Pole masses from one-loop self-energies in the LSMq at finite temperature and magnetic field

66th Cracow School of Theoretical Physics

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Outline

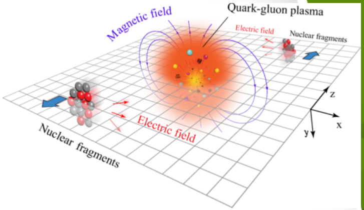
1. Motivation and physical context

2. Pole masses from one-loop self-energies

3. Current status

Physical motivation

- Heavy-ion collisions create strongly interacting matter under extreme conditions.
- Non-central collisions can generate intense magnetic fields.
- Temperature and magnetic fields modify the propagation of particles in the medium.



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In this work, we focus on:

How are the particle masses modified in a hot and magnetized medium?

Our goal is to determine the thermomagnetic pole masses $M_i(T, B)$ of pions (neutral and charged), sigma meson and lightest quarks, within LSMq, for arbitrary values of T and $|eB|$.

Linear Sigma Model with quarks

The LSMq is an effective model for low-energy QCD. The degrees of freedom are the light quarks, $\psi = (u, d)^T$, and the meson fields $\sigma, \pi_0, \pi_\pm$.

$$
\begin{aligned}
\mathcal{L} = &\frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi_0)^2 + D_\mu \pi_- D^\mu \pi_+ + \frac{a^2}{2}(\sigma^2 + \pi_0^2 + 2\pi_-\pi_+) \\
&- \frac{\lambda}{4}\left(\sigma^2 + \pi_0^2 + 2\pi_-\pi_+\right) + i\bar{\psi}\partial\!\!\!/\psi - g\bar{\psi}(\sigma + i\gamma^5 \vec{\tau}\cdot\vec{\pi})\psi.
\end{aligned}
$$

After spontaneous symmetry breaking,

$$
\sigma \to \sigma + v,
$$

the model generates tree-level masses.

$$
m_\sigma^2 = 3\lambda v^2 - a^2, \qquad m_\pi^2 = \lambda v^2 - a^2, \qquad m_f = gv.
$$

These tree-level masses are the starting point for the pole-mass equations, where one-loop self-energies introduce thermal and magnetic corrections.

One-loop self-energies

The pole-mass calculation requires the full one-loop self-energies. These contributions can be classified according to their medium dependence:

$$\Pi_i(p;T,B) = \Pi_i(p;T) + \Pi_i(p;T,B), \qquad i = \sigma, \pi_0, \pi_\pm, f.$$

- $\Pi_i(p;T)$: contributions involving neutral internal fields, sensitive only to the thermal medium.
- $\Pi_i(p;T,B)$: contributions involving charged internal fields, sensitive to both temperature and magnetic field.

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Dressed propagator and pole mass definition

In an interacting theory, the free propagation of a particle is modified by quantum, thermal, and magnetic corrections. These effects are summarized in the self-energy $\Pi(p; T, B)$, leading to the dressed propagator.

The pole mass is obtained from the condition that the inverse propagator vanishes,

$$D(p) = \frac{i}{p^2 - m_0^2 - \Pi(p; T, B)} \rightarrow p^2 - m_0^2 - \Pi(p; T, B) = 0.$$

In the rest frame of the medium, the pole mass is defined from the zero-momentum pole,

$$M_{\text{pole}}^2 = m_0^2 + \Pi(p_0 = M_{\text{pole}}, \vec{p} = 0; T, B).$$

Self-consistent masses

We obtain a system of four coupled equations for the in-medium masses:

$$
\begin{aligned}
M_\sigma^2(T,B) &= m_\sigma^2 + \Pi_\sigma(M_\sigma, M_{\pi_0}; T) + \Pi_\sigma(M_\sigma, M_{\pi_0}, M_{\pi_\pm}, M_f; T, B),\\
M_{\pi_0}^2(T,B) &= m_{\pi_0}^2 + \Pi_{\pi_0}(M_\sigma, M_{\pi_0}; T) + \Pi_{\pi_0}(M_\sigma, M_{\pi_0}, M_{\pi_\pm}, M_f; T, B)\\
M_{\pi_\pm}^2(T,B) &= m_{\pi_\pm}^2 + \Pi_{\pi_\pm}(M_\sigma, M_{\pi_0}; T) + \Pi_{\pi_\pm}(M_\sigma, M_{\pi_0}, M_{\pi_\pm}, M_f; T, B),\\
M_f(T,B) &= m_f + \Pi_f(M_\sigma, M_{\pi_0}, M_{\pi_\pm}, M_f; T, B).
\end{aligned}
$$

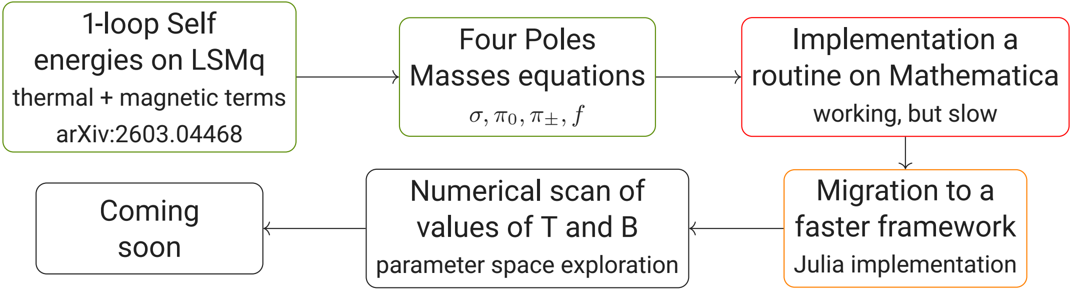
The coupled system has been implemented and can be solved numerically for fixed values of T and B. However, the full self-energies are computationally demanding, so the current work is focused on optimizing the numerical routine before performing a systematic scan of the T-B parameter space.

Self-consistent masses

Full expression to illustrate why the solution is not easy to solve.

$$
\begin{aligned}
M_{\pi_0}^2(T,B) = m_{\pi_0}^2 &+ \frac{3\lambda}{2\pi^2}\int_0^\infty dq \frac{q^2}{\sqrt{q^2+M_{\pi_0}^2}} n_B(E_\pi) + \frac{\lambda}{2\pi^2}\int_0^\infty dq \frac{q^2}{\sqrt{q^2+M_\sigma^2}} n_B(E_\sigma) \\
&+ \frac{\lambda|eB|}{8\pi^2}\int_0^\infty \frac{d\tau}{\tau} e^{-\tau M_{\pi^\pm}^2}\left[\frac{1}{\sinh(|eB|\tau)} - \frac{1}{|eB|\tau} + \frac{|eB|\tau}{6}\right] \\
&+ \frac{\lambda|eB|}{8\pi^2}\int_0^\infty \frac{d\tau}{\tau} \frac{e^{-\tau M_{\pi^\pm}^2}}{\sinh(|eB|\tau)}\left[2\sum_{n=1}^\infty e^{-\frac{n^2}{4T^2\tau}}\right] \\
&- \lambda^2 v^2 \int \frac{d^3q}{(2\pi)^3}\frac{1}{E_1E_2}\left[\Big(n_B(E_1)+n_B(E_2)\Big)\left(\frac{1}{i\omega+E_1+E_2} - \frac{1}{i\omega-E_1-E_2}\right)\right. , \\
&+ \left.\Big(n_B(E_1)-n_B(E_2)\Big)\left(\frac{1}{i\omega-E_1+E_2} - \frac{1}{i\omega+E_1-E_2}\right)\right] \\
&- \frac{g^2}{(2\pi)^2} N_C \sum_f (\textit{fermionic contribution: not shown for everyone's sanity}).
\end{aligned}
$$

Work in progress



The Mathematica routine is already functional, but the full self-energies are lengthy and numerically demanding. The calculation is being migrated to Julia to improve performance before scanning the T-B parameter space.

Summary

- Pole masses are defined from the poles of the dressed propagators.
- The pole-mass equations were formulated using the full one-loop self-energies of the LSMq at finite T and B.
- A first numerical implementation is already working in `Mathematica`.

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Outlook

- Optimize the numerical evaluation by migrating the routine to `Julia`.
- Perform the scan in T and B to obtain M_σ, M_{π^0}, $M_{\pi^\pm}$, and M_f.
- Analyze the behavior of the pole masses near chiral symmetry restoration.

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Thanks...!!

Doubts and Suggestions
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Back up slides

Self-consistent masses

Fermionic contribution:

$$
\begin{aligned}
M^2_{\pi^0}(T,B) = (\cdots) - \frac{g^2}{(2\pi)^2} N_C \sum_f \int_0^\infty du \int_0^1 dv \Bigg\{ & \frac{|q_f B|}{\sinh(|q_f B|u)} e^{-uv(1-v)k_3^2} e^{-m_f^2 u} e^{(i\omega)^2 uv(1-v)} e^{-\frac{k_\perp^2}{|q_f B|}\frac{\sinh(|q_f B|u(1-v))\sinh(|q_f}{\sinh(|q_f B|u)}} \\
& \times \Bigg\{ \cosh(|q_f B|u(1-2v)) \Bigg[\Bigg(1 + 2\sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2}{4T^2 u}} \cosh\left(\frac{(1-v)n(i\omega)}{T}\right)\Bigg) \left(m_f^2 + \frac{1}{u}\left(uv(1-v)k_3^2 - \frac{1}{2}\right)\right) \\
& - i\omega \Bigg[(1-v)(i\omega) + 2\sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2}{4T^2 u}} \left((1-v)(i\omega)\cosh\left(\frac{(1-v)n(i\omega)}{T}\right) - \frac{n}{2Tu}\sinh\left(\frac{(1-v)n(i\omega)}{T}\right)\right)\Bigg] \\
& - \Bigg(\frac{1}{2u} - (1-v)^2(i\omega)^2 + 2\sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2}{4T^2 u}} \Bigg[\left(\frac{1}{2u} - (1-v)^2(i\omega)^2 - \frac{n^2}{4T^2u^2}\right)\cosh\left(\frac{(1-v)n(i\omega)}{T}\right) \\
& + \frac{(1-v)n(i\omega)}{Tu}\sinh\left(\frac{(1-v)n(i\omega)}{T}\right)\Bigg]\Bigg)\Bigg] + \frac{1}{\sinh^2(|q_f B|u)}\Big[k_\perp^2 \sinh(|q_f B|u(1-v))\sinh(|q_f B|uv) - |q_f B|\sinh(|q_f B|u)\Big] \\
& \times \Bigg[1 + 2\sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2}{4T^2 u}} \cosh\left(\frac{(1-v)n(i\omega)}{T}\right)\Bigg]\Bigg\} - e^{-\left(k_\perp^2 + k_3^2 - (i\omega)^2\right)uv(1-v)} e^{-m_f^2 u} \frac{1}{u}\Bigg(m_f^2 + \frac{1}{u}\Bigg(uv(1-v)k_3^2 - \frac{1}{2} \\
& - (i\omega)^2(1-v)x - \frac{1}{2u} + (1-v)^2(i\omega)^2 + \frac{1}{u}\left(k_\perp^2 uv(1-v) - 1\right)\Bigg)\Bigg)\Bigg\}
\end{aligned}
$$

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