dashen, Ma, Bernstein (1969), Weinhold, Friman, Norenberg (1998), Lo (2017, 2020)

Point 3: S-matrix formulation gives consistent values.

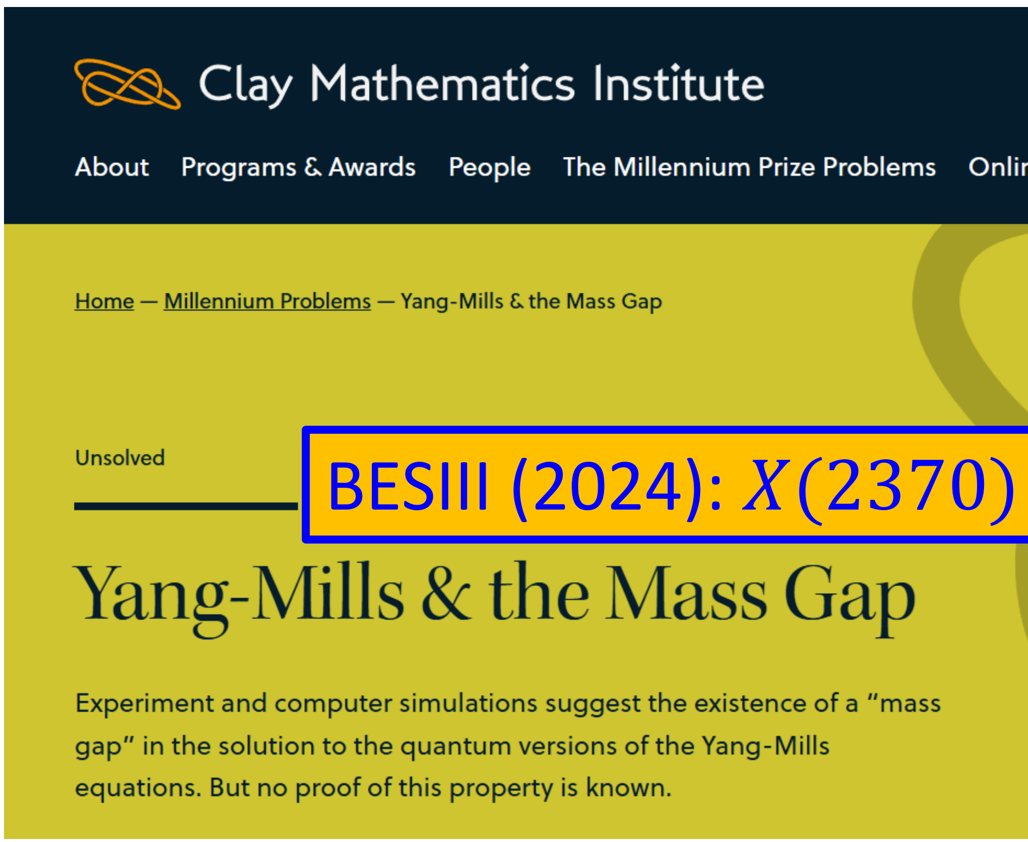
Summary of Part I

- New nonet scheme
  - $f_0(980), a_0(980), K_0^*(1430), f_0(1770)$
- Their production in HICs
  - Statistical model
  - Coalescence model
  - S-matrix model
- $f_0(1500)$ as a scalar glueball

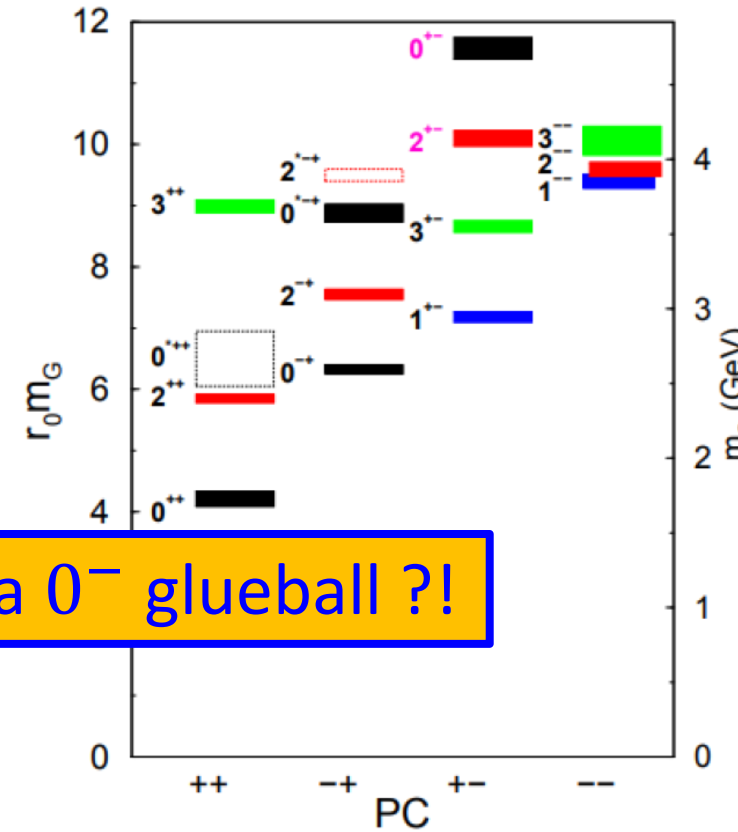
Part II:
Glueballs as topological solitons

Why glueballs?

The key objects to understand the mass gap in pure Yang-Mills theory



BESIII (2024): $X(2370)$ as a 0^{-} glueball ?!



Morningstar & Peardon
PRD **60**, 034509 (1999)

The goal

Topological approach to non-perturbative physics

- ❑IR phenomenology of SU(2) Yang-Mills theory
- ❑Glueballs as topological solitons

  cf. Faddeev and Niemi ('99), the lowest glueball as a Hopfion

➔ Higher-lying states and their structure, comparison to available experimental & lattice data

Based on the work in collaboration with Amari, Nitta, Sasaki, Shigaki, Yano and Yasui, Phys. Lett. B 869, 139805 (2025)

The Skyrme-Faddeev model (SFM)

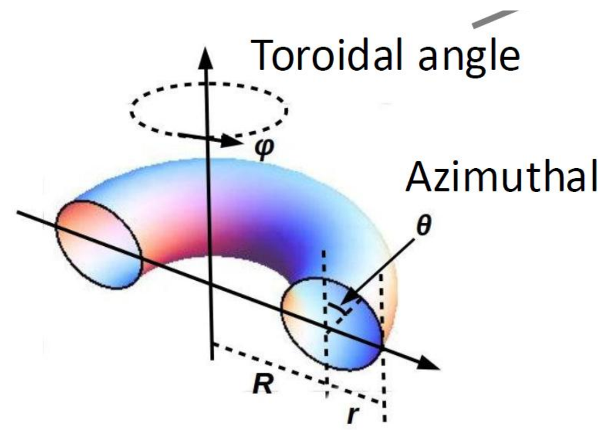
A low-energy effective model of SU(2) YM

[Faddeev & Niemi (99)]

$$\mathcal{L} = \frac{\kappa^2}{4} \mathrm{Tr}\left(\partial_\mu \mathfrak{n} \partial^\mu \mathfrak{n}\right) + \frac{1}{32e^2} \mathrm{Tr}\left[\partial_\mu \mathfrak{n}, \partial_\nu \mathfrak{n}\right]^2$$

$$\mathfrak{n} = \boldsymbol{\tau} \cdot \boldsymbol{n} \qquad \boldsymbol{n} = (n_1, n_2, n_3) \text{ with } |\boldsymbol{n}| = 1$$

- ❑Topological solitons: Hopfions ← $\pi_3(S^2) = \mathbb{Z}$
- ❑Hopf charges: $Q = lm$
  [$l, m \in \mathbb{Z}$ winding numbers]



Energy eigenvalues

$$E = \frac{\kappa}{e} M_{\rm cl}^{*} + \frac{\kappa e^{3}}{2} \left\{ \frac{J(J+1)}{V_{11}^{*}} + \left(\frac{1}{U_{33}^{*}} - \frac{\ell^{2}}{V_{11}^{*}} \right) K_{3}^{2} \right\}$$

Ansatz for a torus-shape Hopfion
$n(l, m; \theta, \varphi)$

$Q_{\rm top} = \ell m$	(ℓ, m)	$M_{\rm cl}^{*}$	V_{11}^{*}	U_{33}^{*}
1	(1,1)	274.79	372.17	227.78
2	(2,1)	447.20	968.33	267.72
2	(1,2)	508.16	1203.11	398.09

Phenomenological parameters

Energy specified by

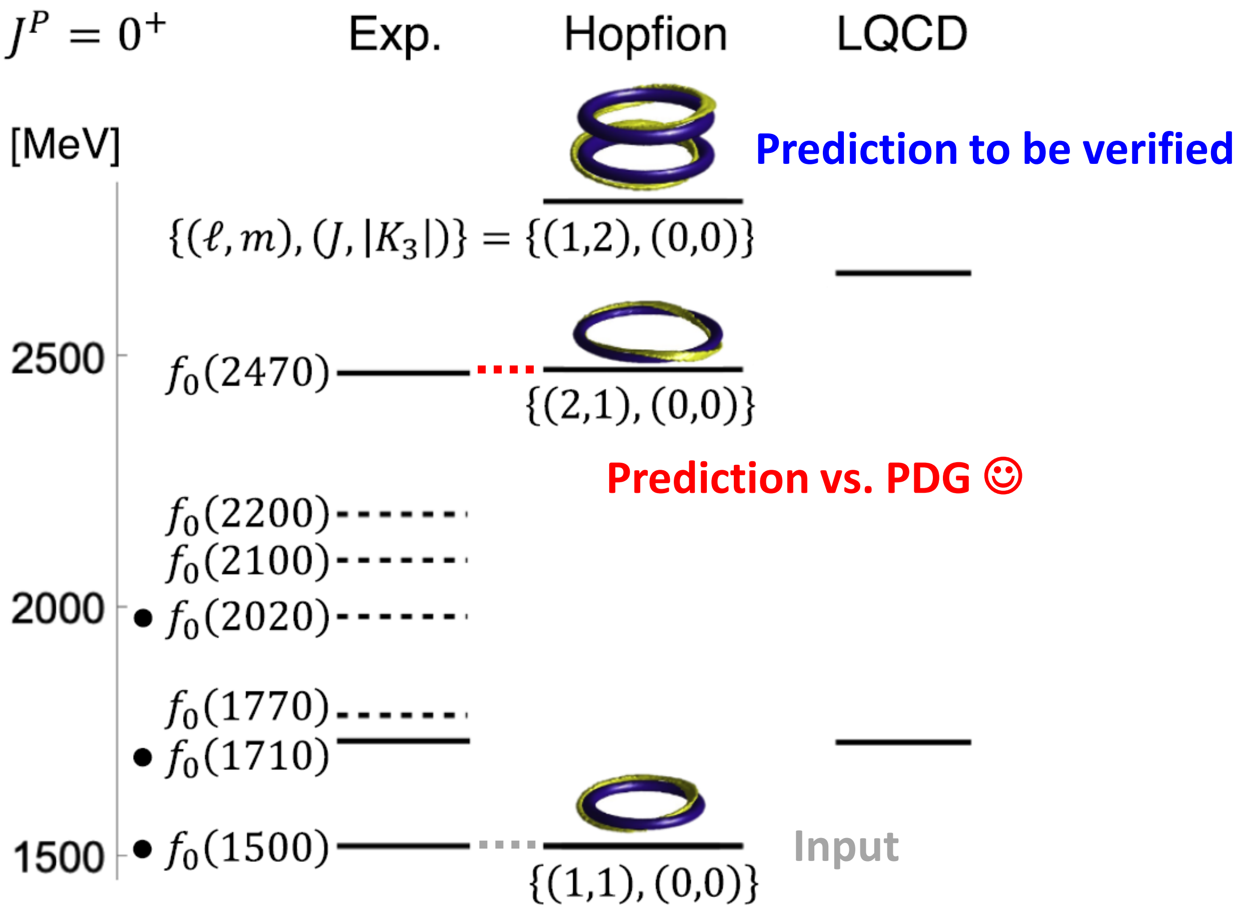
- Winding numbers (l, m)
- Quantum numbers (J, K_3)
- Model parameters (κ, e)

Input (Q=1)

- J=0: $\{(l, m), (J, K_3)\} = \{(1,1), (0,0)\}$ as $f_0(1500)$
- J=2: $\{(1,1), (2,0)\}$ as e.g. $f_2(2300)$

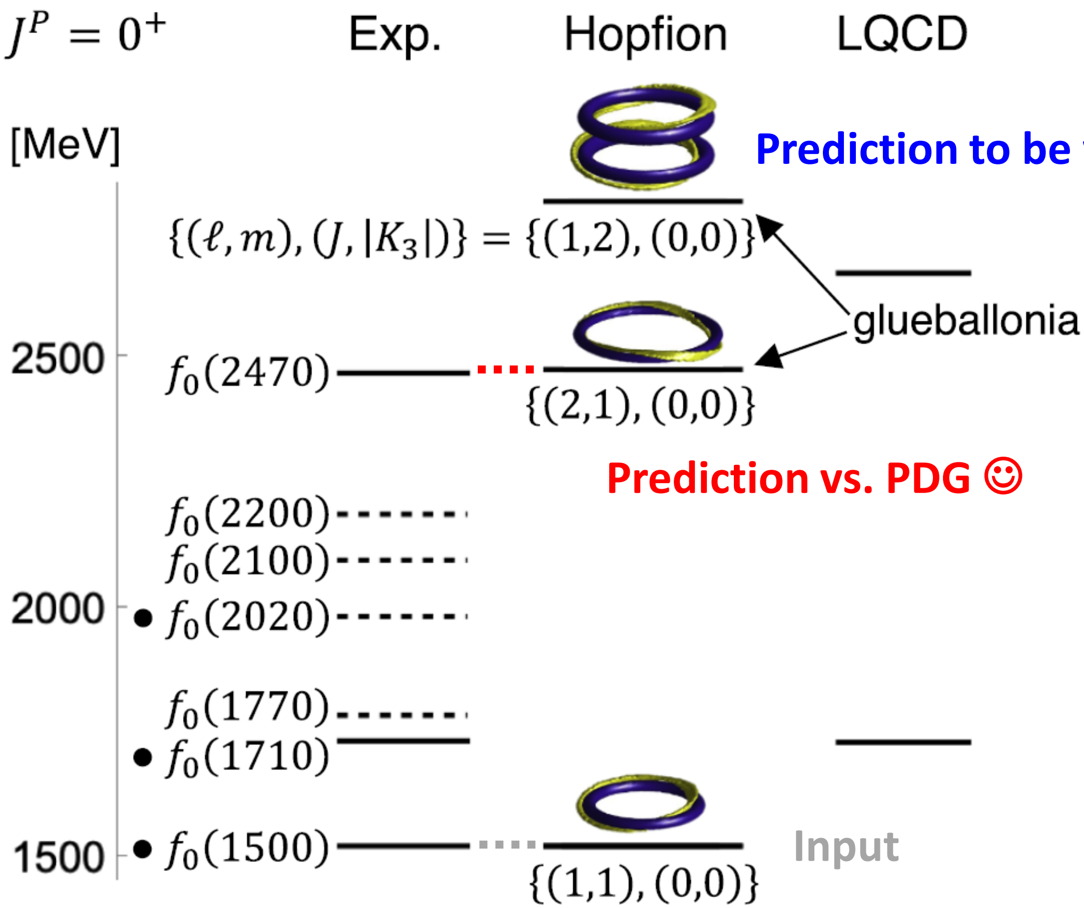
$$\kappa/e = 5.5 \text{ MeV}, \quad \kappa e^3 = 88 \text{ GeV}$$

Mass spectrum of scalar glueballs



● Listed as established states in PDG
Solid: narrow states ($\Gamma < 150$ MeV)
Broken: broad states ($\Gamma > 150$ MeV)

Mass spectrum of scalar glueballs



• Listed as established states in PDG
Solid: narrow states ($\Gamma < 150$ MeV)
Broken: broad states ($\Gamma > 150$ MeV)

Q=1 Hopfion
➔ The lowest glueball

Q=2 Hopfion w/ $(l, m) = (2,1)$
➔ Bound state of 2 Q=1 Hopfions
➔ *Tightly* bound glueballonium
B.E. = 577 MeV

Q=2 Hopfion w/ $(l, m) = (1,2)$
➔ Bound state of 2 Q=1 Hopfions
➔ *Loosely* bound glueballonium
B.E. = 230 MeV

Remarks

❑ Non-topological approach
- A model for dilatons [Giacosa, Pilloni & Trotti, ('22)]
- The lowest glueballonium of the mass 3.4 GeV
- cf. our Hopfion approach predicts 2.4 GeV

On our agenda:

❑ Extension to color SU(3)

❑ Coupling to light quarks

❑ …

❑ GFFs toward EIC

Gravitational form factors

- Encode the gluon momentum fraction and distribution of quantities e.g. the energy inside hadrons. Defined from the matrix elements of $T^{\mu\nu}$.
- In YM for the scalar glueball,

$$\langle G[0^{++}](p')|T^{\mu\nu}|G[0^{++}](p)\rangle$$

$$= 2P^\mu P^\nu A(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{2} D(t)$$

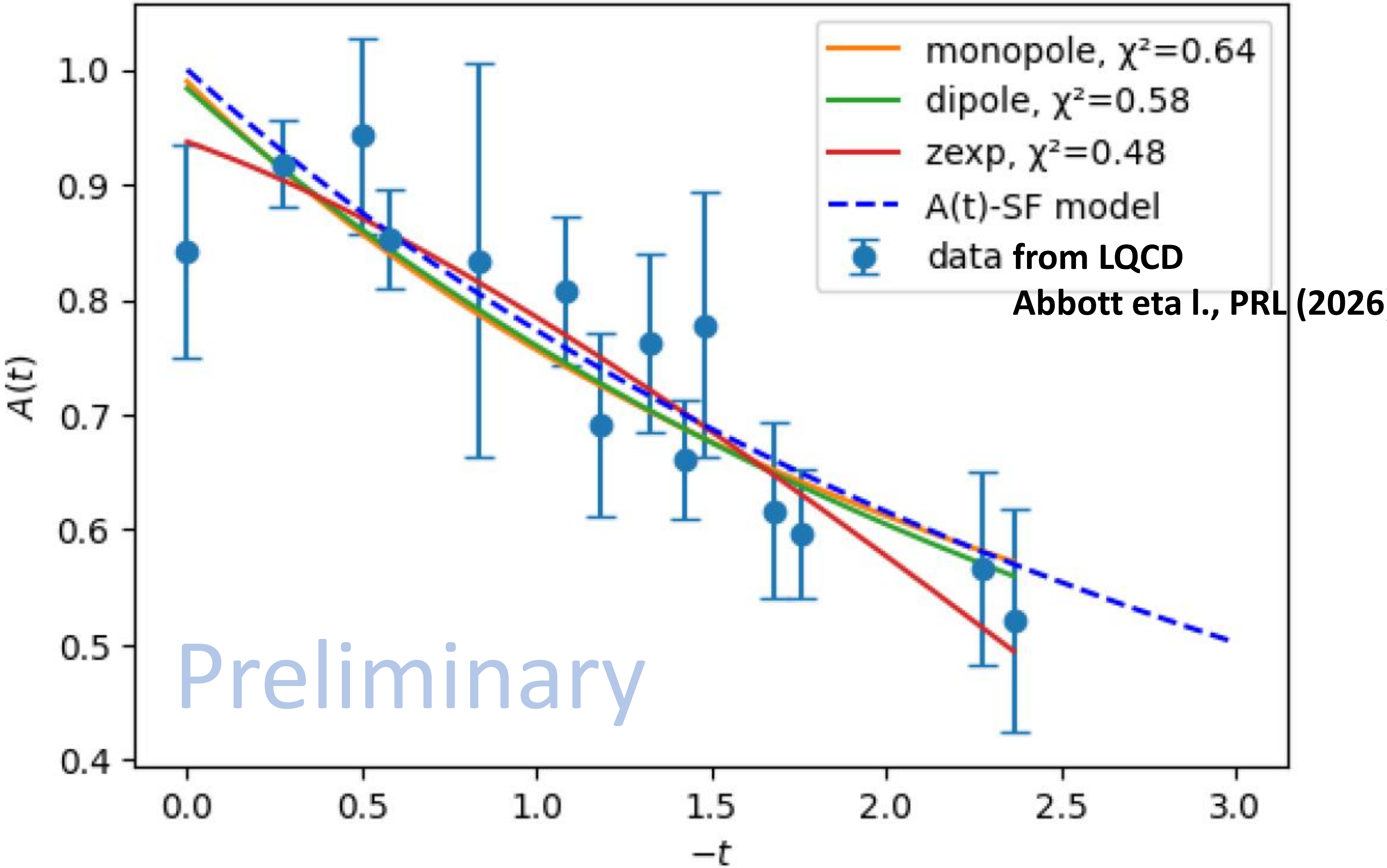
$$P = (p+p')/2,\ \Delta = p' - p,\quad t = \Delta^2$$

- Energy density $\mathcal{E}(n) = T^{00}$
- Pressure & shear forces in T^{ij}

} A- & D-terms

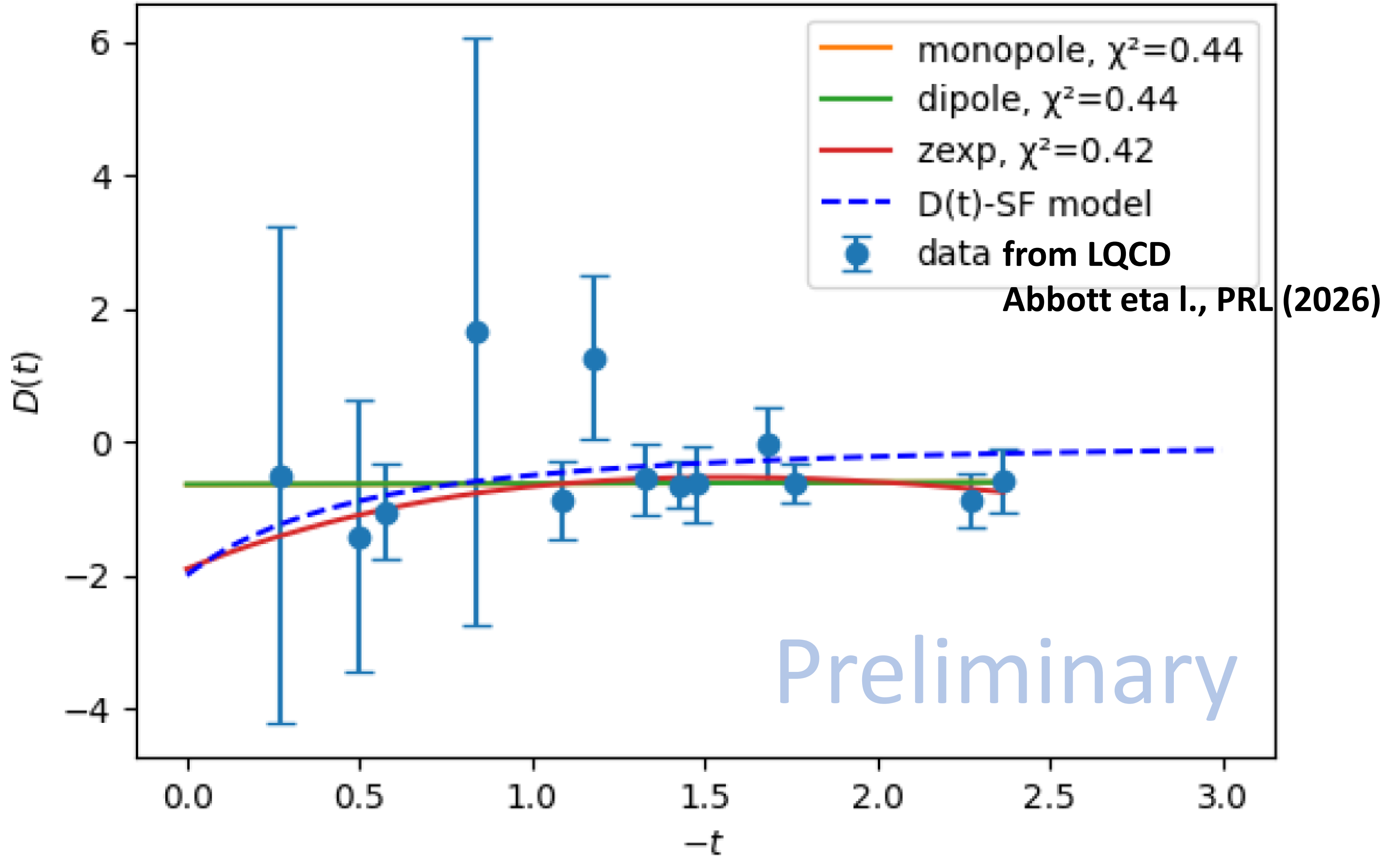
[Hutauruk, Amari, Sasaki, to appear]

Glueball GFFs in the SF model



[Hutauruk, Amari, Sasaki, to appear]

Glueball GFFs in the SF model



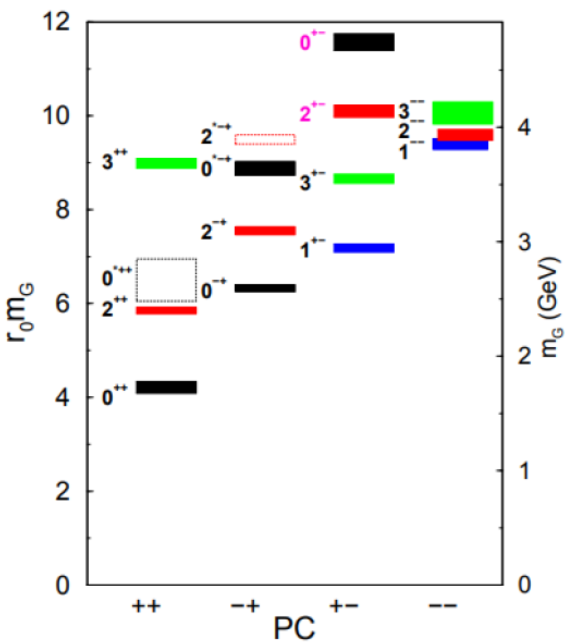
Summary of Part II

- Conjecture: $f_0(2470)$ as a molecule of 2 glueballs (glueballonium) with B.E. = 577 MeV
  - Their internal structures and binding energies
  - Characteristic multiplets of glueballonia (scalar & tensor)
- GFFs of scalar glueball (preliminary)
  - A- & D-terms align with Lattice YM.
  - Toward the nucleon GFFs, in progress
- BESIII X(2370) as another benchmark test
  - So far, only rotational modes [Amari et al., in progress]
  - Vibration modes of a Hopfion → excited states

Summary

Fascinating Light Scalars: Bridging from Quark to Gluon Dynamics

Backup



Morningstar & Peardon
PRD **60**, 034509 (1999)

0^{++} light unflavored mesons from PDG

		Mass[MeV]	Width[MeV]
●	$f_0(980)$	990	$10-100$
●	$f_0(1370)$	$1200-1500$	$200-500$
●	$f_0(1500)$	1522	108
●	$f_0(1710)$	1733	150
	$f_0(1770)$	1784	161
●	$f_0(2020)$	1982	440
	$f_0(2100)$	2095	287
	$f_0(2200)$	2187	210
	$f_0(2330)$		
	$f_0(2470)$	2470	75

● indicates established particles

2^{++} light unflavored mesons from PDG

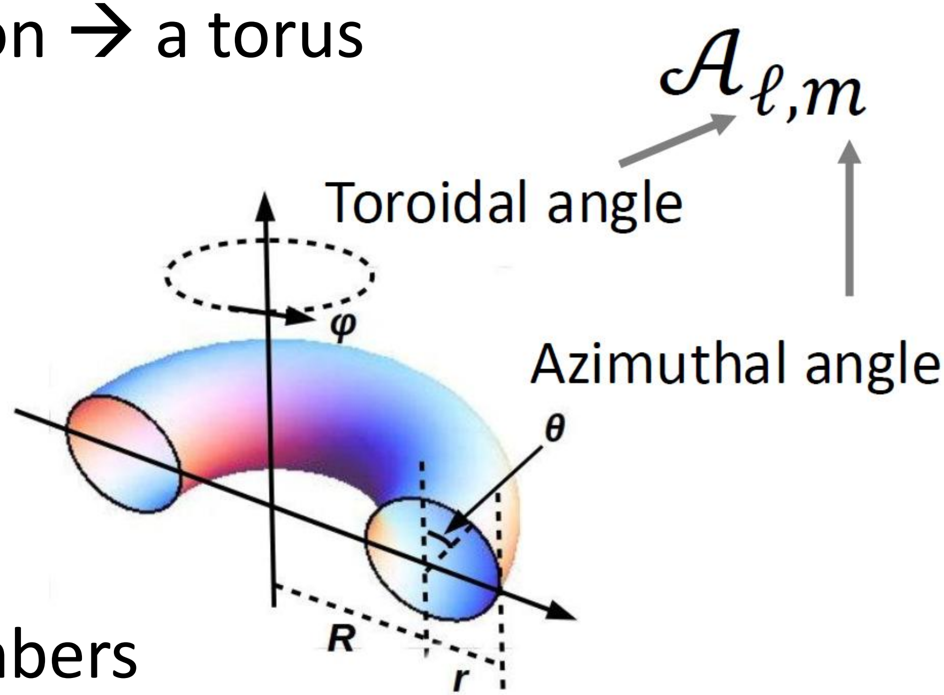
		Mass[MeV]	Width[MeV]
●	$f_2(1270)$	1275	186
	$f_2(1430)$	1430	
●	$f_2'(1525)$	1517	72
●	$f_2(1565)$	1571	132
	$f_2(1640)$	1639	100
	$f_2(1810)$	1815	197
	$f_2(1910)$		
●	$f_2(1950)$	1936	464
●	$f_2(2010)$	2010	200
	$f_2(2150)$		
	$f_J(2220)$	2231	23
●	$f_2(2300)$	2297	150
●	$f_2(2340)$	2346	331

The Skyrme-Faddeev model (SFM)

An axial-symmetric Hopfion → a torus

[Gladikowski & Hellmund (97)]

[Battye & Sutcliffe (98)]



- Hopf charges: $Q = lm$

  $l, m \in \mathbb{Z}$ winding numbers
- Static energy configurations via rational map

Energy eigenvalues

- Classical mass [$M_{\text{cl}} = \frac{\kappa}{e} M_{\text{cl}}^*$]
- Inertia tensors [$X_{jk} = \frac{1}{\kappa e^3} X_{jk}^*$]

$$M_{\text{cl}}^* = \int d^3x \left[\frac{1}{2}(\partial_i \boldsymbol{n})^2 + \frac{1}{4}\left(\partial_i \boldsymbol{n} \times \partial_j \boldsymbol{n}\right)^2\right]$$

$$U_{jk}^* = \int d^3x \left[\delta_{jk} - n_j n_k + \partial_l n_j \partial_l n_k\right]$$

$$V_{jk}^* = \int d^3x \left[\left(i\mathrm{L}_j \boldsymbol{n}\right) \cdot (i\mathrm{L}_k \boldsymbol{n}) + \left(i\mathrm{L}_j \boldsymbol{n} \times \boldsymbol{n}\right) \cdot (i\mathrm{L}_k \boldsymbol{n} \times \boldsymbol{n})\right]$$

with $\mathrm{L}_j = -i\varepsilon_{jkl} x^k \partial_l$