Reconstructing E-by-E fluctuations measured at HADES

MARVIN NABROTH

OUTLINE

- MOTIVATION
- EXPERIMENTAL TECHNIQUE
- VOLUME CORRECTION
- RESULTS
- SUMMARY AND OUTLOOK

HADES

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

FAIR Phase 0 Research Program

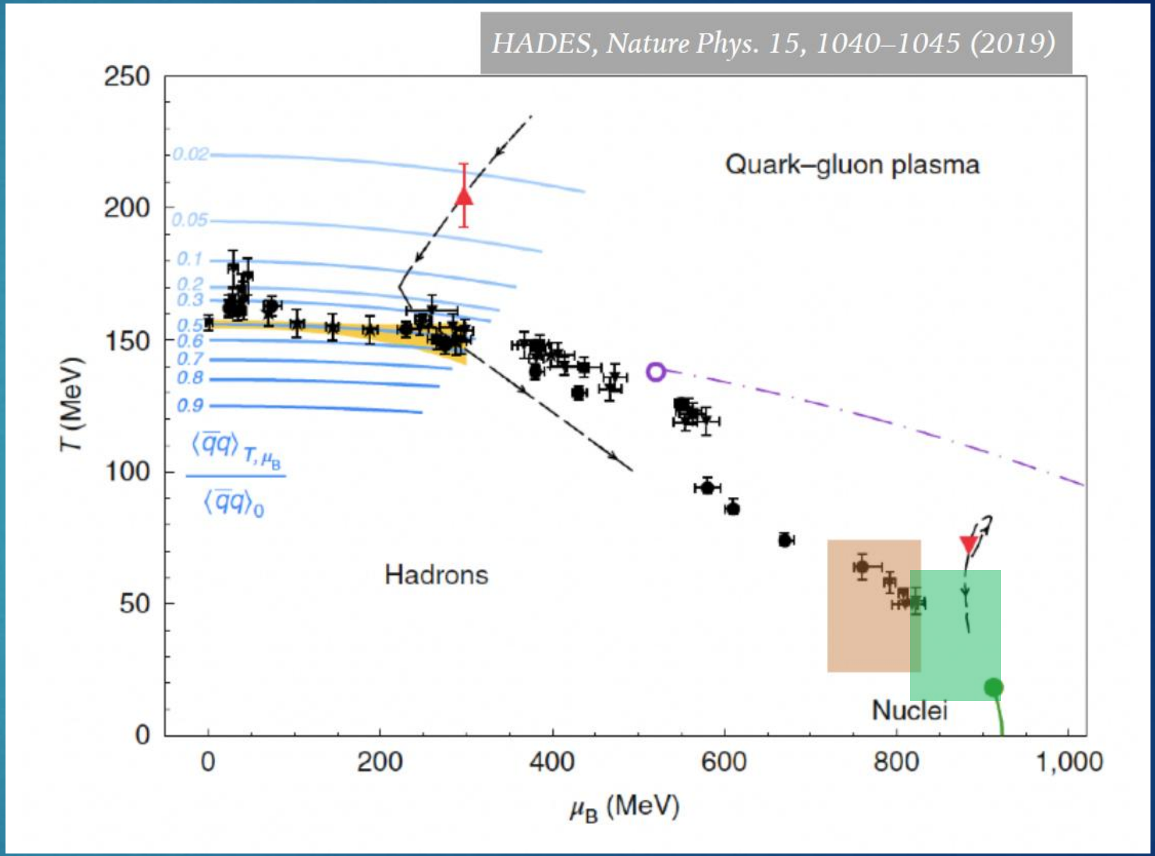
66. Cracow School of Theoretical Physics

HADES

- High μ_b-region is not accessible by Lattice QCD calculations (sign-problem)
  - Effective approaches, functional methods

- **Experimental access: Heavy-Ion collisions at energies of a few GeV**
  - → High μ_b and moderate temperature
  - → **In the vicinity of first order phase transition or CEP**

  Observables to detect critical behaviour:
  - Fluctuations of baryon number, charge, strangeness

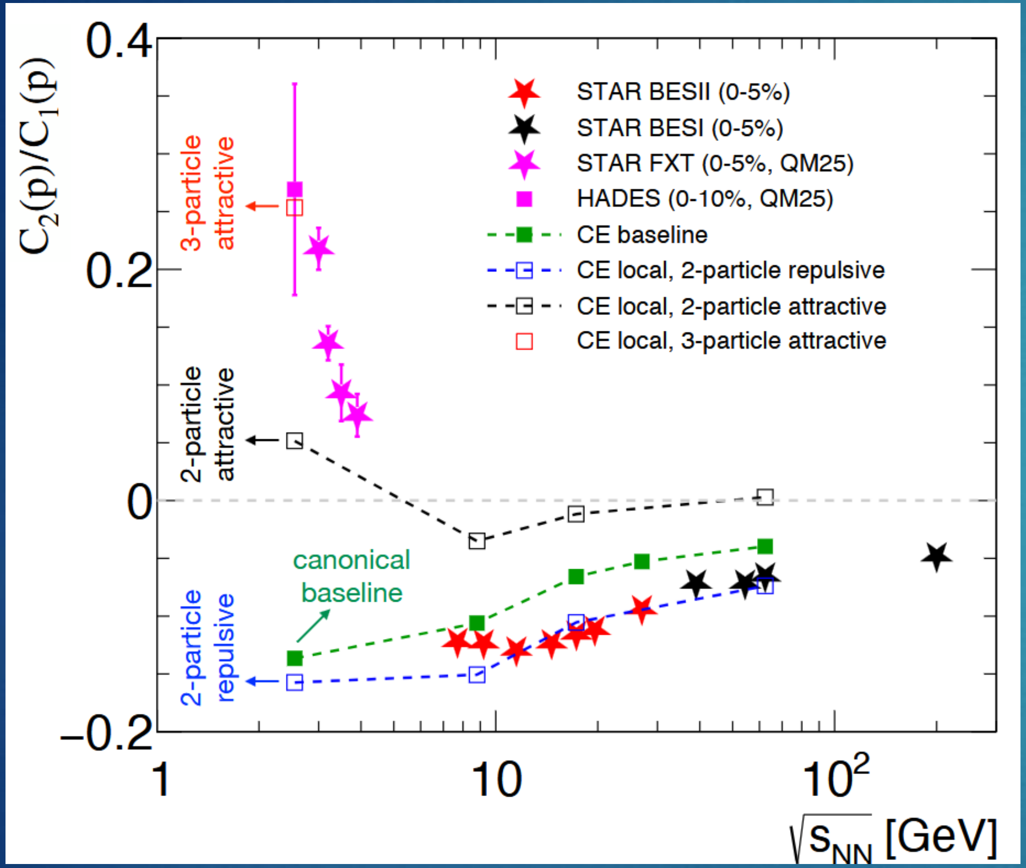


Heavy-ion measurements at HADES:
Au+Au (2.42 GeV), Ag+Ag (2.55, 2.42 GeV), Au+Au (1.96 to 2.24 GeV)

In this talk

New BES data recorded last year

Event-by-Event fluctuations



B. Friman, K. Redlich and A. Rustamov arXiv:2508.18879v1

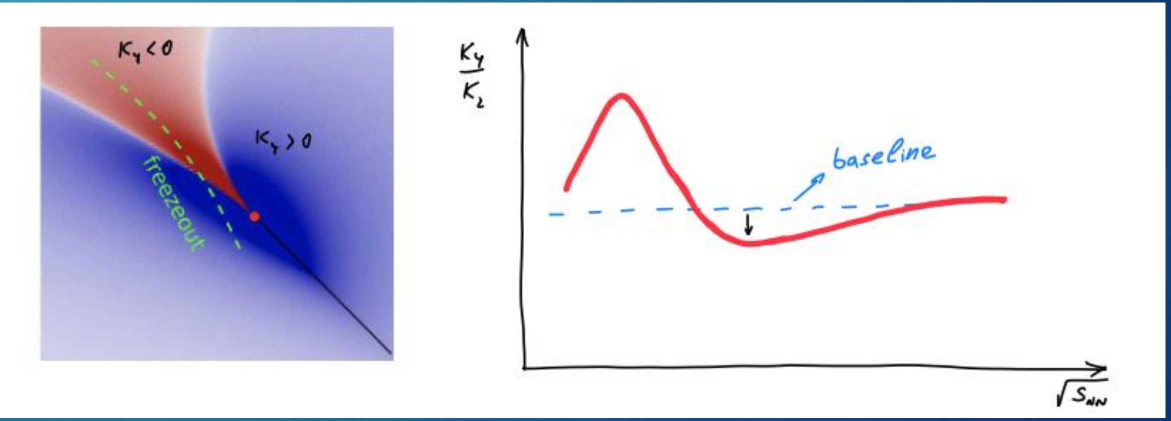
HADES: QM 25 Proc., e-Print: 2510.15353
STAR BESI/II: PRL 135 (2025) 14, 142301
STAR FXT: PRC 107 (2023) 2, 024908 and Quark Matter 2025

- Study QCD phase structure of freeze-out in detail by looking at **higher order moments of particle yields → related to derivates of partition Function Z w.r.t** μ_B

$$\langle N \rangle = \frac{\partial \ln(Z)}{\partial\left(\frac{\mu}{T}\right)} \qquad \langle N^2\rangle - \langle N\rangle^2 = \frac{\partial^2 \ln(Z)}{\partial\left(\frac{\mu}{T}\right)^2} \quad \dots \quad \Rightarrow \quad \kappa_n = \frac{\partial^n \ln(Z)}{\partial\left(\frac{\mu}{T}\right)^n}$$

- Measure energy excitation function of **higher-order (factorial)cumulant ratios of net-baryons** (protons, light nuclei as proxy in experiment)

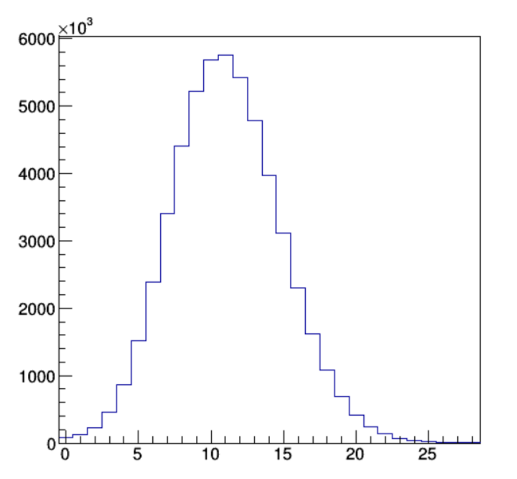
→ Expected non-monotonic trend for critical behaviour



M. Stephanov, PRL102.032301(2009) PRL107.052301(2011)
M.Cheng et al, PRD79.074505(2009)

Event-by-Event fluctuations

- Basic idea: **Count number of baryons (protons as proxy)** emitted per event, extract moments and other distribution measures



$N_{protons}$

Cumulants

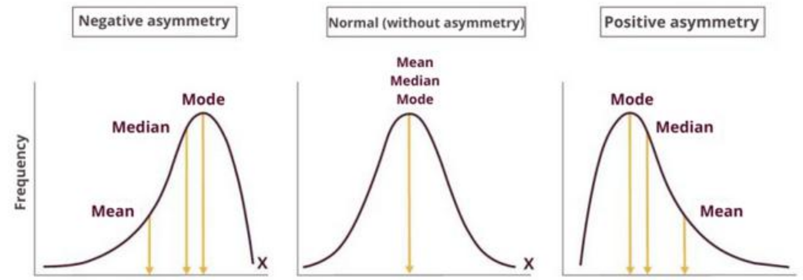
- $\kappa_1 = \langle N \rangle$
- $\kappa_2 = \langle N^2 \rangle - \langle N \rangle^2$
- $\kappa_3 = \langle N^3 \rangle - 3\langle N^2 \rangle\langle N \rangle + 2\langle N \rangle^3$
- $\kappa_4 = \langle N^4 \rangle - 4\langle N^3 \rangle\langle N \rangle - 3\langle N^2 \rangle^2 + 12\langle N^2 \rangle\langle N \rangle^2 - 6\langle N \rangle^4$

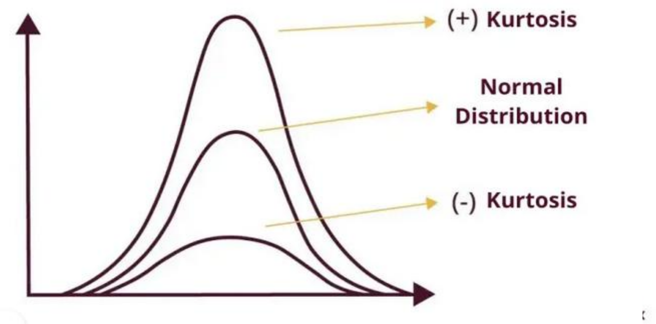
Factorial Cumulants

- $C_2 = \kappa_2 - \kappa_1$
- $C_3 = \kappa_3 - 3\kappa_2 + 2\kappa_1$
- $C_4 = \kappa_4 - 6\kappa_3 + 11\kappa_2 - 6\kappa_1$

Skewness $= \frac{\kappa_3}{\sigma^3}$

Kurtosis $= \frac{\kappa_4}{\sigma^4}$



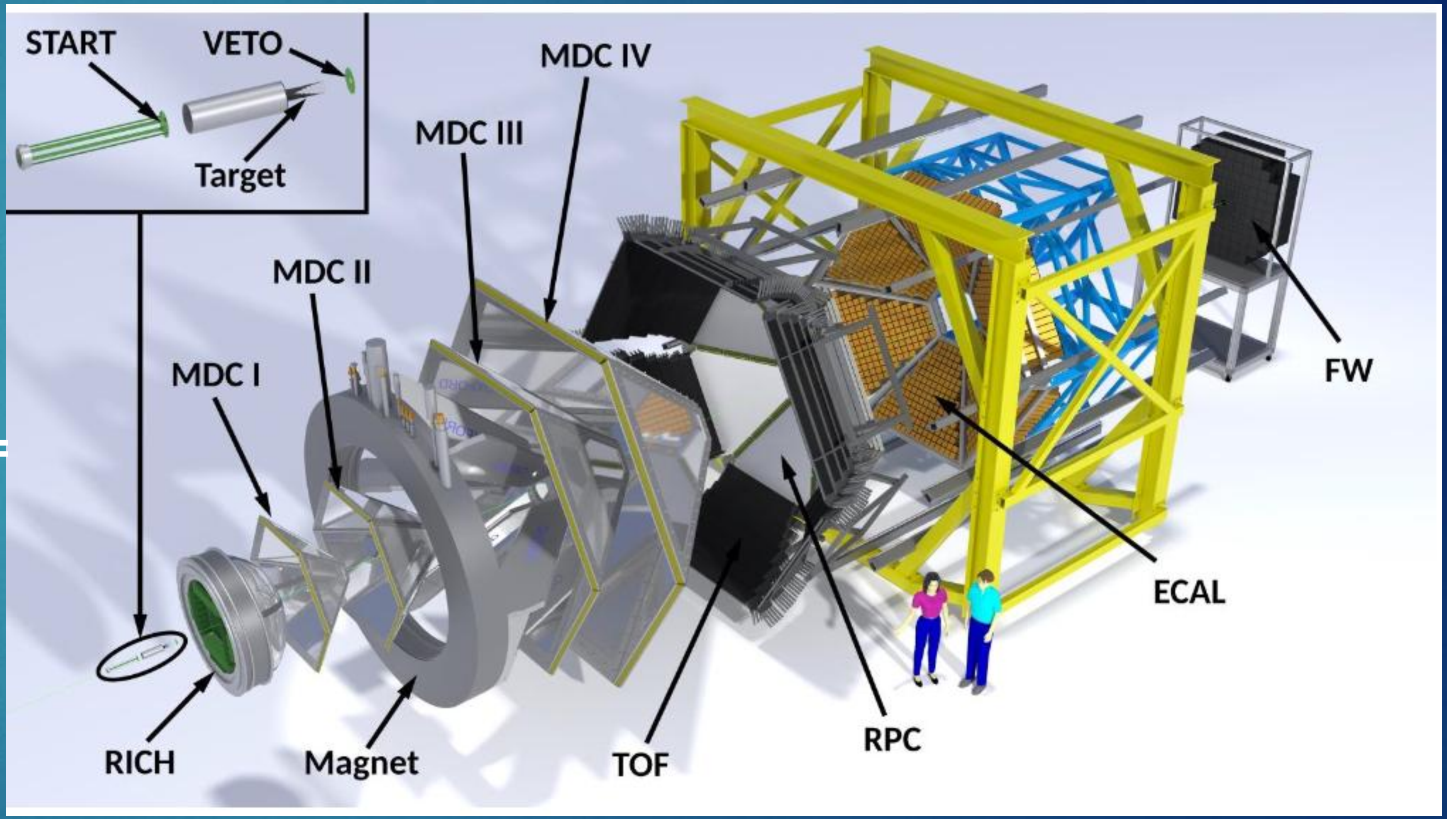


- **Experimental challenge: Disentangle non-critical from critical fluctuations**

  Distortion from
  - **detector fluctuations, impurities in particle identification, volume fluctuations**

Experimental setup - HADES

- ❑ Fixed target experiment at SIS18
- ❑ **Momentum reco.** based on **magnetic spectrometer** (**MDCs and Magnet**)
- ❑ **Time-of-flight** from **START, RPC and TOF**
- ❑ **Energy loss measurement** from **MDC** and **TOF**
- ❑ **Forward Wall** for projectile spectator measurement → Event plane, **centrality selection for fluctuation analysis**



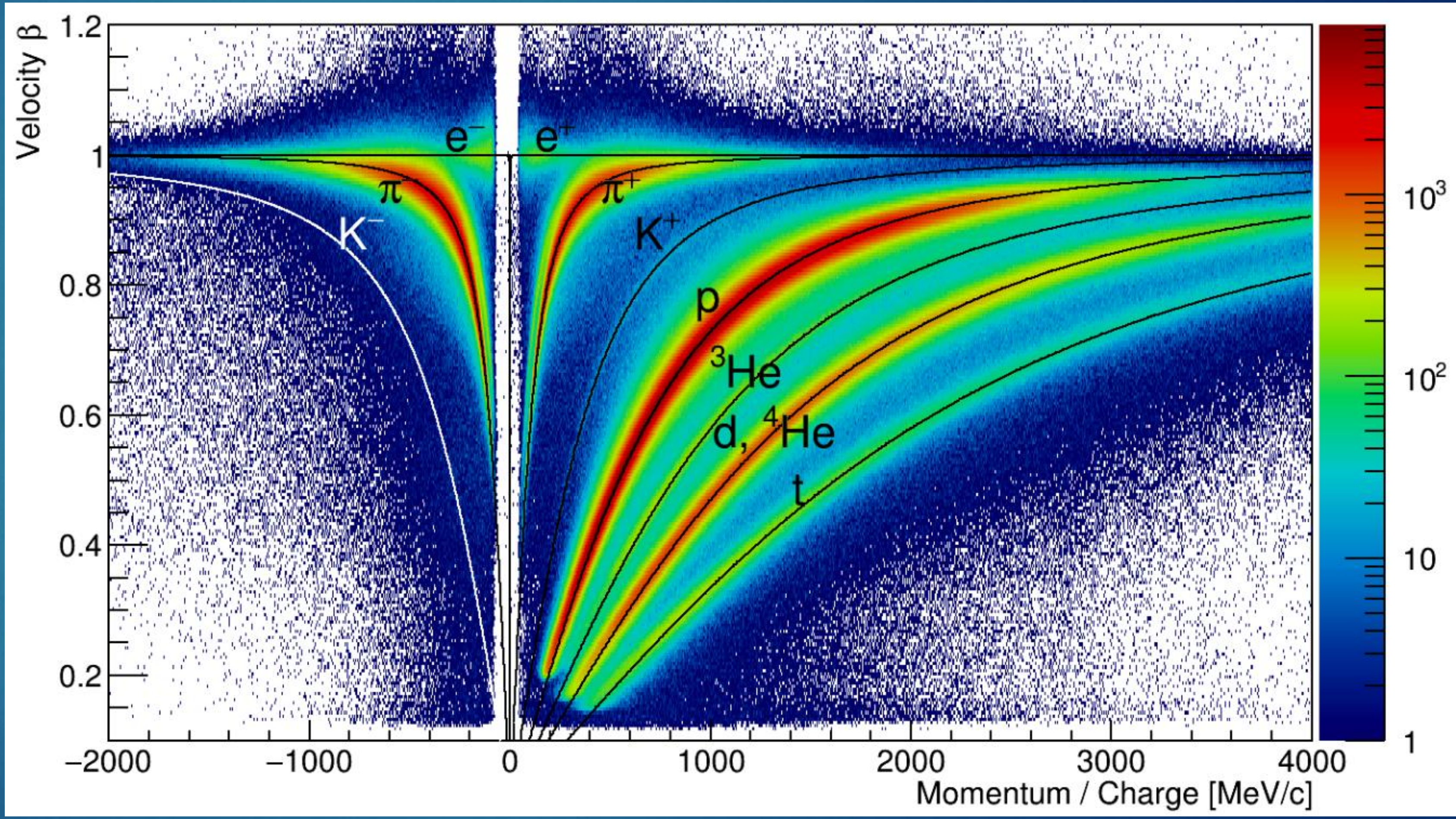
- ❖ Almost full azimuthal coverage
- ❖ Polar angle coverage between 18° and 85°

Particle Identification at HADES

- Particle Identification based on time of flight and momentum measurement

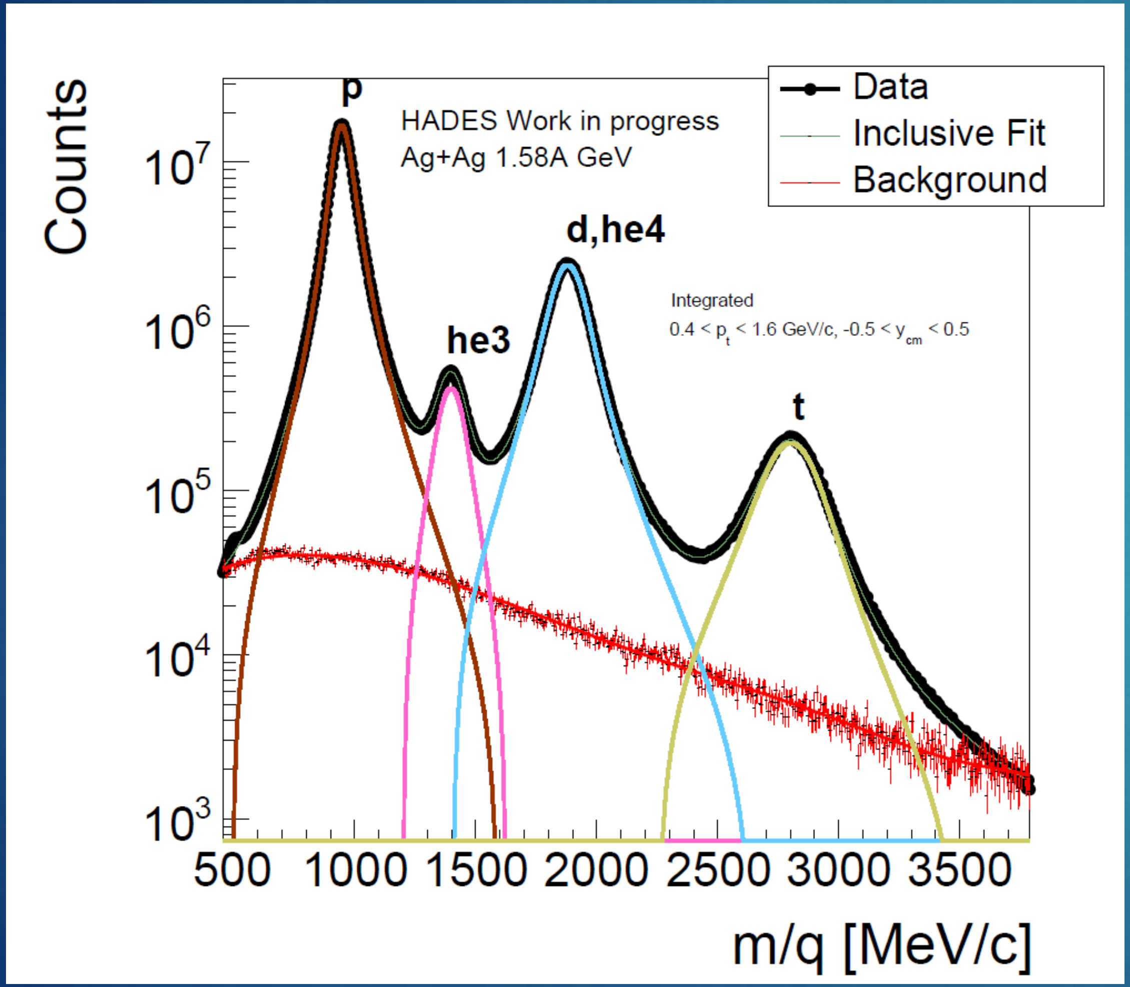
$$\frac{p}{q} = \frac{m\beta c}{\sqrt{1-\beta^2}}$$

- dE/dx measurement from MDCs as additional preselection, in particular to separate d and He4



- ➢ Baryon dominated
- ➢ Approx. 45 % of protons are bound

PID fits and Fuzzy Logic



Mass spectra lines shapes

$$\rho_k(x_i = m)$$

$$\omega_k = \frac{\rho_k(x_i)}{\sum \rho_l(x_i)}$$

Event\Proxy quantities

$$W_k = \sum_{i=1}^{N_{tracks}} \omega_k(x_i)$$

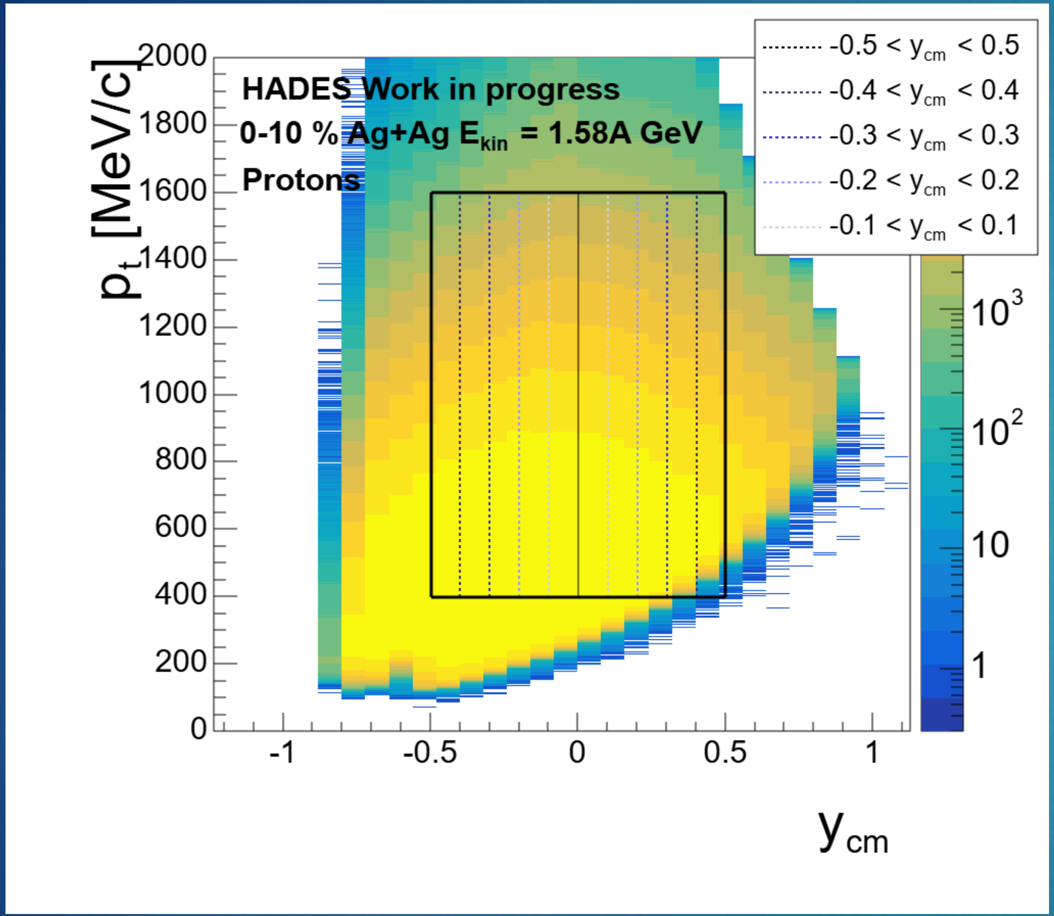
- PID on mass spectra, fitted differentially as function of pt, y, centrality, detector-sector → fully data driven
- Fit composed of Two-Sided Crystal-ball functions
  - Gaussian core for hard cut selection
  - Probability based PID → Fuzzy logic
- **Background estimation by sector-wise rotating technique**

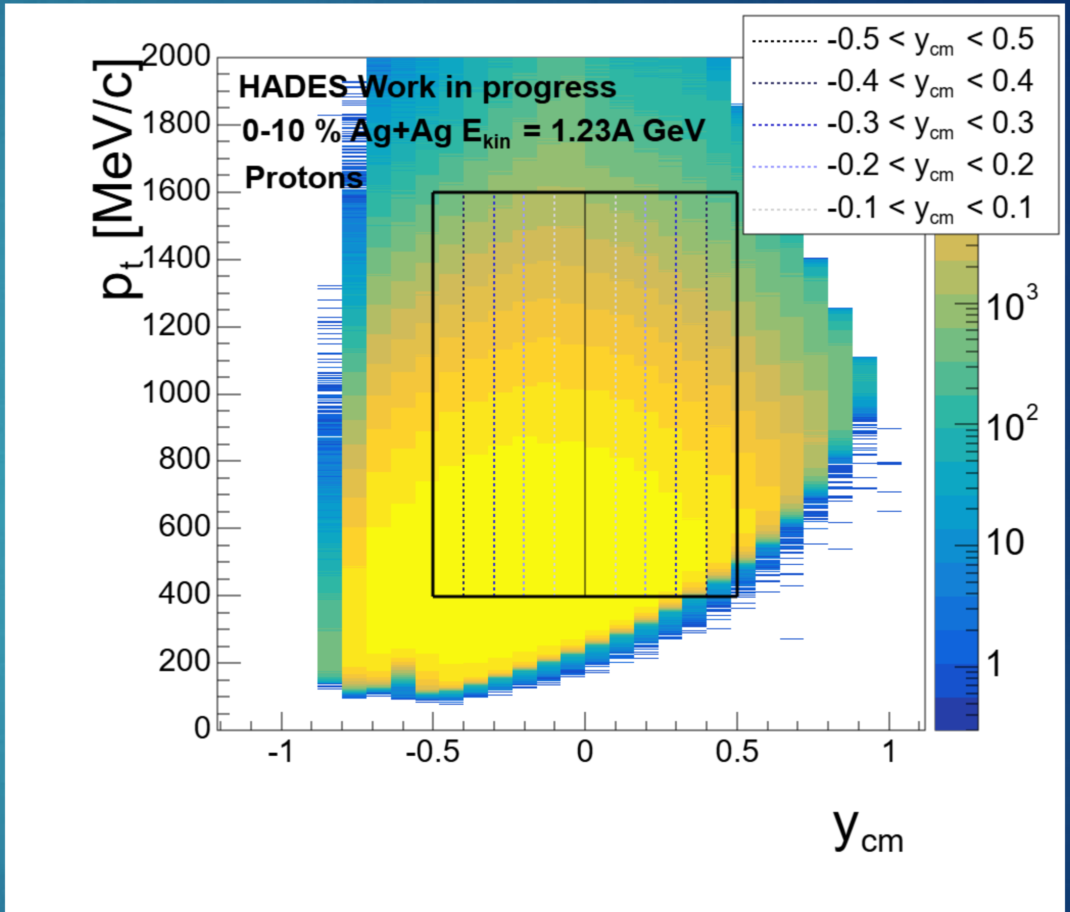
Inversion procedure implemented up to 4th order

$$< N > = R^{-1} < W >$$

A. Rustamov, Phys.Rev.C 110 (2024) 6

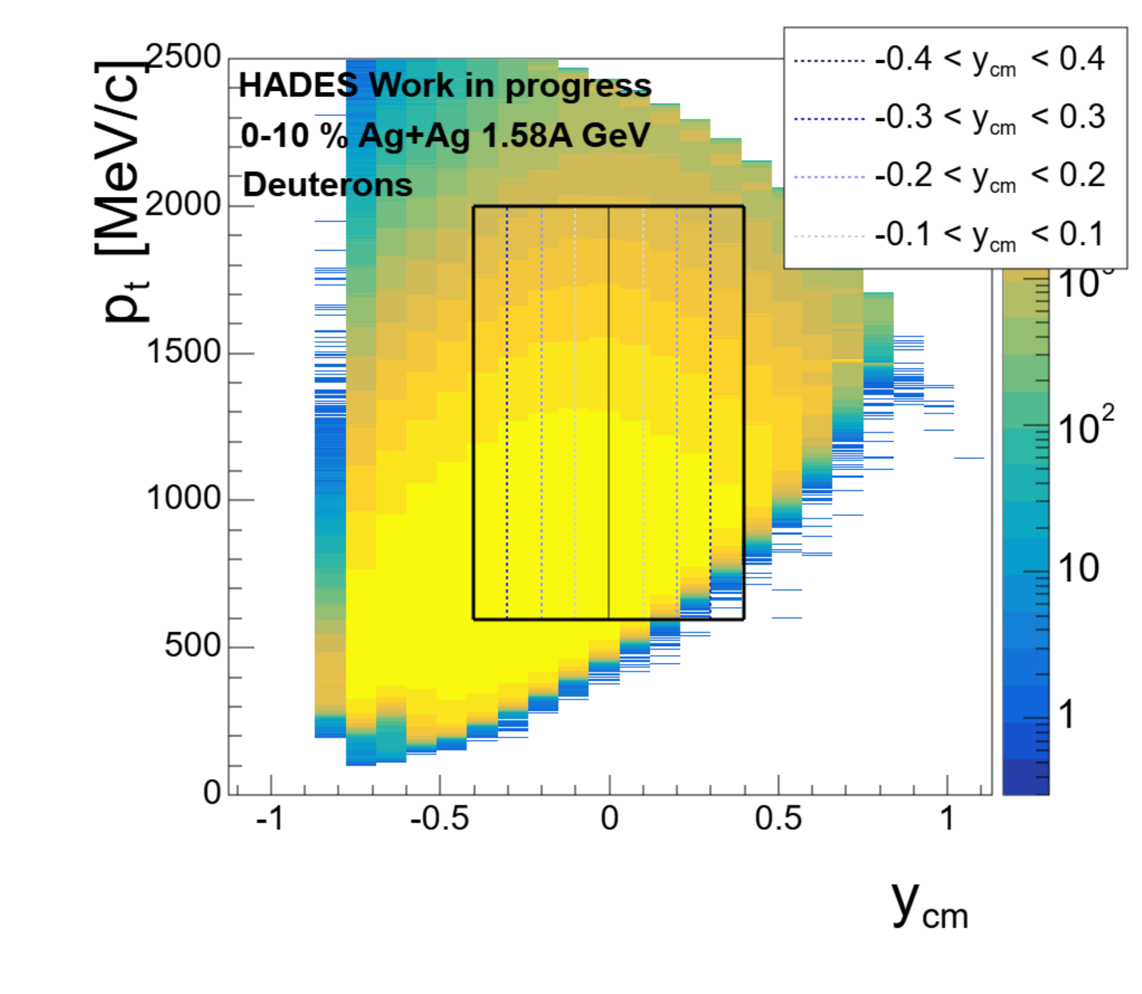
Phase Space Coverage - Protons





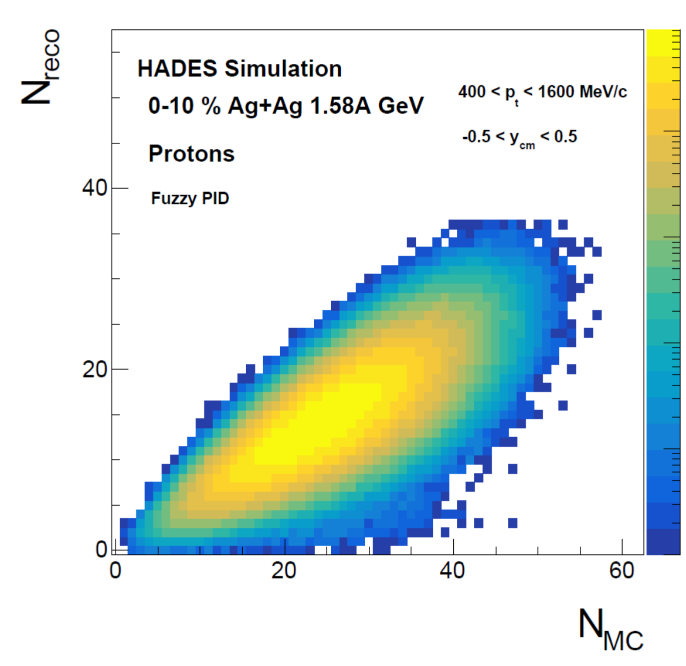
- For protons (Ag+Ag 1.58 AGeV and Ag+Ag 1.23 AGeV) rapidity window up to $\Delta y = \pm 0.5$.

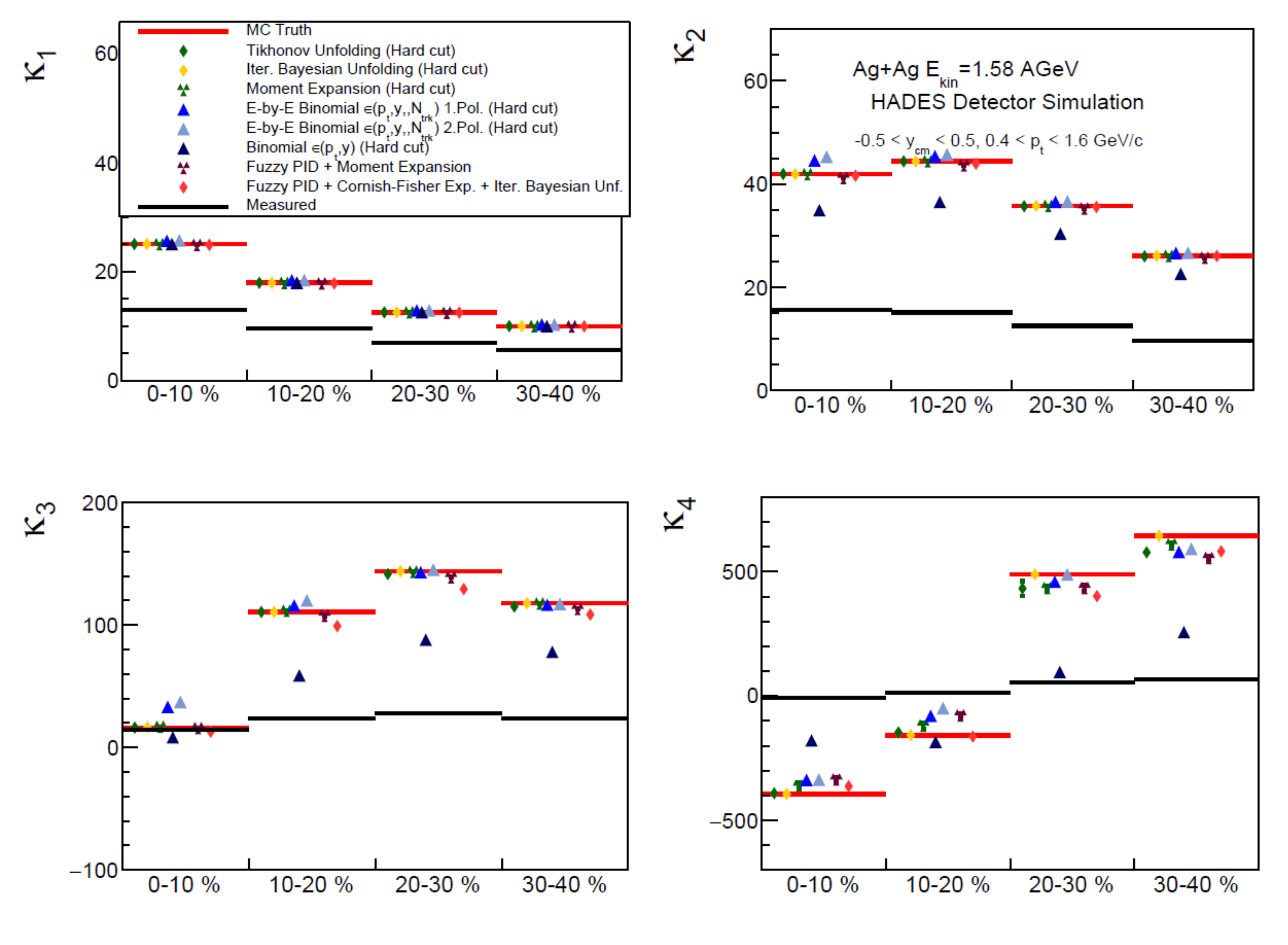
Phase Space Coverage - Deuterons



- Rapidity window for deuterons, symmetric around rapidity: $\Delta y = \pm 0.3$, $(\Delta y = \pm 0.4)$
- Extended p_t range to 600 < p_t < 2000 MeV/c
  - Fuzzy PID allow to go to high momenta

Efficiency correction





- Efficiency correction studied and tuned in detail
- Various methods employed
- Combined with Fuzzy PID

A. Rustamov, PRC 110 (2024) 6, 064910
T. Nonaka, M. Kitazawa, and S. Esumi, Phys. Rev. C 95 (2017) 064912
G. D'Agostini, Nucl. Instrum. Methods Phys. Res., Sect. A 362 (1995) 487
T. Nonaka, M. Kitazawa, and S. Esumi, Nucl. Instrum. Methods Phys. Res., Sect. A 906 (2018) 10–17

Controlling volume fluctuations

- How to describe **contribution of volume fluctuations** in **observed particle cumulants**?
- Model with simplest assumption:
  - **Sources for particle emission** are **statistical independent**
  - Moment generating function factorizes

P. Braun-Munzinger, A. Rustamov, J. Stachel, Nucl.Phys.A 960 (2017), 114-130

$$M_N(t) = [M_N(t)]^{N_{src}} \rightarrow \langle N^m(N_{src})\rangle = \frac{d^m}{dt^m}[M_N(t)]$$

$$k_1(N) = \langle N_{Src}\rangle\langle n\rangle$$

$$k_2(N) = \langle N_{Src}\rangle k_2(n) + \langle n\rangle^2 k_2(N_{src})$$

$$k_3(N) = \langle N_{Src}\rangle k_3(n) + 3\langle n\rangle k_2(n) k_2(N_{src}) + \langle \mathrm{n}\rangle^3 \mathrm{k}_3(\mathrm{N}_{\mathrm{src}})$$

$$k_4(N) = \langle N_{src}\rangle k_4(n) + 4\langle n\rangle k_3(n) k_2(N_{src}) + 3k_2^2(n) k_2(N_{src}) 6\langle n\rangle^2\, k_2(n) k_3(N_{src}) + \langle n\rangle^4\, k_4(N_{src})$$

$$k_m(N_{src}) \coloneqq \mathit{Cumulants\ of\ sources\backslash participants}$$

$$k_m(n) \coloneqq \mathit{Single\ particle\ cumulants\ per\ source}$$

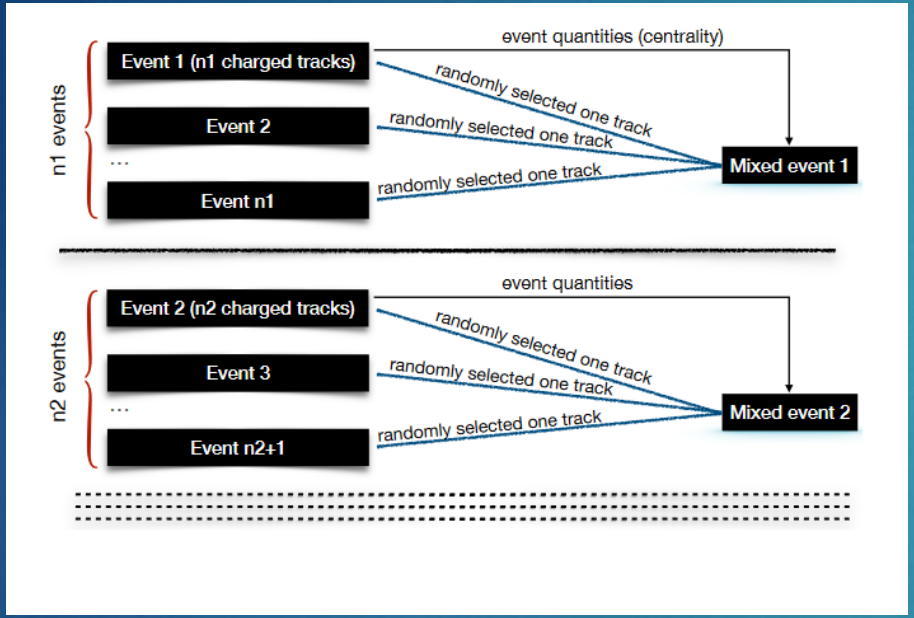
$$k_m(N) \coloneqq \mathit{Oberserved\ particle\ cumulants}$$

Generalized derivation with Bell-Polynomials:

V. Skokov, B. Friman, K. Redlich, Phys. Rev. C 88 (2013) 034911. arXiv:1205.4756, doi:10.1103/PhysRevC.88.034911

→ Need to know volume / source cumulants

Controlling volume fluctuations with event mixing technique



- Event mixing scheme preserves vol. fluctuations by keeping track. mult. constant.

R. Holzmann, A. Rustamov, J. Stroth
Nucl.Phys.A 1034 (2023), 122641

- Event mixing removes correlations between particles
  - In case of **Poisson like behaviour** one expects for the emission per sources: $\boldsymbol{k_m(n) = <n>,\ cov(n_1, n_2) = 0}$

$$k_2(N_{mix}) = \langle N_{Src}\rangle k_2(n) + \langle n\rangle^2 k_2(N_{src})$$

$$k_2(N_{src}) = k_2(N_{mix})/\langle n\rangle^2 - \langle N_{Src}\rangle/\langle n\rangle$$

Based on assumption of independent sources!

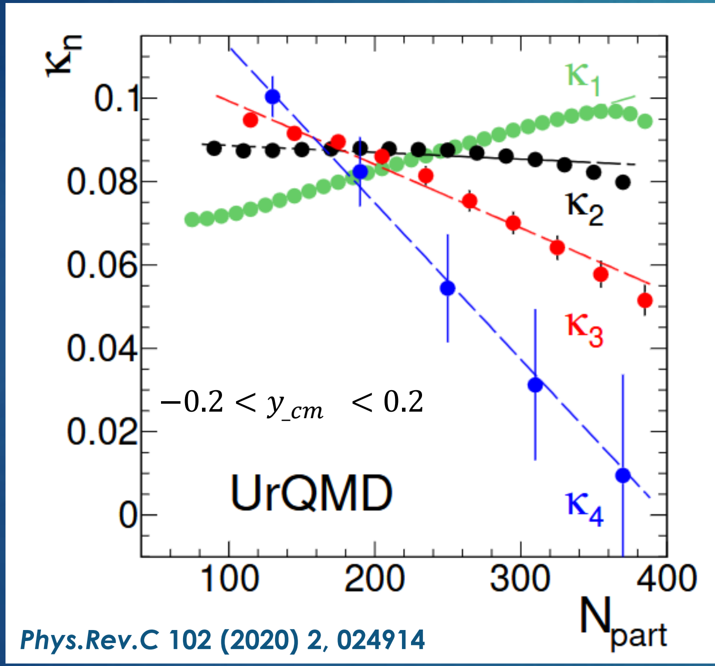
- Same argument for higher order
- Mathematical framework has been genialized → correction can be expressed in terms of factorial cumulants of mixed events

$$\frac{k_n(N_{Src})}{\langle N_{Src}\rangle^n} = \frac{C_n(N_{mixed})}{\langle N_{protons}\rangle^n} + \mathrm{Bias}(\mathrm{C_n}(\mathrm{N_{fixed}}))$$

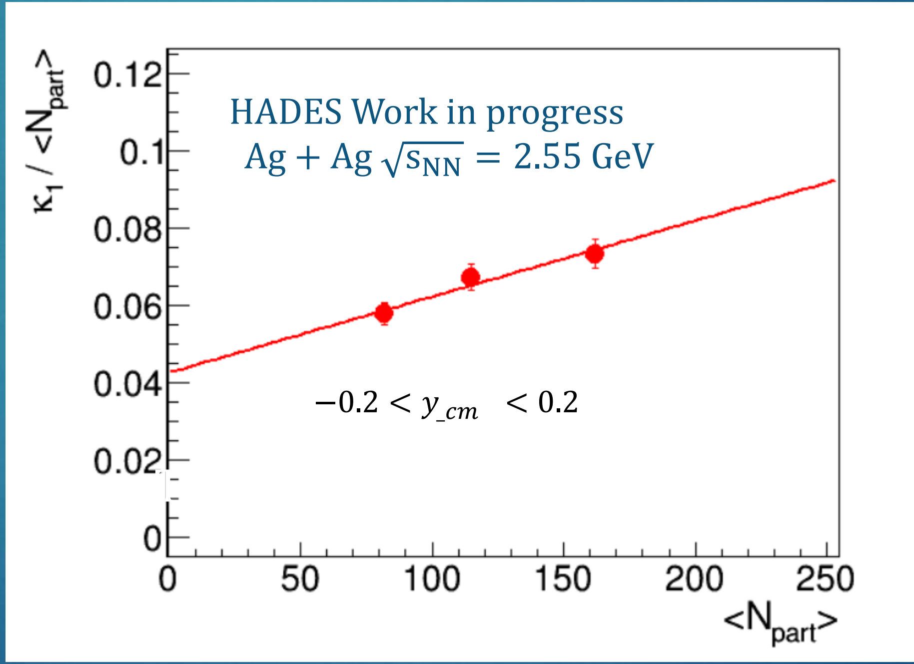
R.Holzmann, V.Koch, A. Rustamov, J. Stroth
Nucl.Phys.A 1050 (2024), 122924

But what is with the independent source assumption?
Is it applicable to lower energy-regime?

Assumption:
Statistical independent sources → Linear scaling of cumulants as function of fixed volume → $\frac{k_n(N_{protons})}{N_{part}}$ = const.



Au+Au
1.23AGeV



Ag+Ag
1.58AGeV

- **Non-constant pattern observed in transport models**
- **For real data, we can only test** $\frac{k_1(N_{protons})}{N_{part}}$
- General problem: Definition of volume
  - N_w, N_{coll}, N_{eff}?

At lower energy:

- Significant part of the volume are the protons itself
- Interaction of produced particles with pole caps (spectators)

Modification of the independent source model

***Phys.Rev.C* 102 (2020) 2, 024914:**

Replace source cumulants by a volume dependent Taylor expansion around the mean <Npart>=<V>

LO NLO N2LO & Ansatz in **V. Skokov, B. Friman, K. Redlich, Phys. Rev. C 88 (2013) 034911. arXiv:1205.4756, doi:10.1103/PhysRevC.88.034911**

$$\kappa_n(V) = \kappa_n + \kappa'_n(V - \langle V\rangle) + \kappa''_n(V - \langle V\rangle)^2$$

	NLO	**N2LO**
	… 28 terms	… 128 terms for 3. order
	… 84 terms	… 527 terms for 4. order

NLO

$$k_1(N) = f_1(k_1(N_{part}), k_2(N_{part}), \kappa_{1,0}, \kappa'_1)$$

$$k_2(N) = f_2(k_1(N_{part}), k_2(N_{part}), k_3(N_{part}), k_4(N_{part}), \kappa_{1,0}, \kappa'_1, \kappa_{2,0}, \kappa'_2)$$

$$k_3(N) = f_3(k_1(N_{part}), k_2(N_{part}), k_3(N_{part}), k_4(N_{part}), k_5(N_{part}), \kappa_{1,0}, \kappa'_1, \kappa_{2,0}, \kappa'_2, \kappa_{3,0}, \kappa'_3)$$

$$k_4(N) = f_4(k_1(N_{part}), k_2(N_{part}), k_3(N_{part}), k_4(N_{part}), k_5(N_{part}) \ldots k_8(N_{part}), \kappa_{1,0}, \kappa'_1, \kappa_{2,0}, \kappa'_2, \kappa_{3,0}, \kappa'_3, \kappa_{4,0}, \kappa'_4)$$

- f_n are polynomials in terms of $v_n = \frac{k_n(N_{part})}{<N_{part}>}$
- Higher order vol. cumulants affect lower order proton cumulants!
- Vol. cumulants up to 8th order occur

LO+NLO – volume cumulants with event mixing

$$k_1(N_{mix}) = f_1(k_1(N_{part}), k_2(N_{part}), \kappa_{1,0}, \kappa'_1)$$

$$k_2(N_{mix}) = f_2(k_1(N_{part}), k_2(N_{part}), k_3(N_{part}), k_4(N_{part}), \kappa_{1,0}, \kappa'_1, \kappa_{2,0}, \kappa'_2)$$

$$k_3(N_{mix}) = f_3(k_1(N_{part}), k_2(N_{part}), k_3(N_{part}), k_4(N_{part}), k_5(N_{part}), \kappa_{1,0}, \kappa'_1, \kappa_{2,0}, \kappa'_2, \kappa_{3,0}, \kappa'_3)$$

$$k_4(N_{mix}) = f_4(k_1(N_{part}), k_2(N_{part}), k_3(N_{part}), k_4(N_{part}), k_5(N_{part}) \ldots k_8(N_{part}), \kappa_{1,0}, \kappa'_1, \kappa_{2,0}, \kappa'_2, \kappa_{3,0}, \kappa'_3, \kappa_{4,0}, \kappa'_4)$$

- **For event mixing, suppose Poisson like behaviour**

→ offset and slope parameter should be the same:

$$\kappa_{n,o} = \kappa_{1,0}$$

$$\kappa'_n = \kappa'_1$$

LO+NLO – volume cumulants with event mixing

$$k_1(N_{mix}) = f_1(k_1(N_{part}), k_2(N_{part}), \kappa_{1,0}, \kappa'_1)$$

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$$k_4(N_{mix}) = f_4(k_1(N_{part}), k_2(N_{part}), k_3(N_{part}), k_4(N_{part}), k_5(N_{part}) \ldots k_8(N_{part}), \kappa_{1,0}, \kappa'_1)$$

- **For event mixing, suppose Poisson like behaviour**
  → offset and slope parameter should be the same:

$$\kappa_{n,o} = \kappa_{1,0}$$

$$\kappa'_n = \kappa'_1$$

- **Goal: solve set of equations for $k_n(N_{part})$**
  - **Not possible analytically!**

LO+NLO – volume cumulants with event mixing

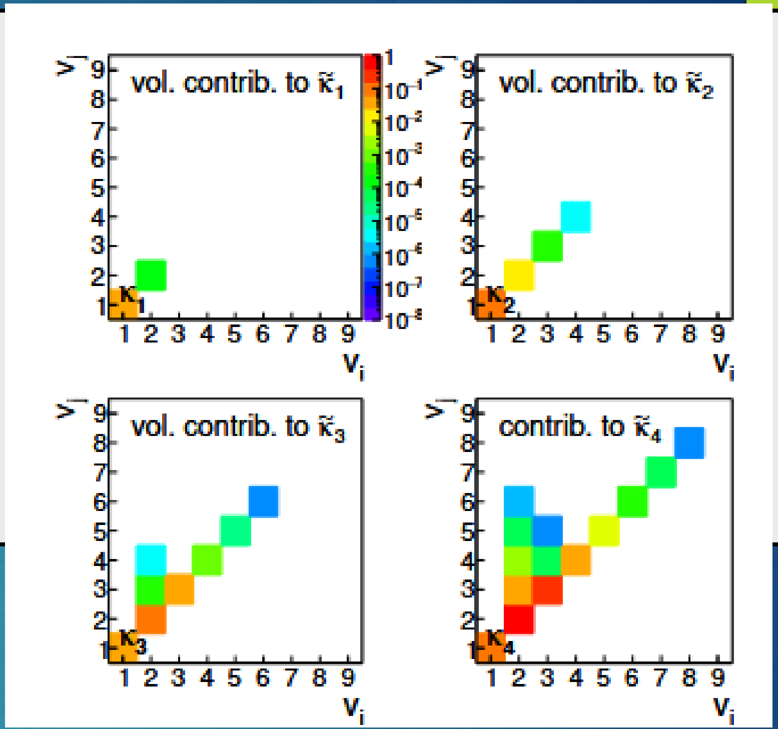
Phys.Rev.C 102 (2020) 2, 024914

$$k_1(N_{mix}) = f_1(k_1(N_{part}), \kappa_{1,0}, \kappa'_1)$$

$$k_2(N_{mix}) = f_2(k_1(N_{part}), k_2(N_{part}), \kappa_{1,0}, \kappa'_1))$$

$$k_3(N_{mix}) = f_3(k_1(N_{part}), k_2(N_{part}), k_3(N_{part}), \kappa'_1)$$

$$k_4(N_{mix}) = f_4(k_1(N_{part}), k_2(N_{part}), k_3(N_{part}), k_4(N_{part}), \kappa_{1,0}, \kappa'_1)$$



→ Impact of higher order vol. cumulants in lower order proton cumulants small

- For event mixing, suppose Poisson like behaviour
→ offset and slope parameter should be the same:

$$\kappa_{n,0} = \kappa_{1,0}$$

$$\kappa'_n = \kappa'_1$$

- Goal: solve set of equations for $k_n(N_{part})$
  - Not possible analytically!

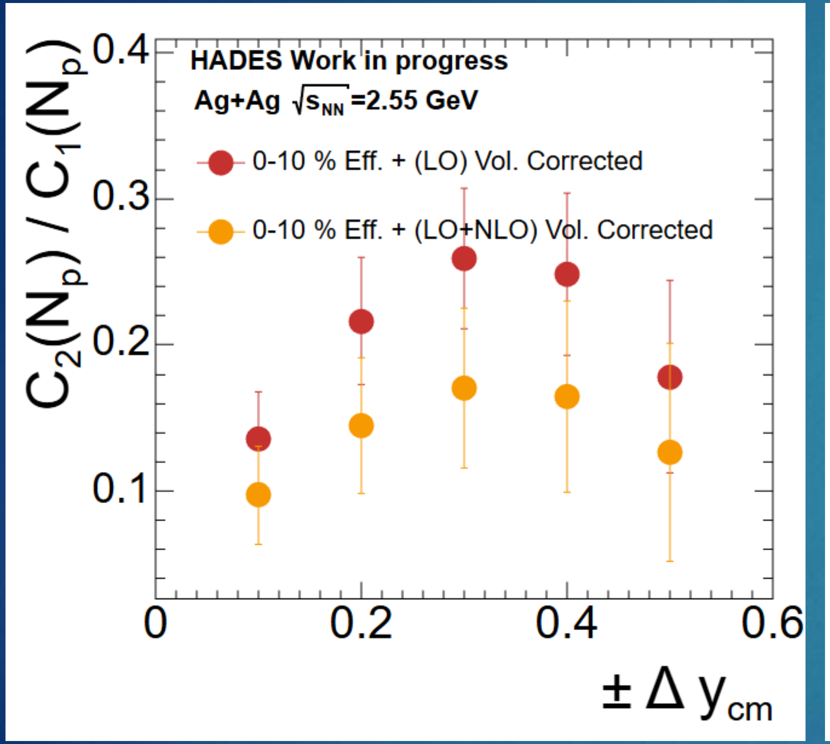
Proposal of approximate solution:

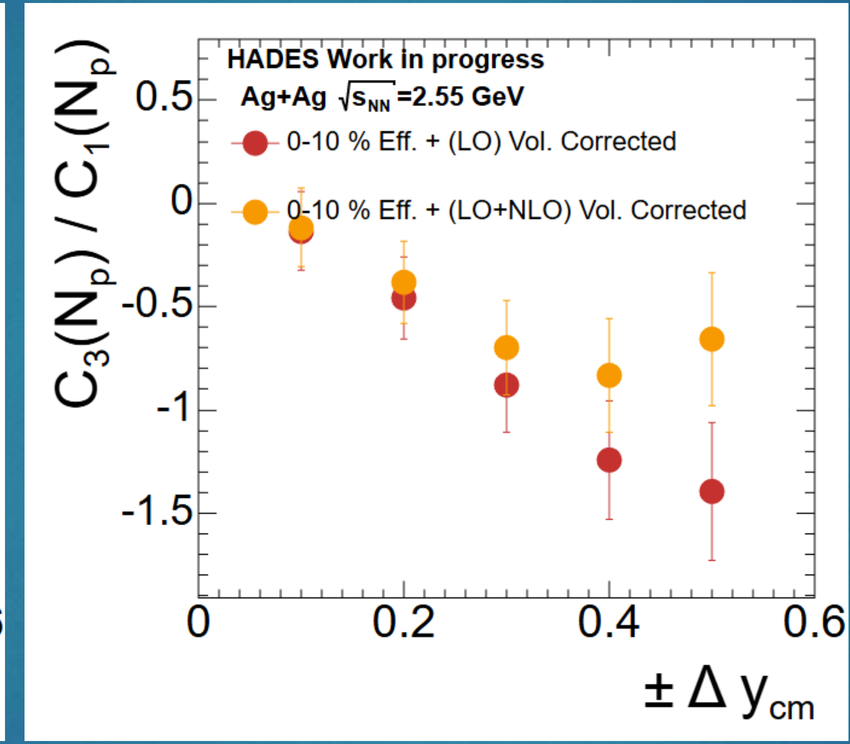
- **Ignore dependence of higher volume cumulants in lower order completely**
- **Solve iteratively order-wise**

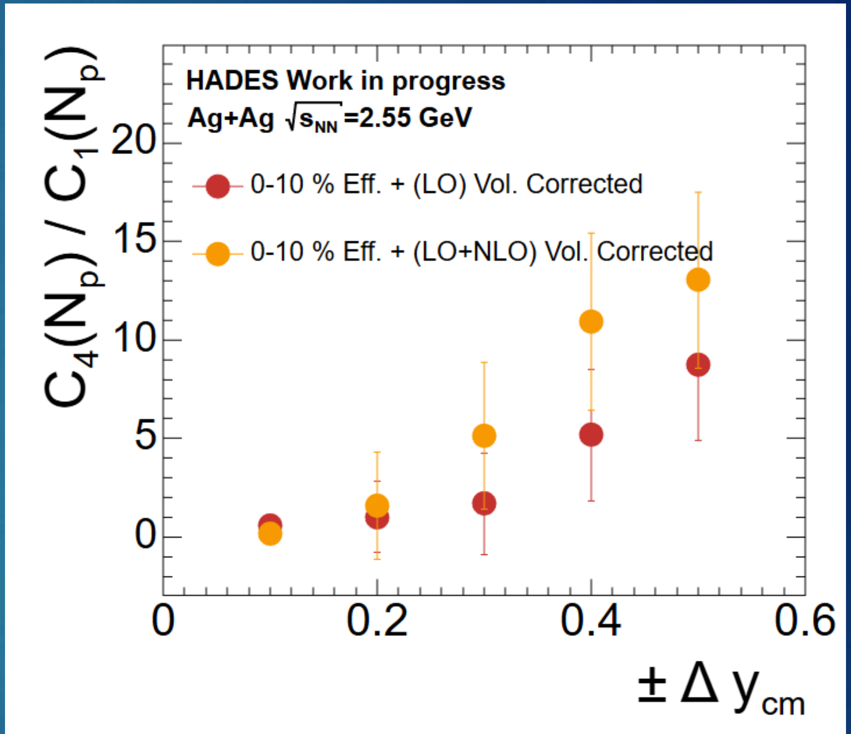
- Slopes of same event sample by multi-order fit to $<N_{part}>$ dependence of eff. corrected cumulants

Volume correction - LO vs LO+NLO

$\mathbf{Ag + Ag}\ \sqrt{s_{NN}} = \mathbf{2.55\ GeV}$



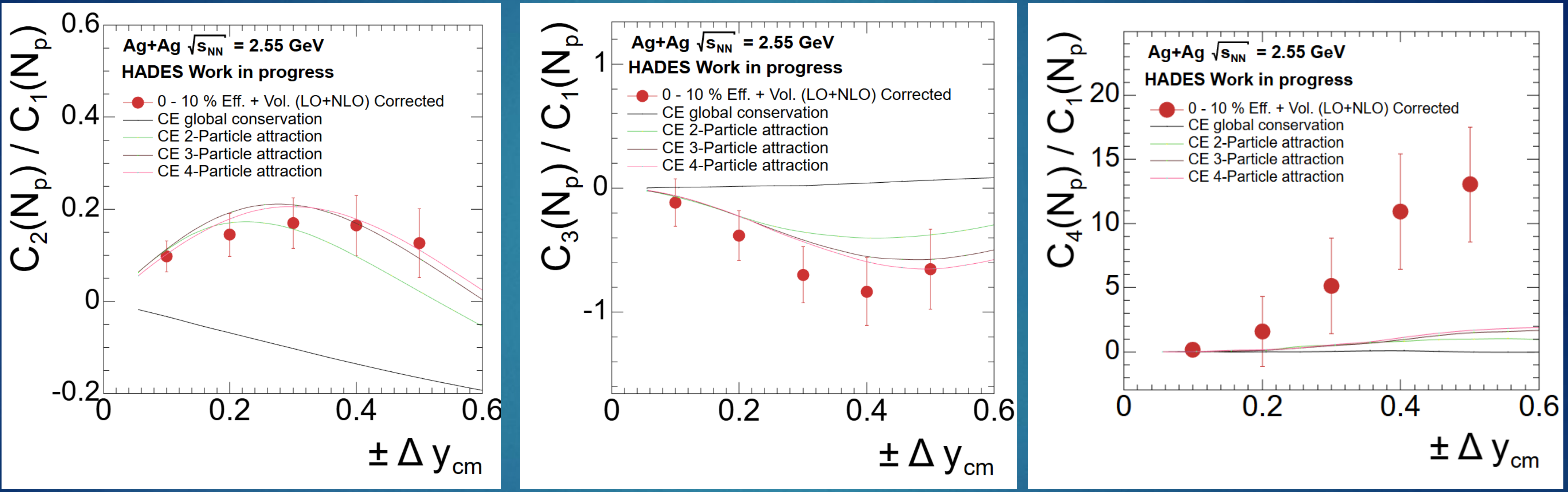




- ❑ LO+NLO volume correction yields same trend behaviour as LO correction
- ❑ Lower magnitude of factorial cumulants for C2/C1, C3/C1 with LO+NLO correction
- ❑ Higher magnitude for C4/C1

Volume correction - LO+NLO

$\mathbf{Ag + Ag}\ \sqrt{s_{NN}} = \mathbf{2.55\ GeV}$



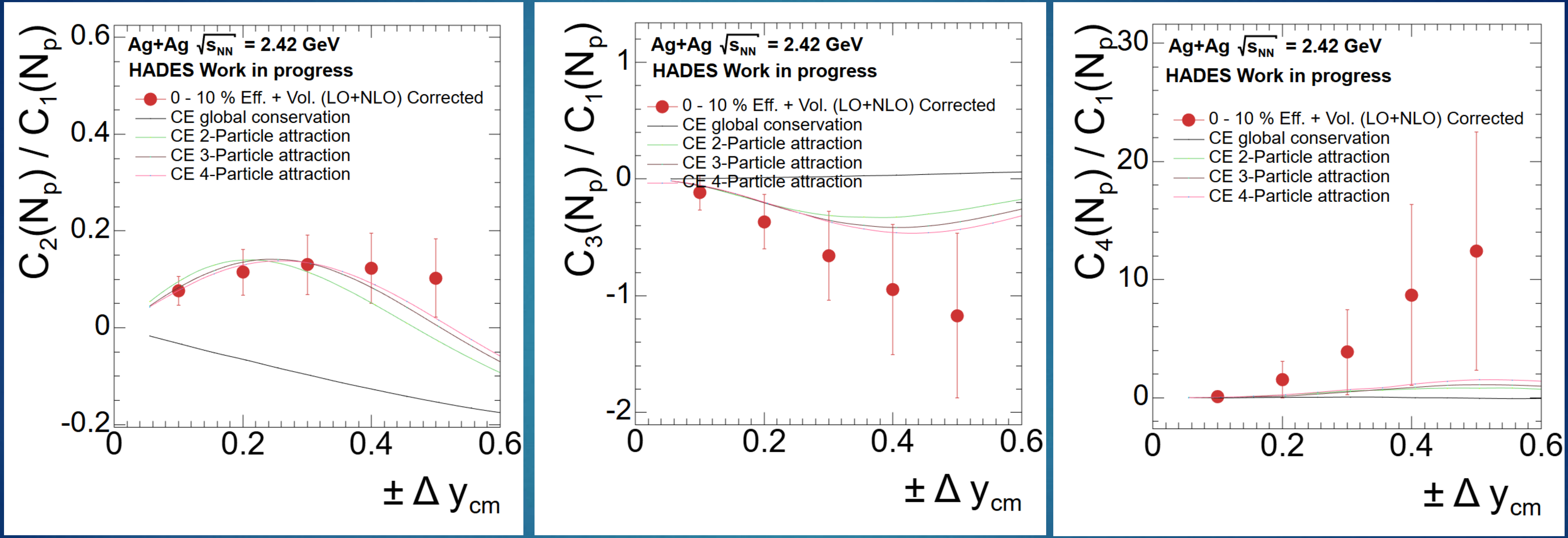
- ❑ CE baseline considering baryon number conservation only can not describe data, different sign
- ❑ 2-Particle correlations does not match the maximum of C2/C1 trend
- ❑ **Multi-particle interactions necessary to describe data within CE framework**

2-Particle corr. : $\rho_{corr} = 0.89$
3-Particle corr. : $\rho_{corr} = 0.78$
4-Particle corr. : $\rho_{corr} = 0.69$

B. Friman, K. Redlich and A. Rustamov **arXiv:2508.18879v1**

Eff. + Volume corrected – **proton** factorial **cumulant ratios**

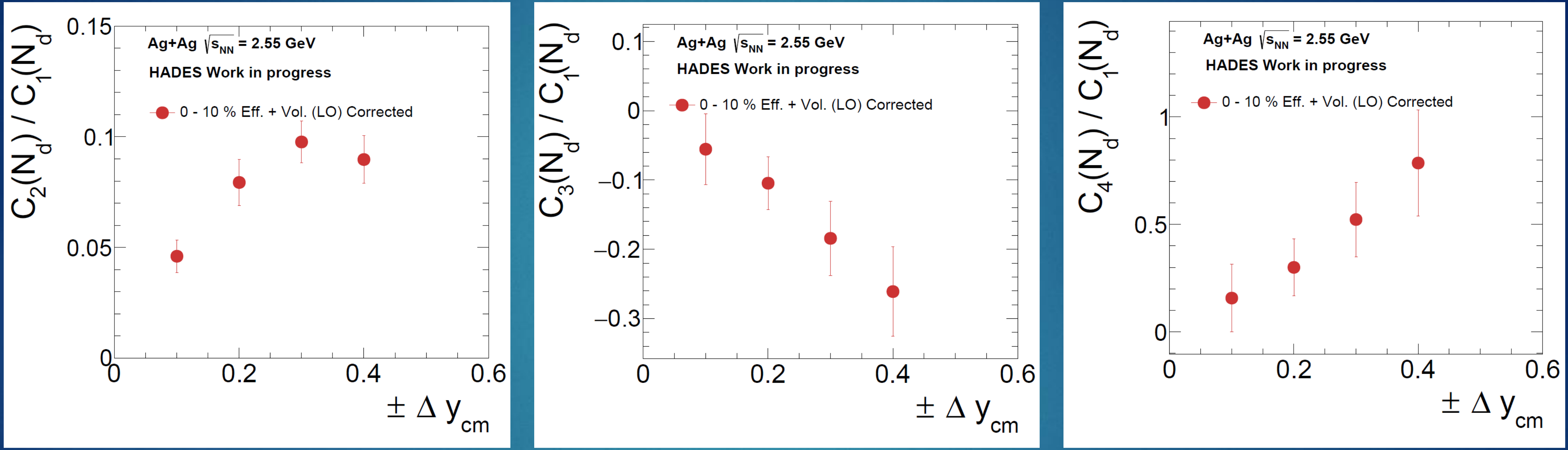
$\mathbf{Ag + Ag}\ \sqrt{s_{NN}} = \mathbf{2.42\ GeV}$



- ❑ For Ag+Ag 1.23 AGeV same trend observed as for higher energy
  - ➢ Data explained by CE model with attractive interaction **B. Friman, K. Redlich and A. Rustamov arXiv:2508.18879v1**
- ❑ Slightly lower absolute values than for Ag+Ag 1.58 AGeV
- ❑ Systematic uncertainty investigation ongoing

Eff. + Volume corrected – **deuteron** factorial **cumulant ratios**

Ag + Ag $\sqrt{s_{NN}} = 2.55$ GeV



- ❑ Deuteron factorial cumulants show same trend as protons, but with lower absolute values
- ❑ NLO correction to be done

- ❑ Need to investigate baseline, effect of clustering…

Summary and Outlook

- HADES probes the QCD phase diagram at highest net-baryon densities
- Presented PID and efficiency correction techniques
- Controlling volume fluctuations challenging:

  Discussed approximate LO+NLO volume correction using event mixing
- Rapidity trend of data for both LO+NLO and LO vol. correction show significant deviation from ideal gas behaviour. Data best described if multi-body attractive proton interactions are included

Outlook

- Understand light nuclei fluctuations
- Cross-cumulants
- Analysis of new HADES BES data…


HADES 50th Collaboration Meeting
GSI February 2026
Thanks for your attention!
THE HADES COLLABORATION
HADES

Back-Up

Fuzzy PID – Correction for purity and in-complete particle identification

Mass spectra lines shapes

$$\rho_k(x_i = m)$$

$$\omega_k = \frac{\rho_k(x_i)}{\sum \rho_l(x_i)}$$

Event\Proxy quantities

$$W_k = \sum_{i=1}^{N_{tracks}} \omega_k(x_i)$$

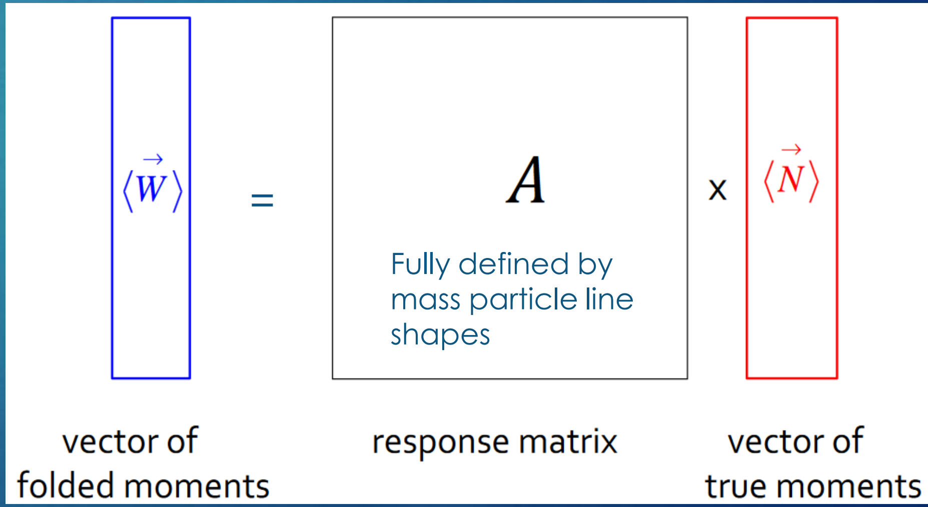
- **A. Rustamov** derived generalized relation between moments of W and N

$$M_W(t_1, t_2, ..., t_n) = \sum_{N_1, N_2, ..., N_n=0}^{\infty} P(N_1, N_2, ..., N_n) \prod_{i=1}^{n} \left[\int_{-\infty}^{+\infty} e^{\sum_{j=1}^{n} t_j \omega_j(x)} \mathcal{P}_i(x) dx \right]^{N_i}$$

***Phys.Rev.C* 110 (2024) 6**

- Multi. variate derivatives of moment generating function yield a linear relation between proxy moments and moments corrected for mis-identification

- Inversion procedure to get from <W> to <N> implemented up to 4. order, including all possible cross-correlations



For details see QM Poster:
Fuzzy logic for reconstructing moments of multiplicity distributions
Anar Rustamov, Joachim Stroth, Marvin Nabroth

- NIM Paper in preparation

Efficiency correction methods - Overview

- Binomial Correction
  - Averaged binomial eff.
  - Multiple Efficiency bins ↔ E-by-E Correction

Toshihiro Nonaka, Masakiyo Kitazawa, and Shinichi Esumi Phys. Rev. C 95, 064912

- Distribution unfolding
  - Iter. Bayesian Unfolding
    - **G. D'Agostini, Nucl. Instrum. Methods Phys. Res. A A362, 487 (1995)**
  - Regularized matrix inversion
    - **S. Schmitt, J. Instrum. 7, T10003 (2012)]**
    - **A. Höcker and V. Kartvelishvili, Nucl. Instrum. Methods Phys. Res. A 372, 469 (1996)**

For multiple observables, cross cumulants:
Iter. Neural Network Unfolding

M. Backes, A. Butter, M. Dunford and B. Malaescu, SciPost Phys.Core 7 (2024) 1, 007

- Direct correction of moments
  - Moment expansion
    - **T.Nonaka et. al.Nucl.Instrum.Meth.A 906 (2018) 10-17**

For Fuzzy PID → Output: PID corrected moments

- Fuzzy output + Moment Expansion
- Cornish Fisher Exp.(Fuzzy output) + Distribution unfolding
- To be combined with E-by-E binomial correction

LO + NLO Vol. Model Equations

Phys.Rev.C 102 (2020) 2, 024914

LO+NLO

$$\tilde{\kappa}_1 = \kappa_1 + v_2\kappa_1' \,, \tag{B1}$$

$$\tilde{\kappa}_2 = \kappa_2 + \kappa_1^2 v_2 + \kappa_2' v_2 + 2\kappa_1\kappa_1' V_2 + 2\kappa_1\kappa_1' v_3 + 2\kappa_1'^2 v_2 V_2 + \kappa_1'^2 V_1 V_2 + 2\kappa_1'^2 V_3 + \kappa_1'^2 v_4 \,, \tag{B2}$$

$$\begin{aligned}\tilde{\kappa}_3 = {} & \kappa_3 + \kappa_1^3 v_3 + 3\kappa_1\kappa_2 v_2 + 3(\kappa_1\kappa_2' + \kappa_1'\kappa_2)v_3 + 6\kappa_1'(\kappa_1^2 + \kappa_2')v_2 V_2 + 3\kappa_1'(\kappa_1^2 + 2\kappa_2')V_3 \\ & + 3\kappa_1'(\kappa_1^2 + \kappa_2')v_4 + 12\kappa_1\kappa_1'^2 V_2^2 + 3\kappa_1\kappa_1'^2 V_1 V_3 + 24\kappa_1\kappa_1'^2 v_2 V_3 + 6\kappa_1\kappa_1'^2 V_4 + 3\kappa_1\kappa_1'^2 v_5 + \kappa_3' v_2 \\ & + 3(\kappa_1\kappa_2' + \kappa_1'\kappa_2)V_2 + 8\kappa_1'^3 v_2 V_2^2 + 6\kappa_1'^3 V_1 V_2^2 + 10\kappa_1'^3 v_3 V_3 + \kappa_1'^3 V_1^2 V_3 + 24 V_2 V_3 \kappa_1'^3 \\ & + 3\kappa_1'^3 V_1 V_4 + 12\kappa_1'^3 v_2 V_4 + 3\kappa_1'^3 V_5 + \kappa_1'^3 v_6 + 3\kappa_1'\kappa_2' V_1 V_2 \,,\end{aligned} \tag{B3}$$

$$\begin{aligned}\tilde{\kappa}_4 = {} & \kappa_4 + \kappa_1^4 v_4 + 6\kappa_1^2\kappa_2 v_3 + (4\kappa_1\kappa_3 + 3\kappa_2^2)v_2 + 24(\kappa_1^3\kappa_1' + 4\kappa_1\kappa_1'\kappa_2' + 2\kappa_1'^2\kappa_2)v_2 V_3 \\ & + 4(\kappa_1^3\kappa_1' + 6\kappa_1\kappa_1'\kappa_2' + 3\kappa_1'^2\kappa_2)V_4 + 2(2\kappa_1^3\kappa_1' + 6\kappa_1\kappa_1'\kappa_2' + 3\kappa_1'^2\kappa_2)v_5 \\ & + 48(\kappa_1^2\kappa_1'^2 + \kappa_1'^2\kappa_2')v_2 V_2^2 + 12(4\kappa_1^2\kappa_1'^2 + 5\kappa_1'^2\kappa_2')v_3 V_3 + 72(\kappa_1^2\kappa_1'^2 + 2\kappa_1'^2\kappa_2')V_2 V_3 \\ & + 6(\kappa_1^2\kappa_1'^2 + 3\kappa_1'^2\kappa_2')V_1 V_4 + 72(\kappa_1^2\kappa_1'^2 + \kappa_1'^2\kappa_2')v_2 V_4 + 6(2\kappa_1^2\kappa_1'^2 + 3\kappa_1'^2\kappa_2')V_5 \\ & + 6(\kappa_1^2\kappa_1'^2 + \kappa_1'^2\kappa_2')v_6 + 2(6\kappa_1^2\kappa_2' + 12\kappa_1\kappa_1'\kappa_2 + 4\kappa_1'\kappa_3' + 3\kappa_2'^2)v_2 V_2 \\ & + 2(3\kappa_1^2\kappa_2' + 6\kappa_1\kappa_1'\kappa_2 + 4\kappa_1'\kappa_3' + 3\kappa_2'^2)V_3 + 2(3\kappa_1^2\kappa_2 + 2\kappa_1\kappa_3' + 2\kappa_1'\kappa_3 + 3\kappa_2\kappa_2')v_3 \\ & + (6\kappa_1^2\kappa_2' + 12\kappa_1\kappa_1'\kappa_2 + 4\kappa_1'\kappa_3' + 3\kappa_2'^2)v_4 + 96\kappa_1\kappa_1'^3 V_2^3 + 96\kappa_1\kappa_1'^3 V_3^2 + 288\kappa_1\kappa_1'^3 v_3 V_2^2 \\ & + 72\kappa_1\kappa_1'^3 V_1 V_2 V_3 + 4\kappa_1\kappa_1'^3 V_1^2 V_4 + 144\kappa_1\kappa_1'^3 V_2 V_4 + 128\kappa_1\kappa_1'^3 v_3 V_4 + 12\kappa_1\kappa_1'^3 V_1 V_5 \\ & + 72\kappa_1\kappa_1'^3 v_2 V_5 + 12\kappa_1\kappa_1'^3 V_6 + 4\kappa_1\kappa_1'^3 v_7 + 24(2\kappa_1\kappa_1'\kappa_2' + \kappa_1'^2\kappa_2)V_2^2 + 6(2\kappa_1\kappa_1'\kappa_2' + \kappa_1'^2\kappa_2)V_1 V_3 \\ & + 2(2\kappa_1\kappa_3' + 2\kappa_1'\kappa_3 + 3\kappa_2\kappa_2')V_2 + 48\kappa_1'^4 v_2 V_2^3 + 48\kappa_1'^4 V_1 V_2^3 + 48\kappa_1'^4 V_1 V_3^2 + 240\kappa_1'^4 v_2 V_3^2 \\ & + 32\kappa_1'^4 v_4 V_4 + 288\kappa_1'^4 V_2^2 V_3 + 24\kappa_1'^4 V_1^2 V_2 V_3 + \kappa_1'^4 V_1^3 V_4 + 144\kappa_1'^4 v_4 V_2^2 + 72\kappa_1'^4 V_1 V_2 V_4 \\ & + 128\kappa_1'^4 V_3 V_4 + 4\kappa_1'^4 V_1^2 V_5 + 72\kappa_1'^4 V_2 V_5 + 56\kappa_1'^4 v_3 V_5 + 6\kappa_1'^4 V_1 V_6 + 24 V_2 V_6 \kappa_1'^4 v_2 V_6 + 4\kappa_1'^4 V_7 \\ & + \kappa_1'^4 v_8 + 36\kappa_1'^2\kappa_2' V_1 V_2^2 + 6\kappa_1'^2\kappa_2' V_1^2 V_3 + 4\kappa_1'\kappa_3' V_1 V_2 + 3\kappa_2'^2 V_1 V_2 + \kappa_4' v_2 \,.\end{aligned} \tag{B4}$$

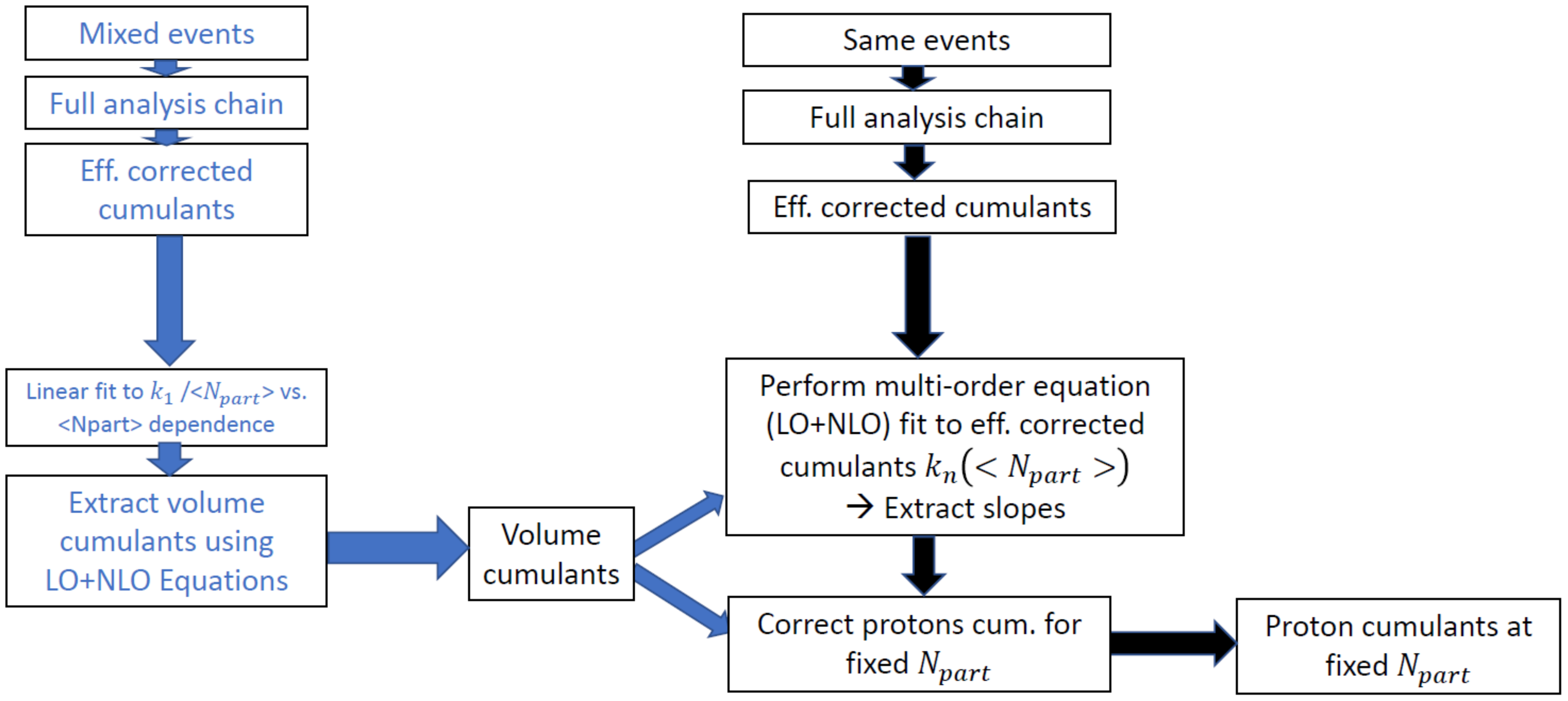
LO+NLO+N2LO

$$\begin{aligned}\tilde{\kappa}_1 = {} & \kappa_1 + v_2\kappa_1' + (V_2 + v_3)\kappa_1'' \,, \\ \tilde{\kappa}_2 = {} & \kappa_2 + \kappa_1^2 v_2 + \kappa_2' v_2 + 2\kappa_1\kappa_1' V_2 + 2\kappa_1\kappa_1' v_3 \\ & + 2\kappa_1'^2 v_2 V_2 + \kappa_1'^2 V_1 V_2 + 2\kappa_1'^2 V_3 + \kappa_1'^2 v_4 \\ & + 6\kappa_1\kappa_1'' v_2 V_2 + 2\kappa_1\kappa_1''(V_3 + v_4) + \kappa_2''(V_2 + v_3) \\ & + 10\kappa_1'\kappa_1'' V_2^2 + 18\kappa_1'\kappa_1'' v_3 V_2 + 2\kappa_1'\kappa_1'' V_1 V_3 \\ & + 4\kappa_1'\kappa_1'' V_4 + 2\kappa_1'\kappa_1'' v_5 + 15\kappa_1''^2 v_2 V_2^2 \\ & + 2\kappa_1''^2 V_1 V_2^2 + 18\kappa_1''^2 V_2 V_3 + 15\kappa_1''^2 v_4 V_2 \\ & + 9\kappa_1''^2 v_3 V_3 + \kappa_1''^2 V_1 V_4 + 2\kappa_1''^2 V_5 + \kappa_1''^2 v_6 \,.\end{aligned} \tag{10}$$

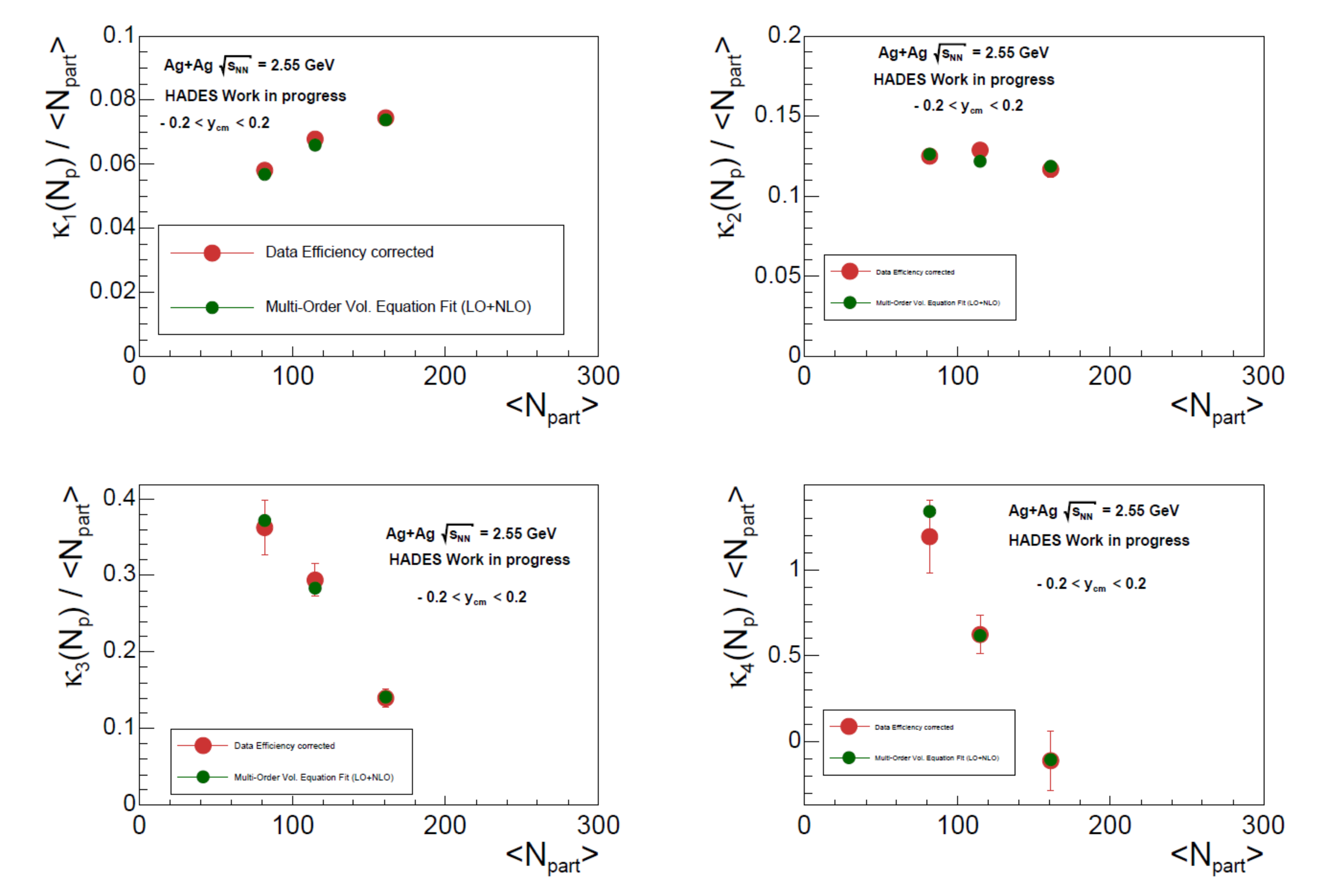
… 128 terms for 3. order

… 527 terms for 4. order

Procedure for volume correction

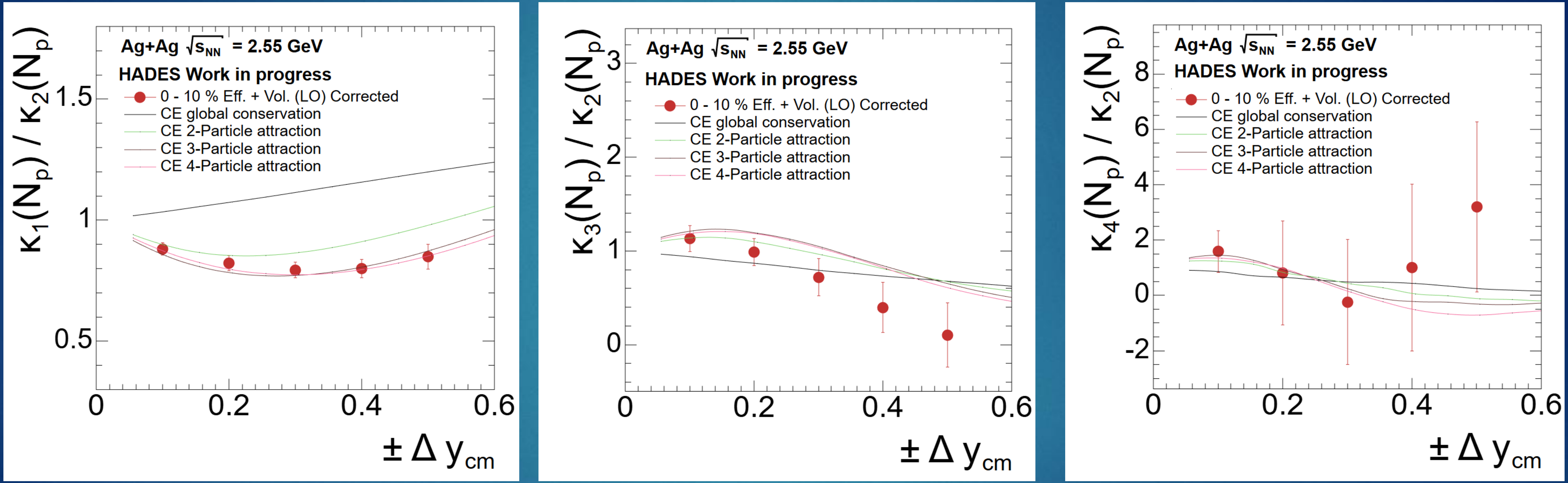


Multi-order Vol. Equation Fit to Data (LO-NLO)



Eff. + Volume corrected – **proton cumulant ratios**

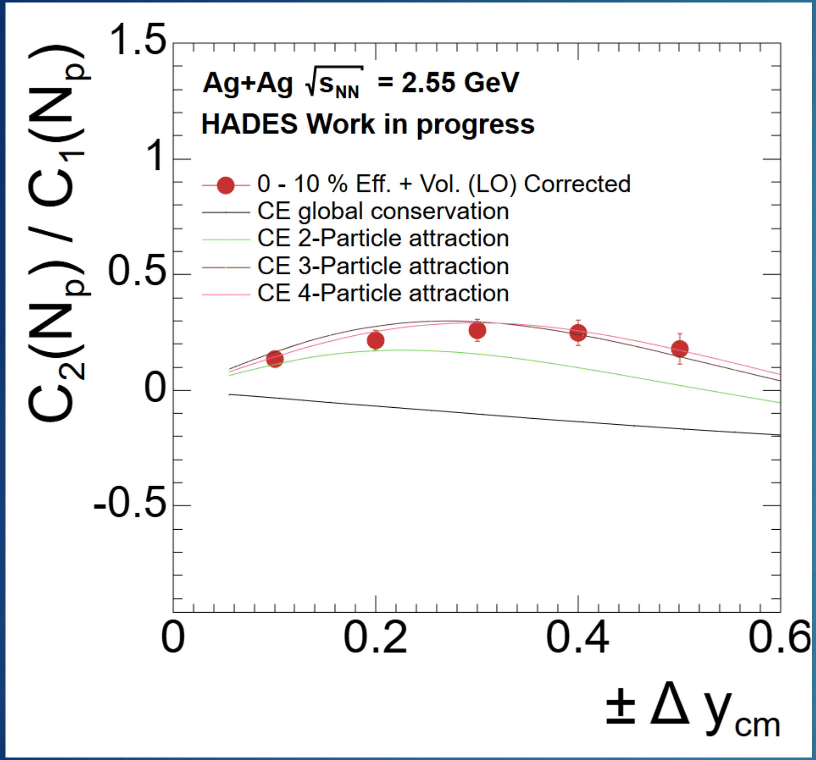
$Ag + Ag \sqrt{s_{NN}} = 2.55$ GeV

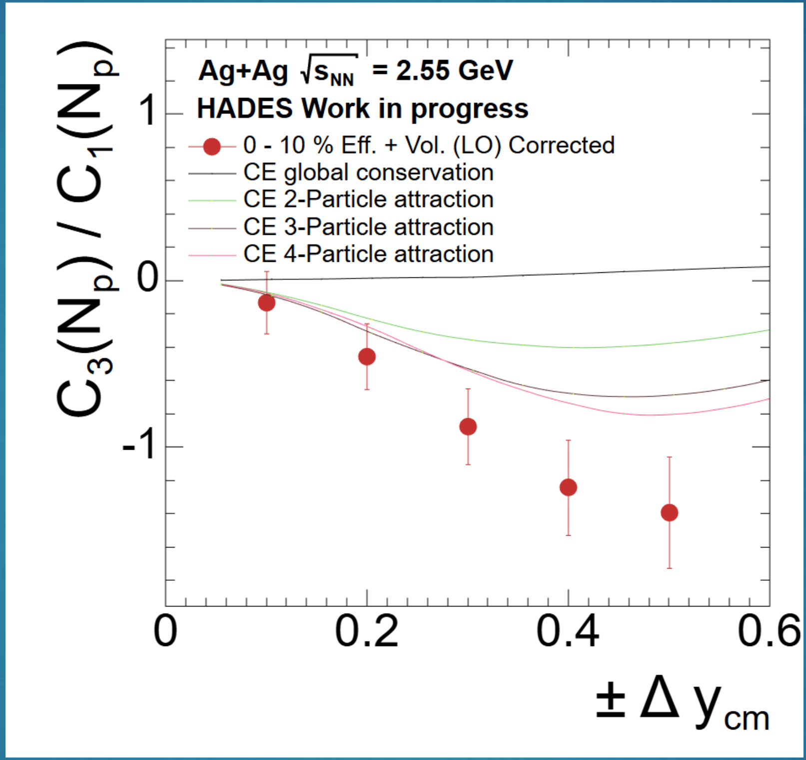


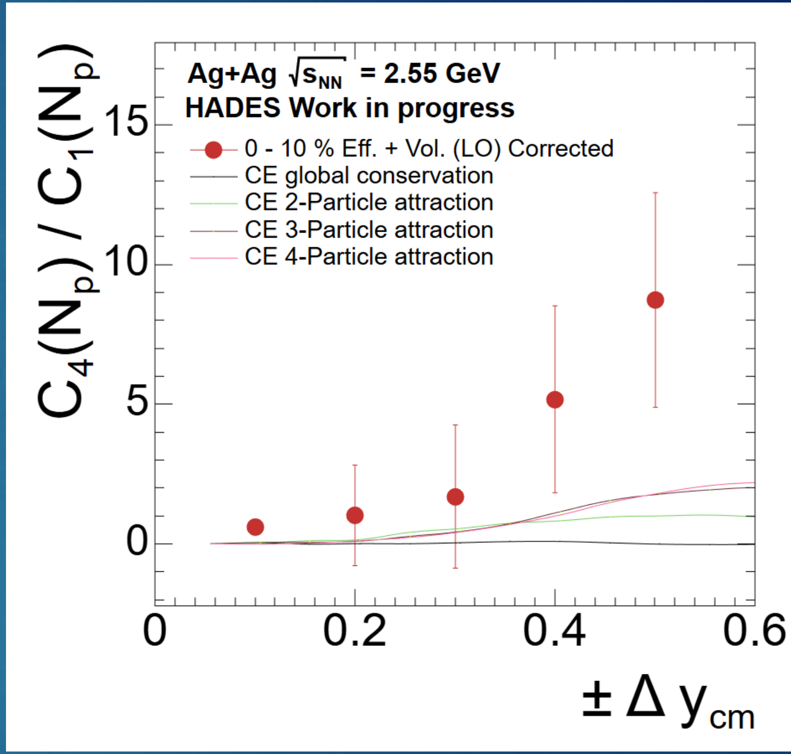
- ❑ CE baseline considering baryon number conservation only can not describe data, different trend for κ_1/κ_2
- ❑ **Trend of rapidity window dependence described by Canonical baselines considering correlations with an attractive potential**

CE simulation implemented as in:
B. Friman, K. Redlich and A. Rustamov arXiv:2508.18879v1

Eff. + Volume corrected – $\mathbf{Ag + Ag\ \sqrt{s_{NN}} = 2.55\ GeV}$ **proton factorial cumulant ratios**







- ❑ Also trend of factorial cumulant ratios described by Canonical baseline considering correlations with an attractive potential

- ❑ 2-Particle correlations does not match the maximum of C2/C1 trend
- ❑ **Multi-particle clusters necessary to describe data within CE framework**

χ^2 Scan performed

2-Particle corr. : $\rho_{corr} = 0.89$

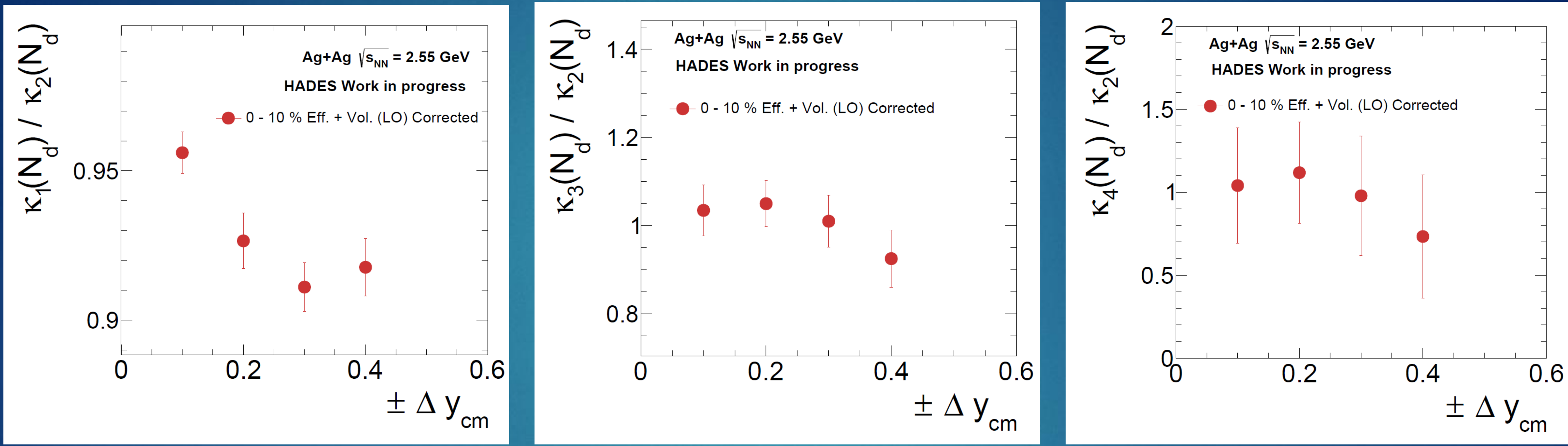
3-Particle corr. : $\rho_{corr} = 0.81$

4-Particle corr. : $\rho_{corr} = 0.74$

B. Friman, K. Redlich and A. Rustamov arXiv:2508.18879v1

Eff. + Volume corrected – **deuteron cumulant ratios**

$\mathbf{Ag + Ag}\ \sqrt{s_{NN}} = \mathbf{2.55\ GeV}$



- ❑ Deuteron cumulants exhibit same sign-trend as protons
- ❑ k3/k2 and k4/k2 are within errors consistent with Poisson limit