Fluctuations, correlations and what have we learned so far from RHIC BES II

"A theory is something nobody believes, except the person who made it.

An experiment is something everybody believes, except the person who made it."

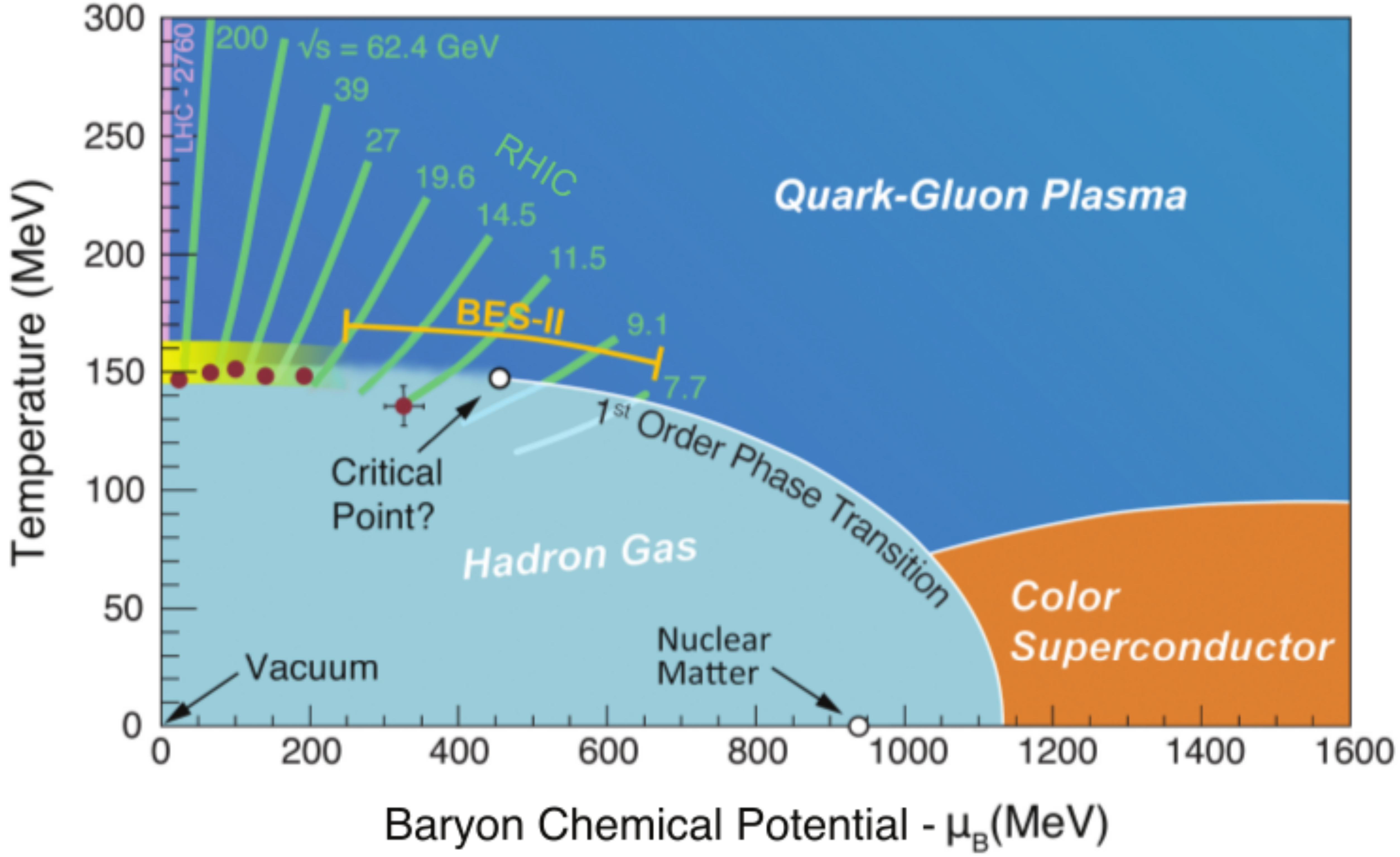
A. Einstein

Fluctuations, correlations and what have we learned so far from RHIC BES II

- Introduction
- Revisit net charge fluctuations
- Cumulants and Correlations
  - Cumulants theory vs measured cumulants
- Non critical baseline
- Comparison with RHIC BES data
  - what have we learned so far
  - next steps
- Conclussions

Thanks to: Volodymyr Vovchenko, Jonathan Parra, Adam Bzdak

The phase diagram



Increase chemical potential by lowering the beam energy

In reality, we add baryons (nucleons) from target and projectile to mid-rapidity

What we know about the Phase Diagram

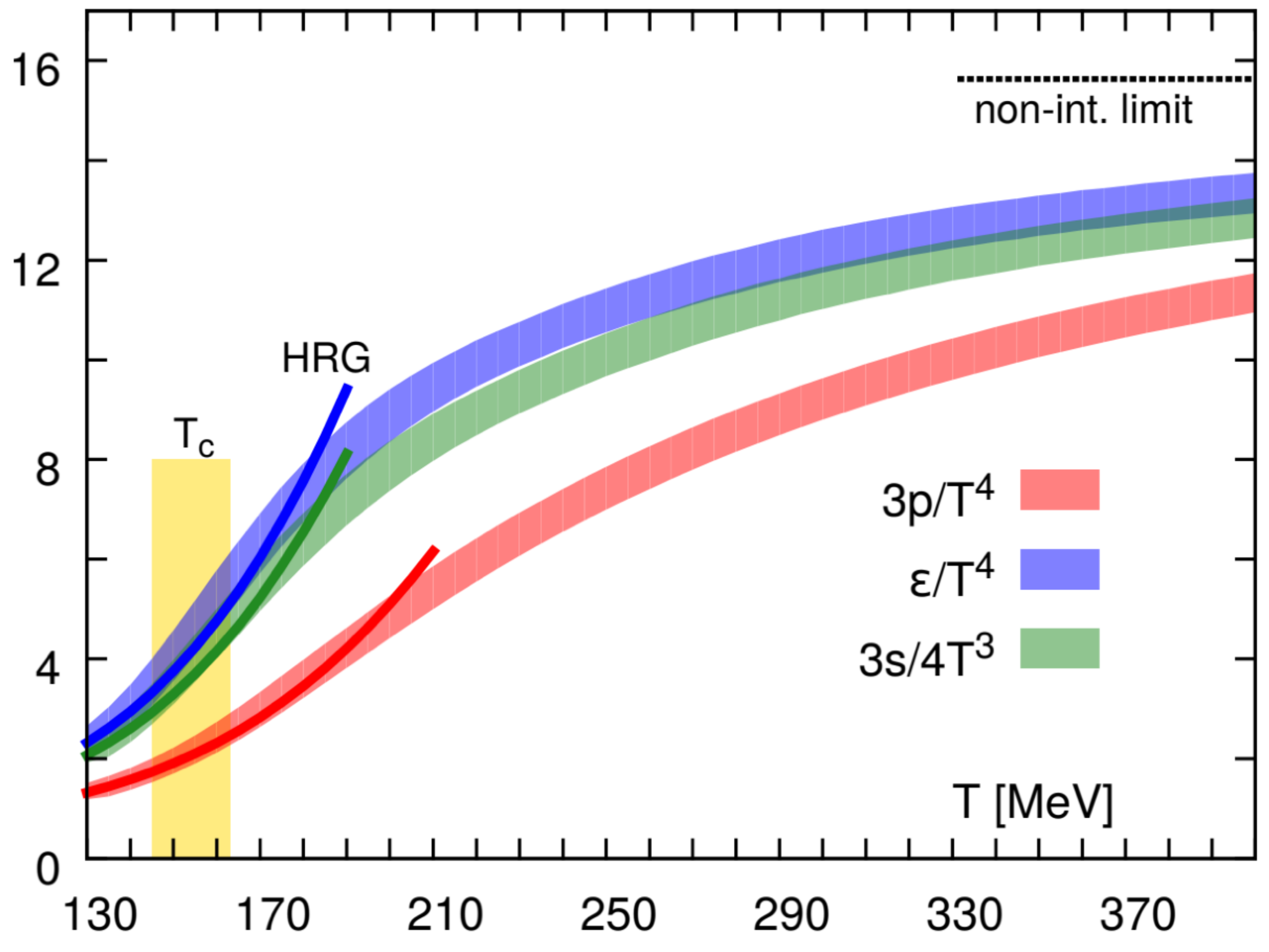
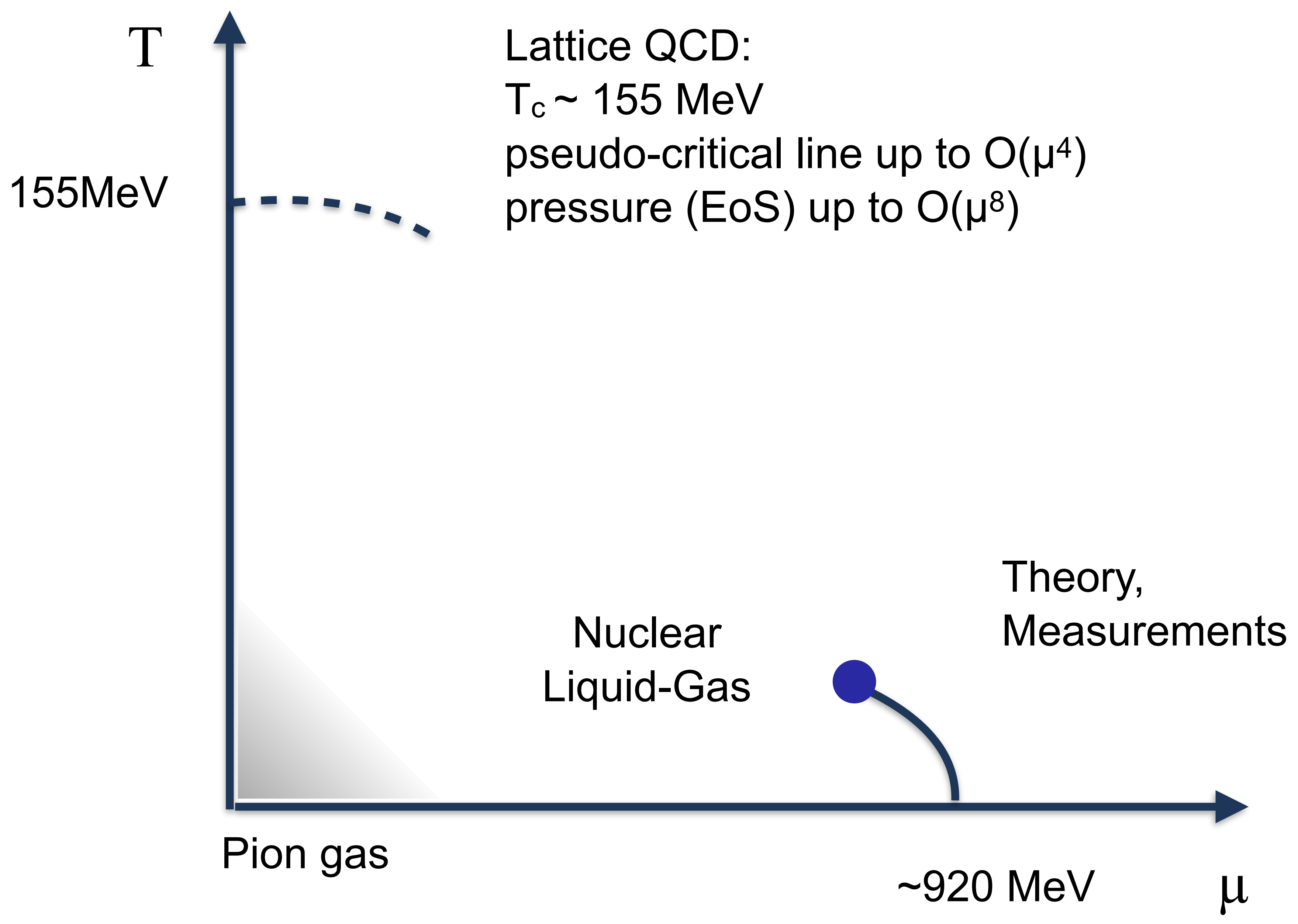
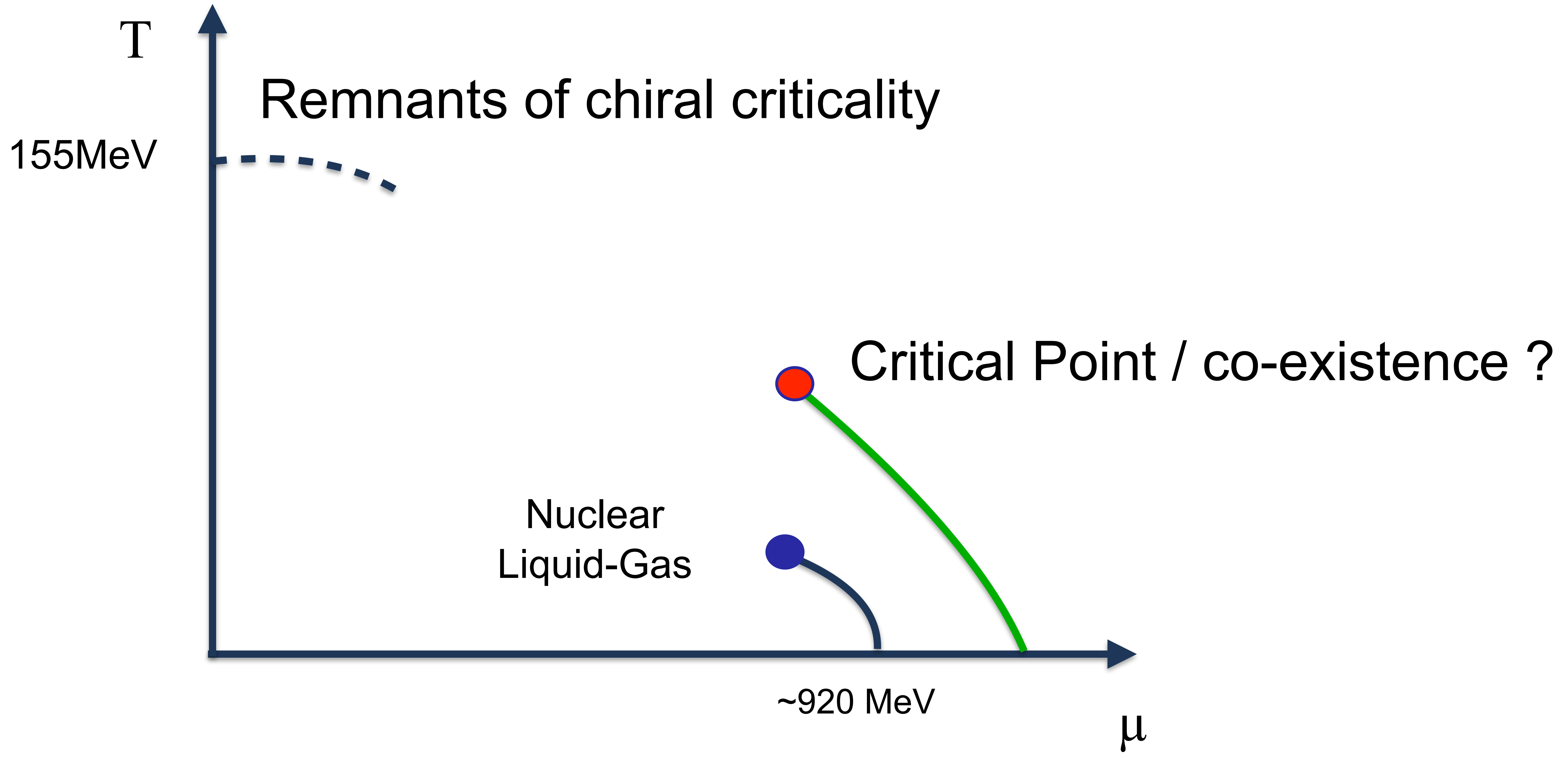


Figure from HotQCD coll., PRD ‘14



What we are looking for



We are dealing with small system of finite lifetime

NO real singularities!

Recent results on CP

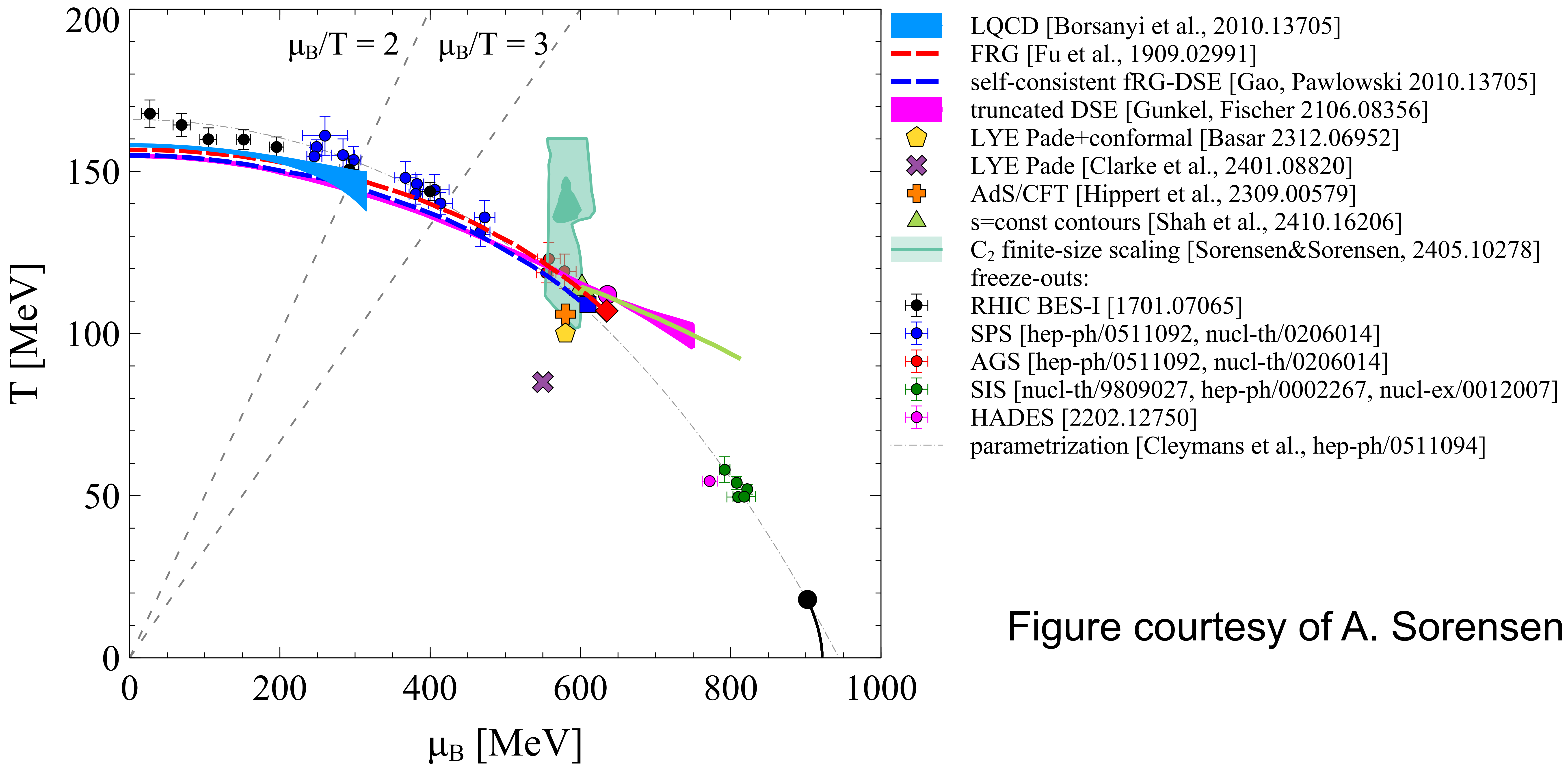


Figure courtesy of A. Sorensen

Fluctuations and the phase diagram

Fluctuations of conserved charges measure derivatives of the partition function (pressure)

$$\kappa_n(q) = \frac{\partial^n}{\partial(\mu_q/T)^n} \ln(Z); \quad q = B, Q, S$$

The are also sensitive to the degrees of freedom (fractional charges, correlations):

$$\kappa_2(B) \sim b^2 N_{\text{quarks}} = \frac{1}{9} N_{\text{quarks}}$$

Asakawa, Heinz and Müller, PRL 86, 2000

$$\kappa_2(Q) \sim q^2 N_{\text{quarks}} = \frac{1}{9} N_{up} + \frac{4}{9} N_{down}$$

Jeon, V.K. PRL 86, 2000

Update on Charge fluctuations

Proposed observable: (Jeon, V.K. PRL 86, 2000) $D = 4\frac{\kappa_2(Q)}{\langle N_{ch}\rangle} = 4\frac{\chi_2(Q)}{s/T^3}\frac{S}{\langle N_{ch}\rangle}$

$$\omega = \frac{\kappa_2[Q]}{\langle N_{\rm ch}^{\rm prim}\rangle}$$

variance at hadronization

charged multiplicity

Hadron gas: $\omega_{HG} \approx 1$ (Poisson statistics + Bose)

Free QGP*: $\omega_{QGP} \approx 0.36$ (Stefan-Boltzmann limit)

More generally:

$$\omega = \frac{\kappa_2[Q]}{\langle N_{\rm ch}^{\rm prim}\rangle} = \frac{VT^3\chi_2^Q}{S}\frac{S}{\langle N_{\rm ch}^{\rm prim}\rangle}$$

$$= \frac{\chi_2^Q}{s/T^3}\frac{S}{\langle N_{\rm ch}\rangle}\frac{\langle N_{\rm ch}\rangle}{\langle N_{\rm ch}^{\rm prim}\rangle}$$

The EoS

e.g. lattice QCD

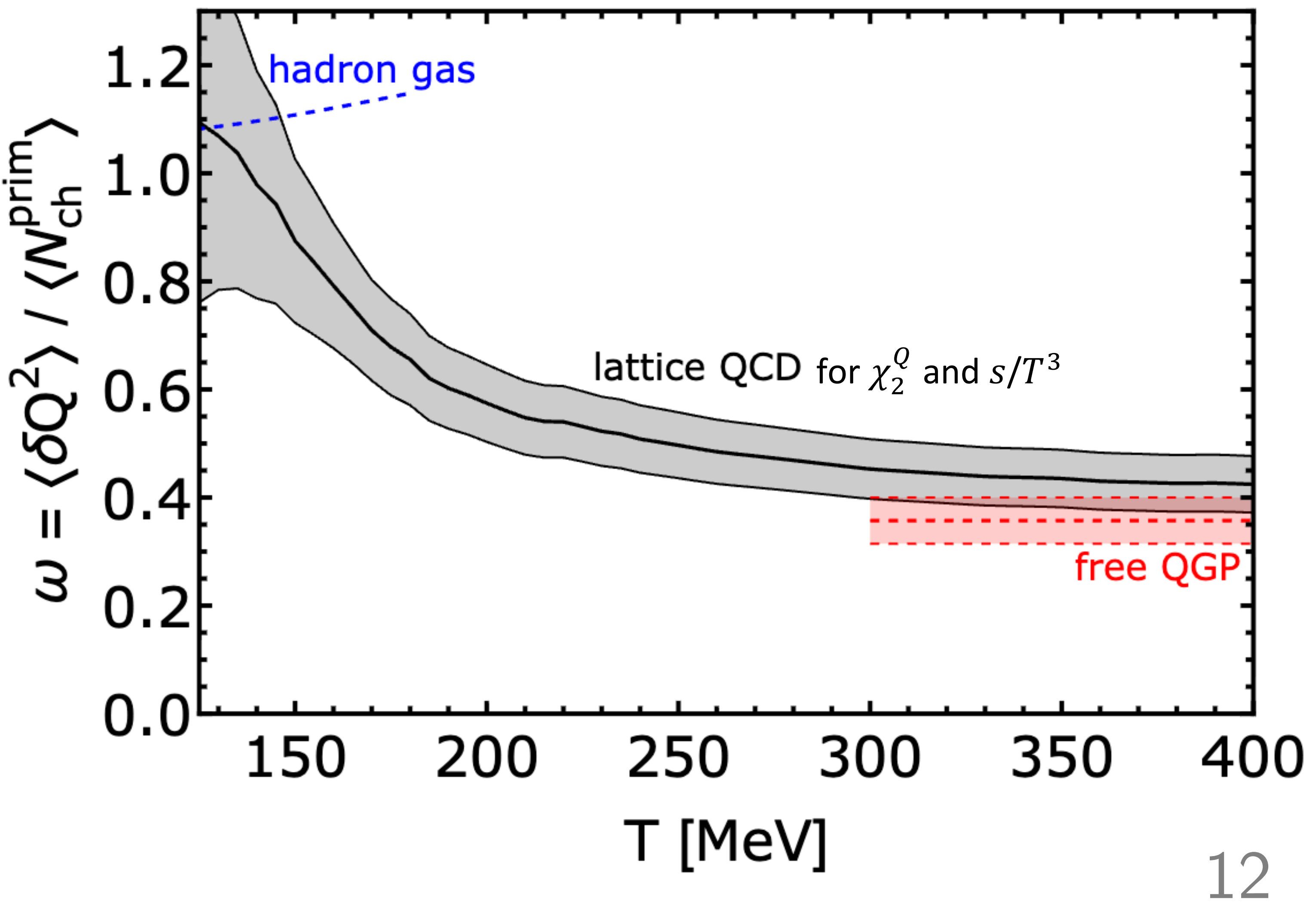
$S/N_{\rm ch} = 6.7 \pm 0.8$ (LHC)

Data-driven [P. Hanus, A. Mazeliauskas, K. Reygers, PRC (2019)]

$\gamma_Q \approx 1.67$ (decays)

from thermal model

ω from lattice QCD



$$D = 4\frac{\omega}{\gamma_Q}$$

*Same/similar for SQGB scenario of Fujimoto et al., PRD 112, 074006 (2025)

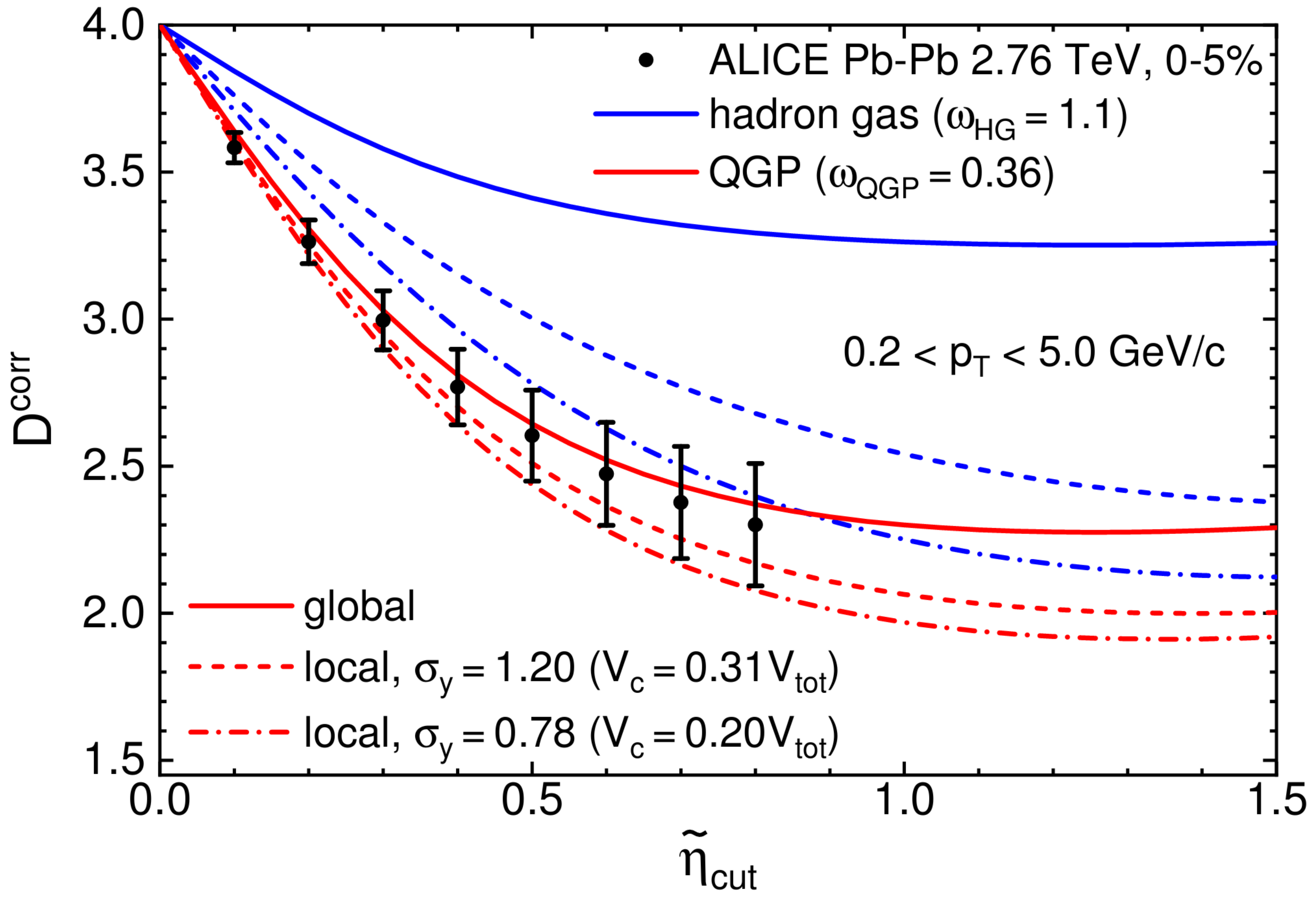
Courtesy V. Vovchenko

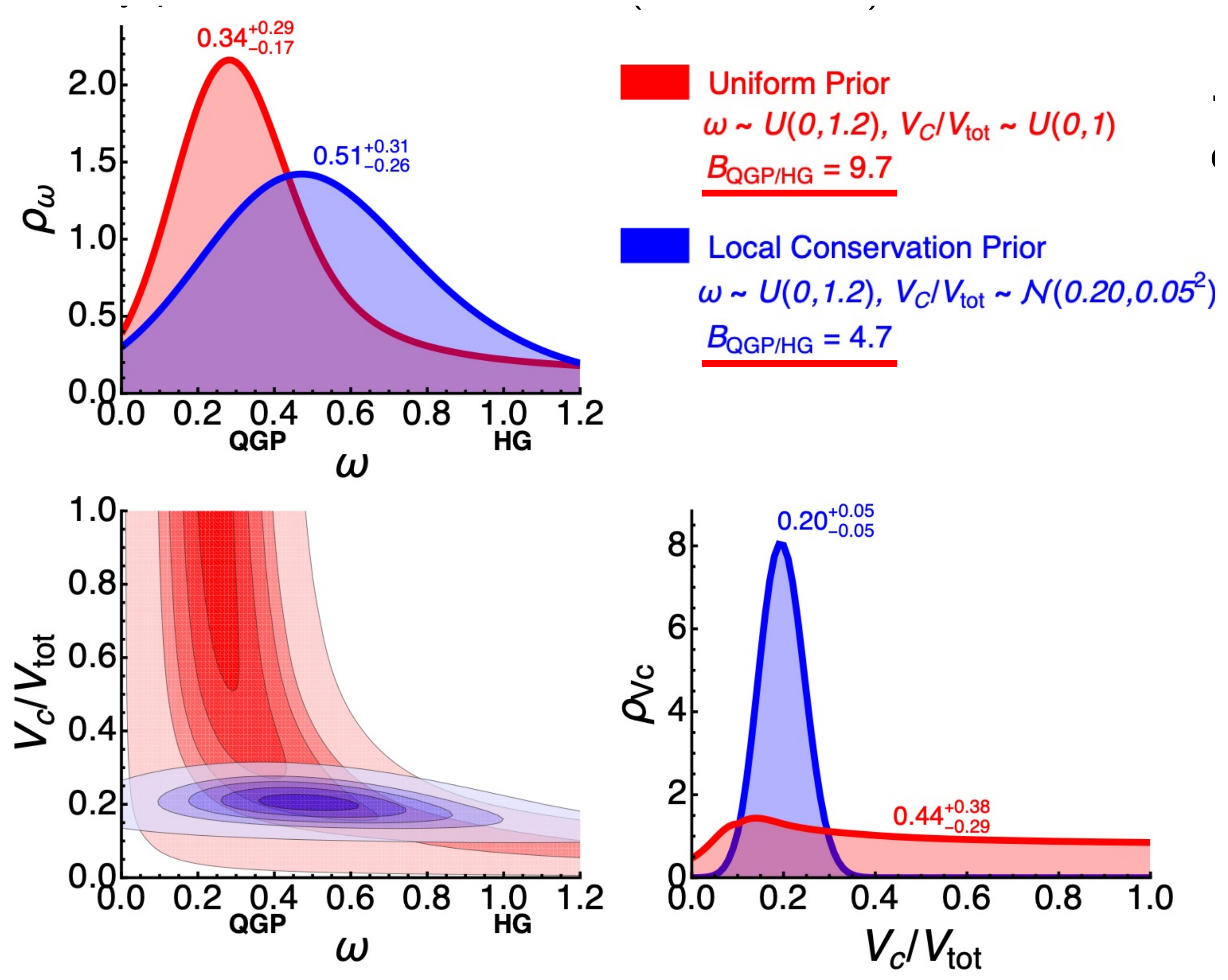
Update on Charge fluctuations

J. Parra et al, PRL 135 (2025)

$$D = 4\frac{\kappa_2(Q)}{\langle N_{ch} \rangle}$$

D_{corr} : simple corrections for charge conservation included





"Moderate" evidence for fractional charges

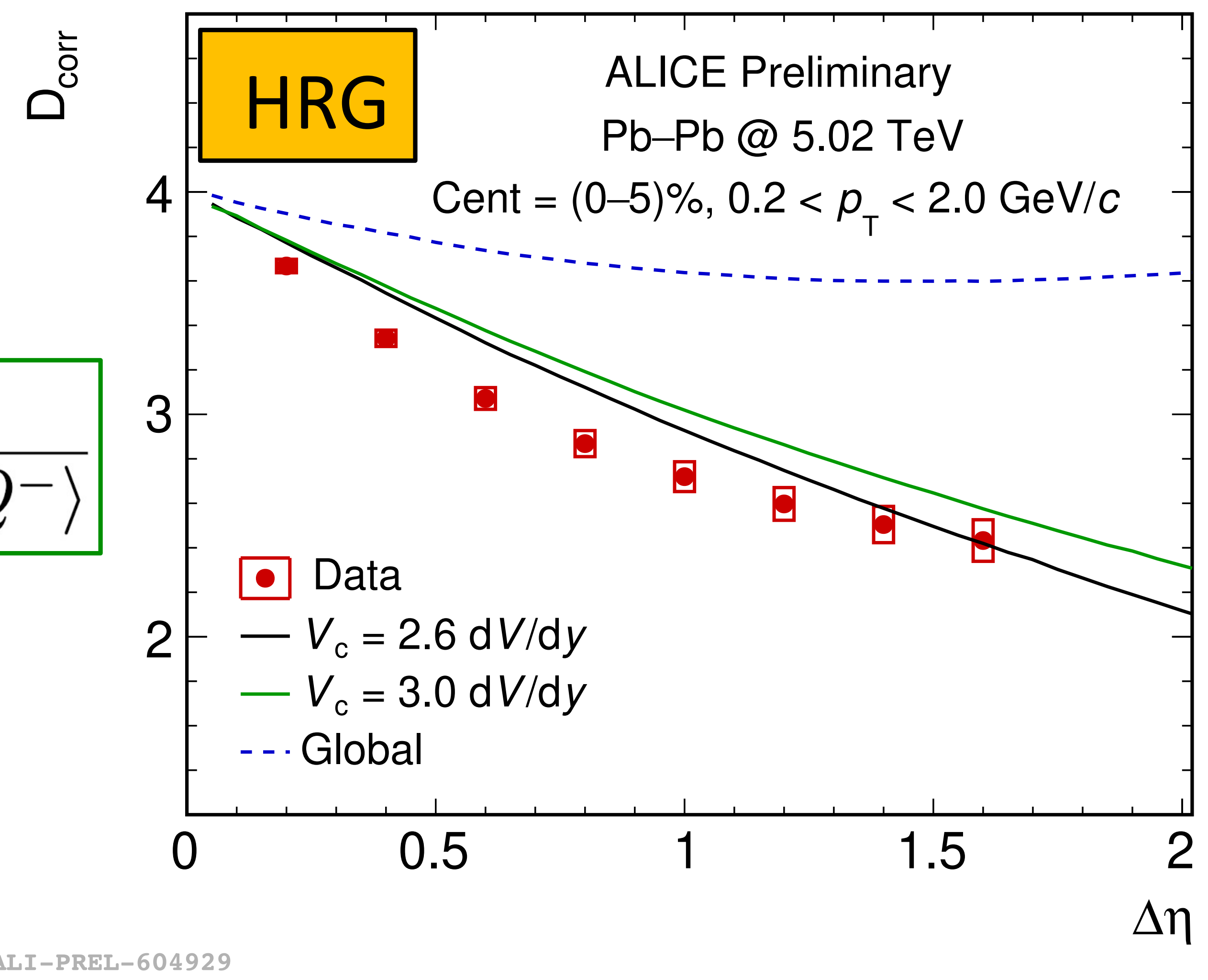
2^{nd} order net-Q: Subvolume vs Correlation length

ALICE: RUN 2

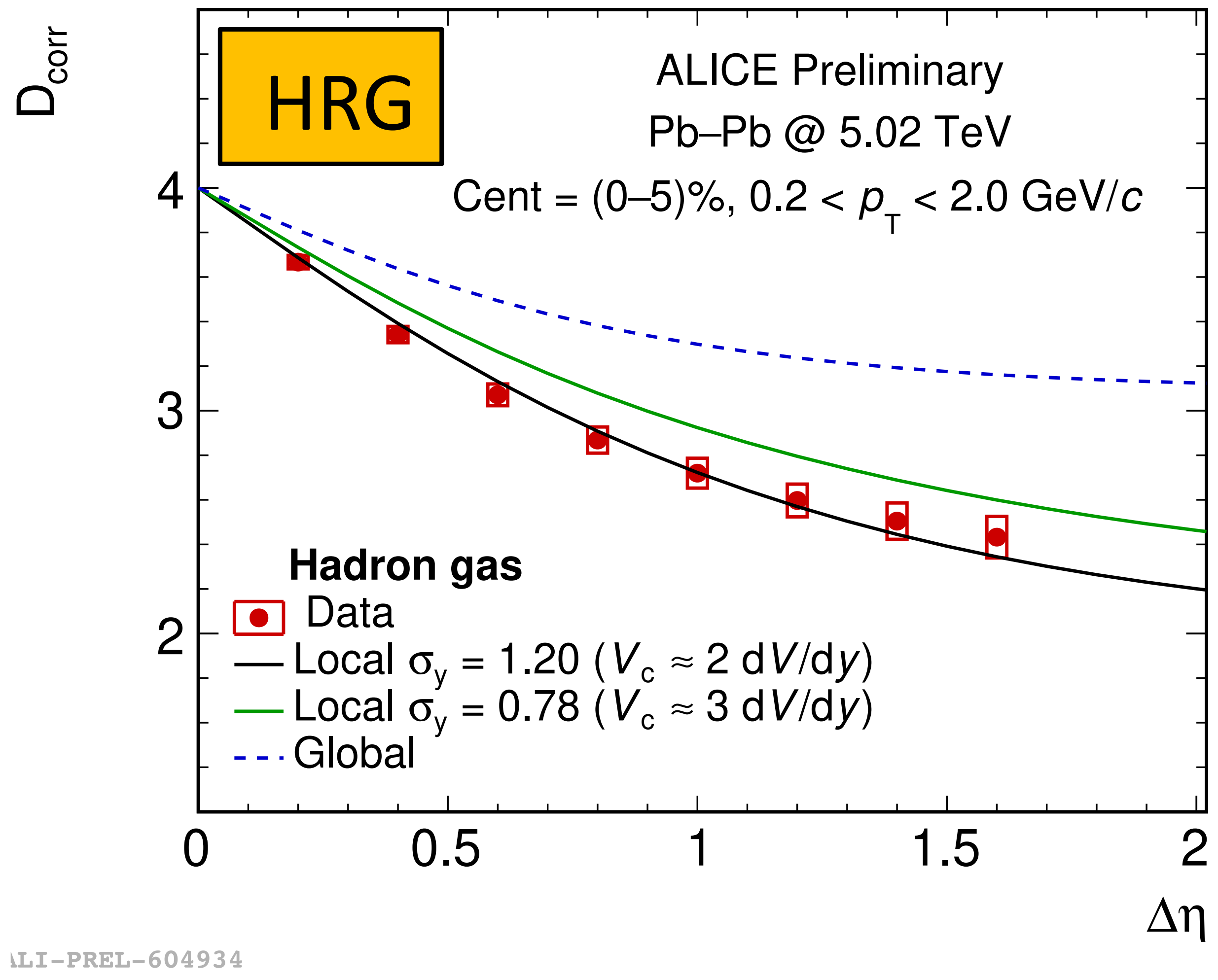
➢ **Question:** What is the right modeling of charge conservation?

$$D = 4\frac{\kappa_2[Q]}{\langle Q^+ + Q^- \rangle}$$

Subvolume approach

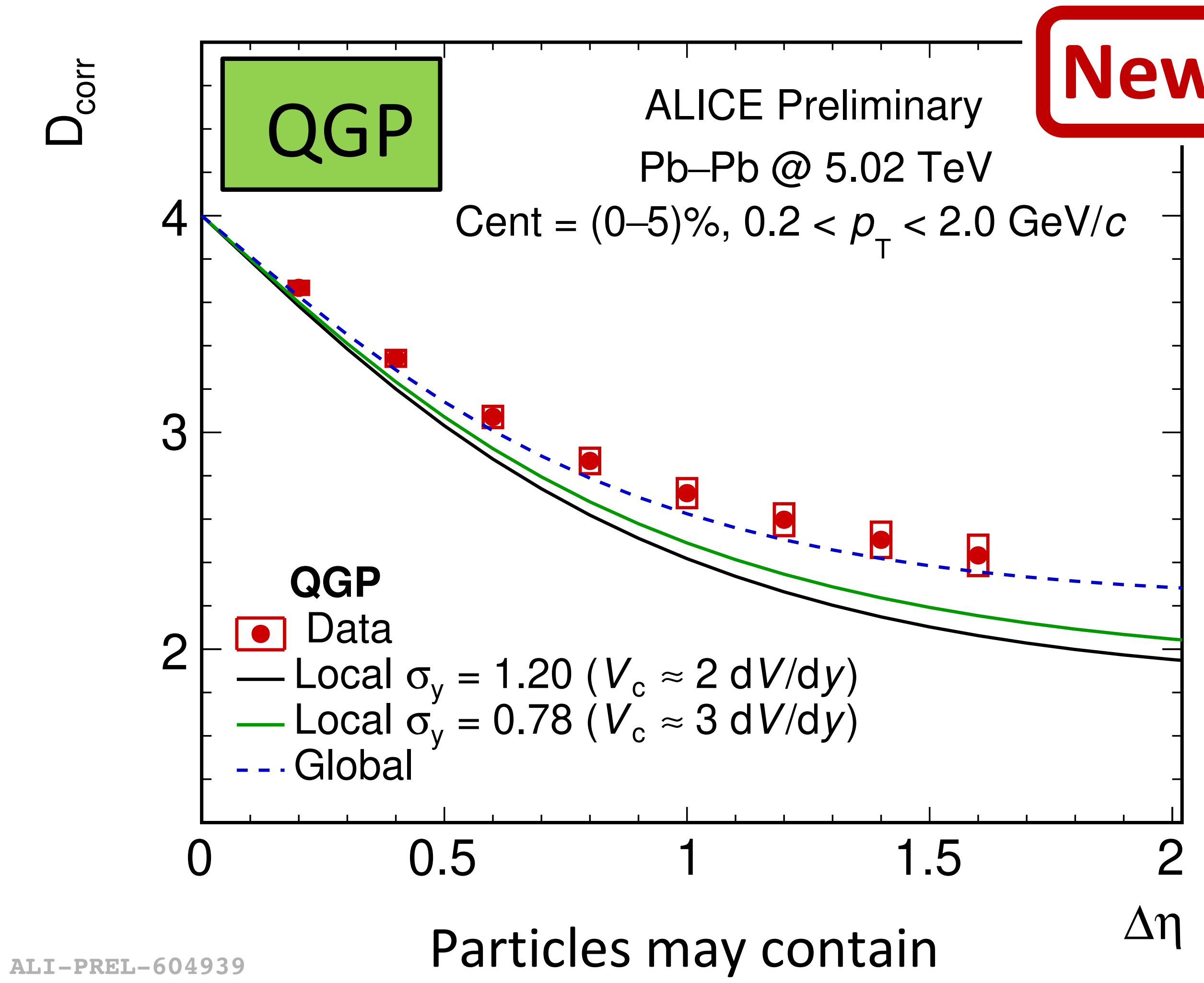


Gaussian correlation



Gaussian correlation

New



Particles may contain the memory of where the Q-fluctuations freeze out

$$D^{\mathrm{corr}} = \frac{D' + D''}{2}$$

$$D' = D + 4\langle p(\eta)\rangle, \quad D'' = \frac{D}{1 - \langle p(\eta)\rangle}$$

direct comparison with UNCORRECTED D-measure preferred

Back to Critical Point

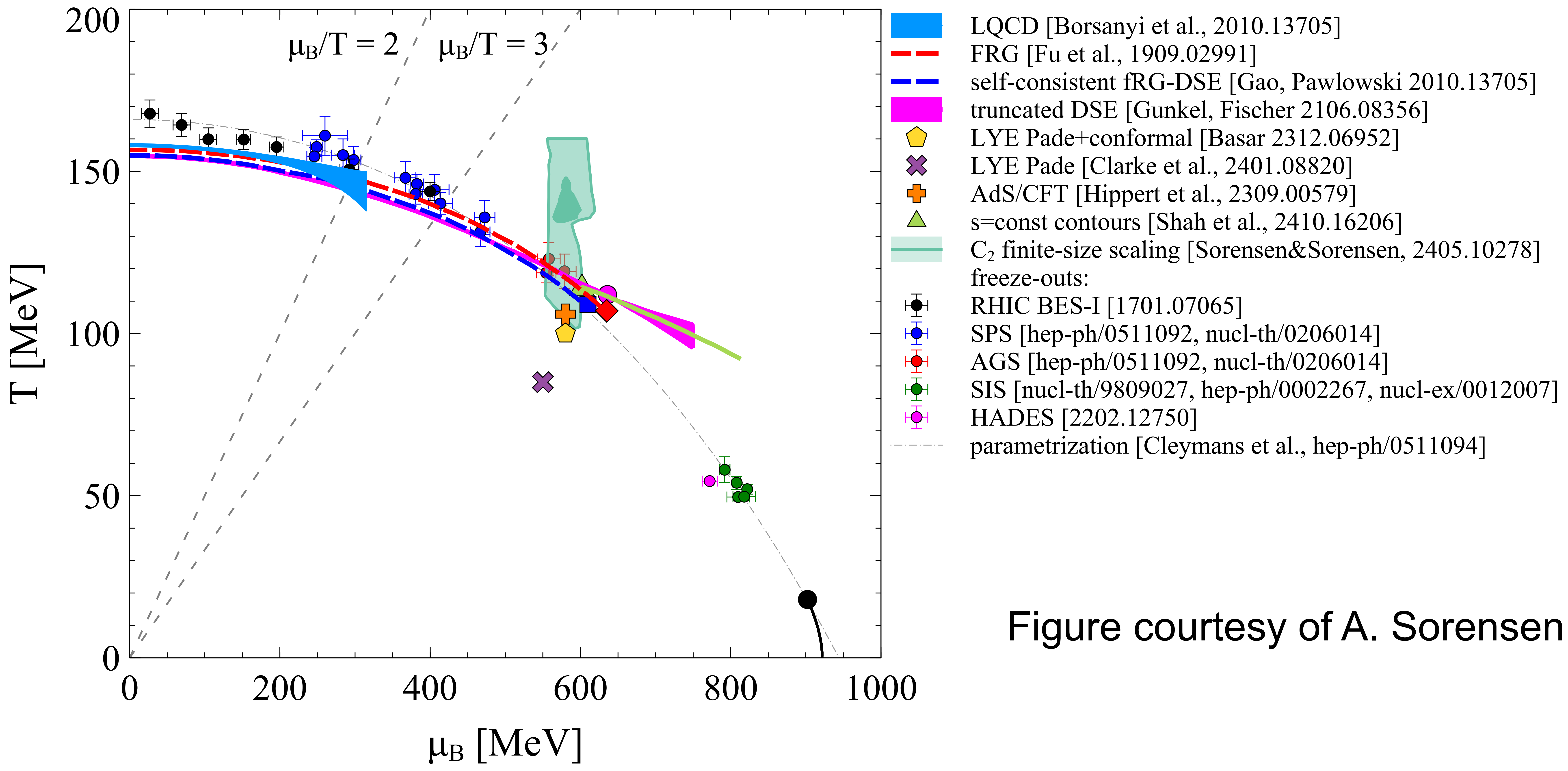
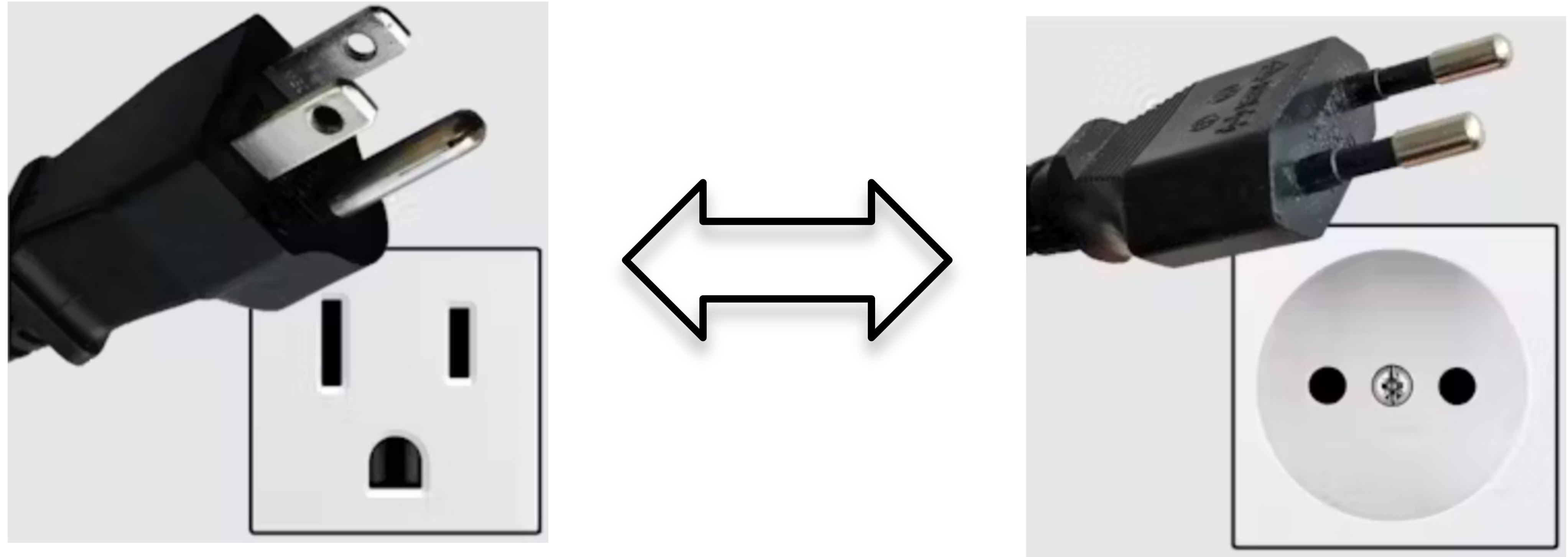


Figure courtesy of A. Sorensen

Conventions, conventions conventions

STAR
Cumulants (C)
Factorial cumulants (κ)

Others
Cumulants (κ), (K)
Factorial cumulants (C), (FC)



Arslandok, QM25

People love conventions…

That's why we have so many of them!!

Cumulants of (Baryon) Number

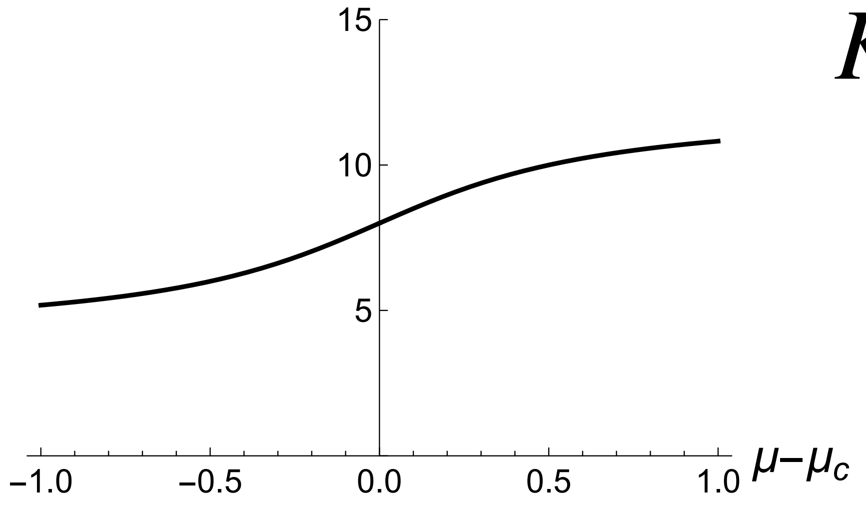
$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

$$K_1 = \langle N \rangle\,, \;\; K_2 = \langle N - \langle N \rangle \rangle^2\,, \;\; K_3 = \langle N - \langle N \rangle \rangle^3$$

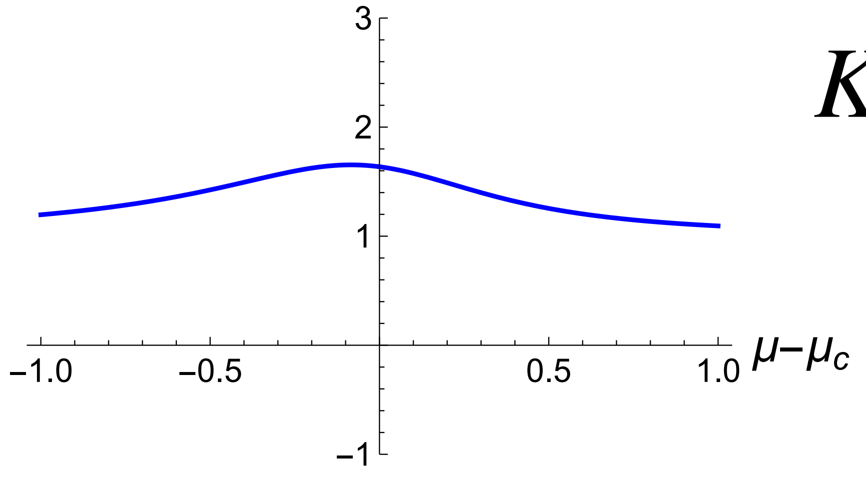
Cumulants scale with volume (extensive): $K_n \sim V$

Volume not well controlled in heavy ion collisions

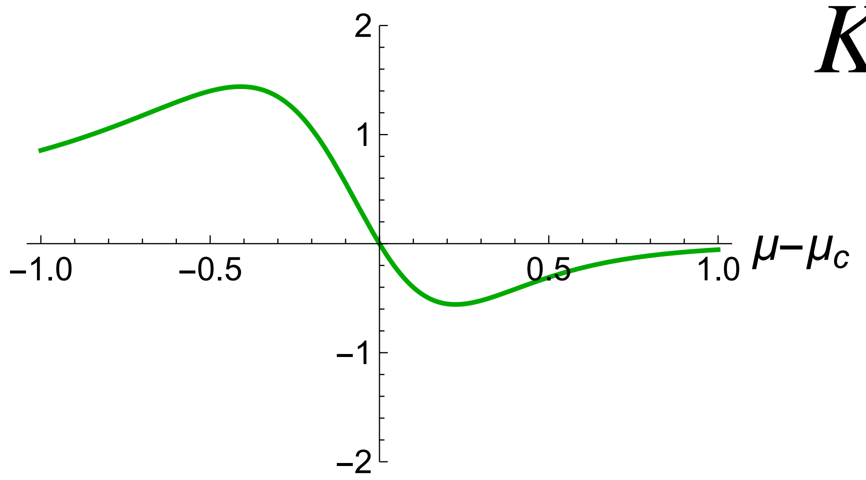
Cumulant Ratios: $\frac{K_2}{\langle N \rangle}, \;\; \frac{K_3}{K_2}, \;\; \frac{K_4}{K_2}$



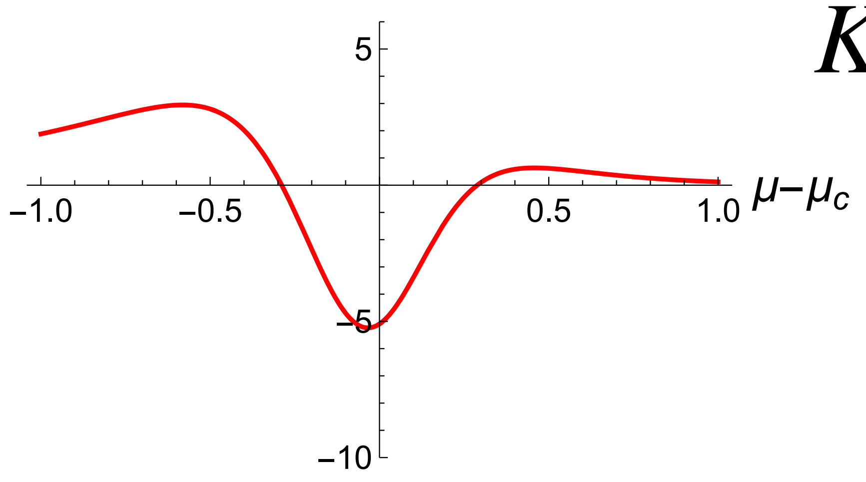
$K_1 = \langle N \rangle$



$K_2 = \frac{\partial}{\partial(\mu/T)} K_1$

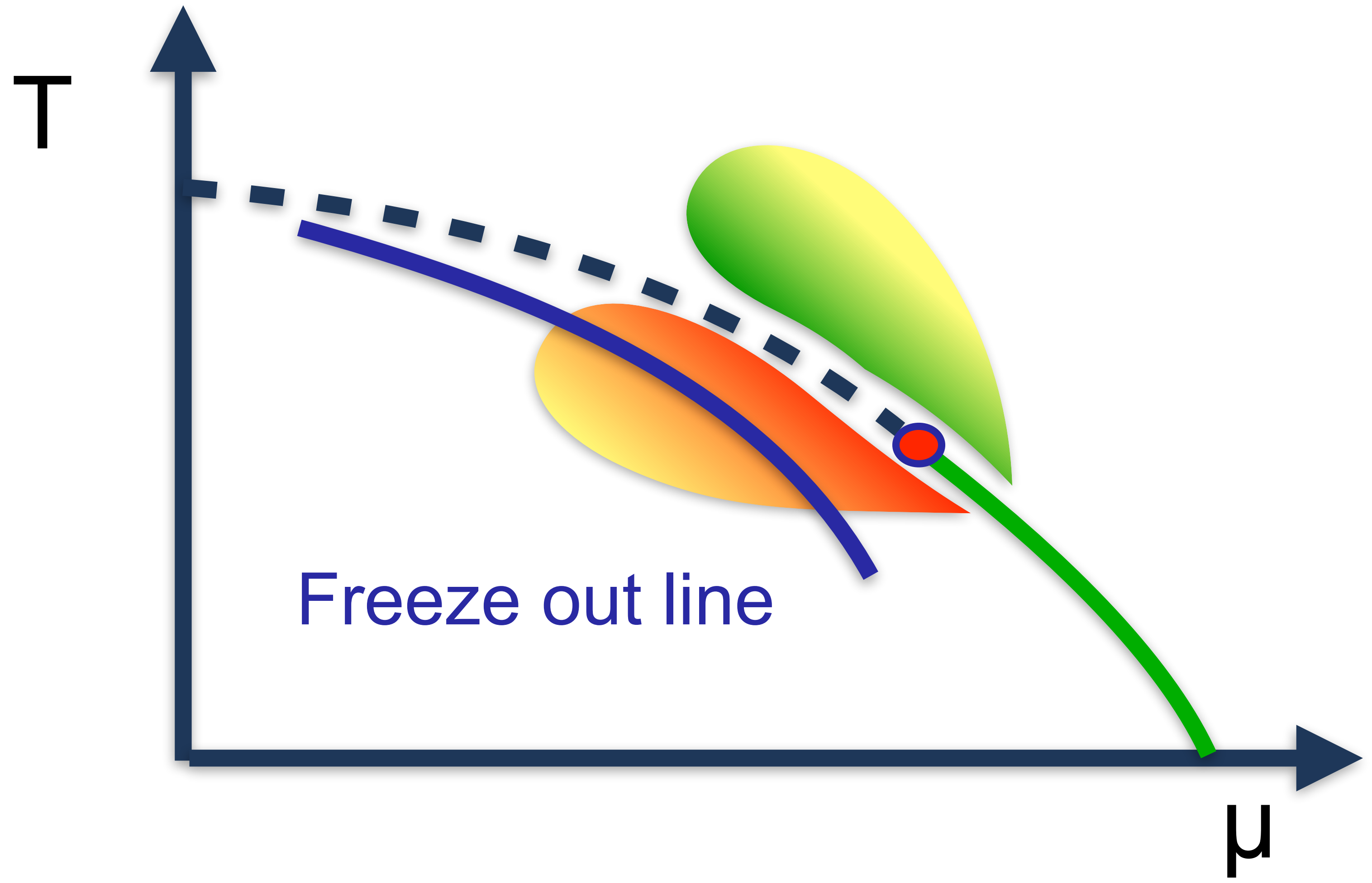


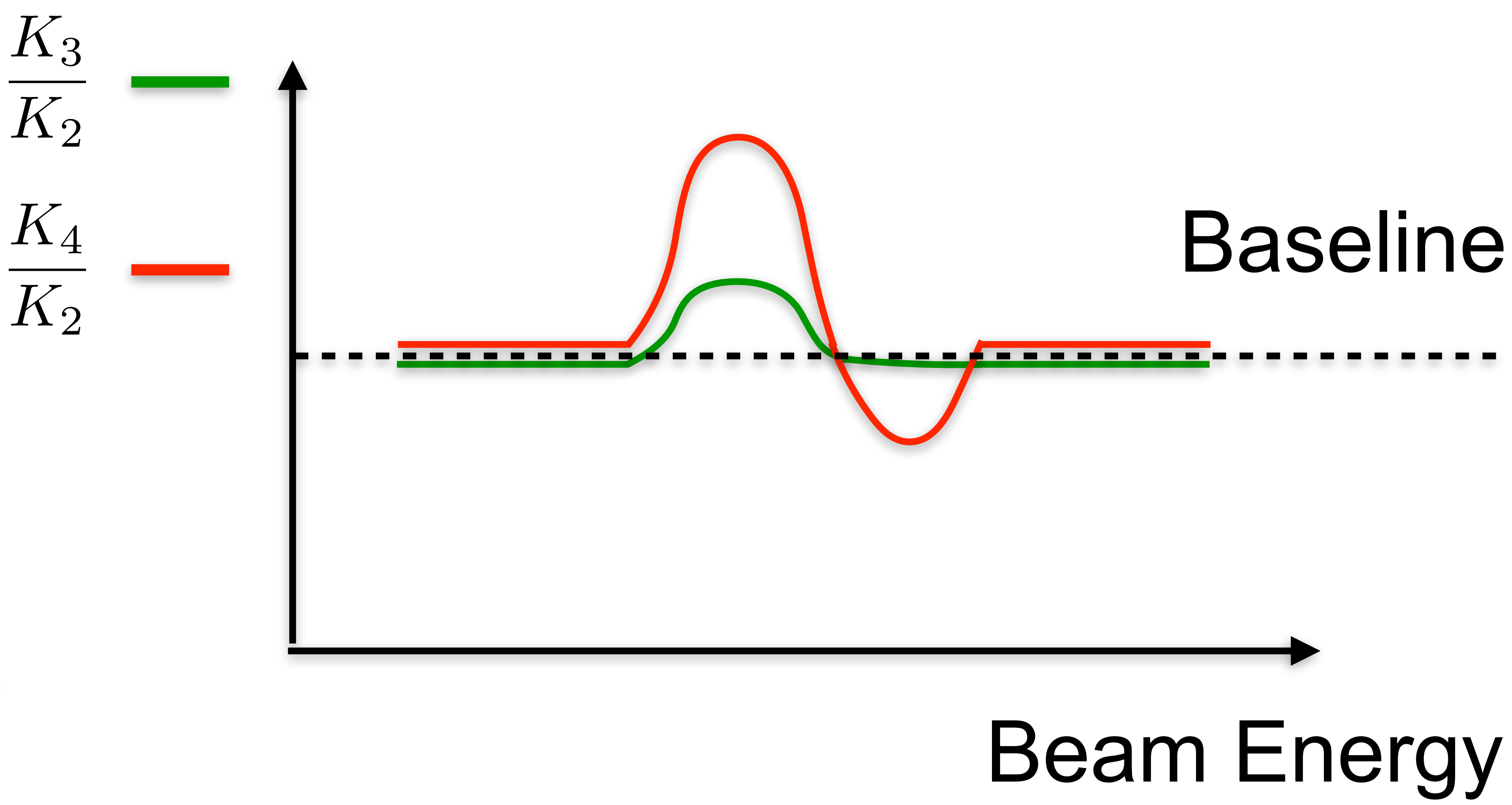
$K_3 = \frac{\partial}{\partial(\mu/T)} K_2$

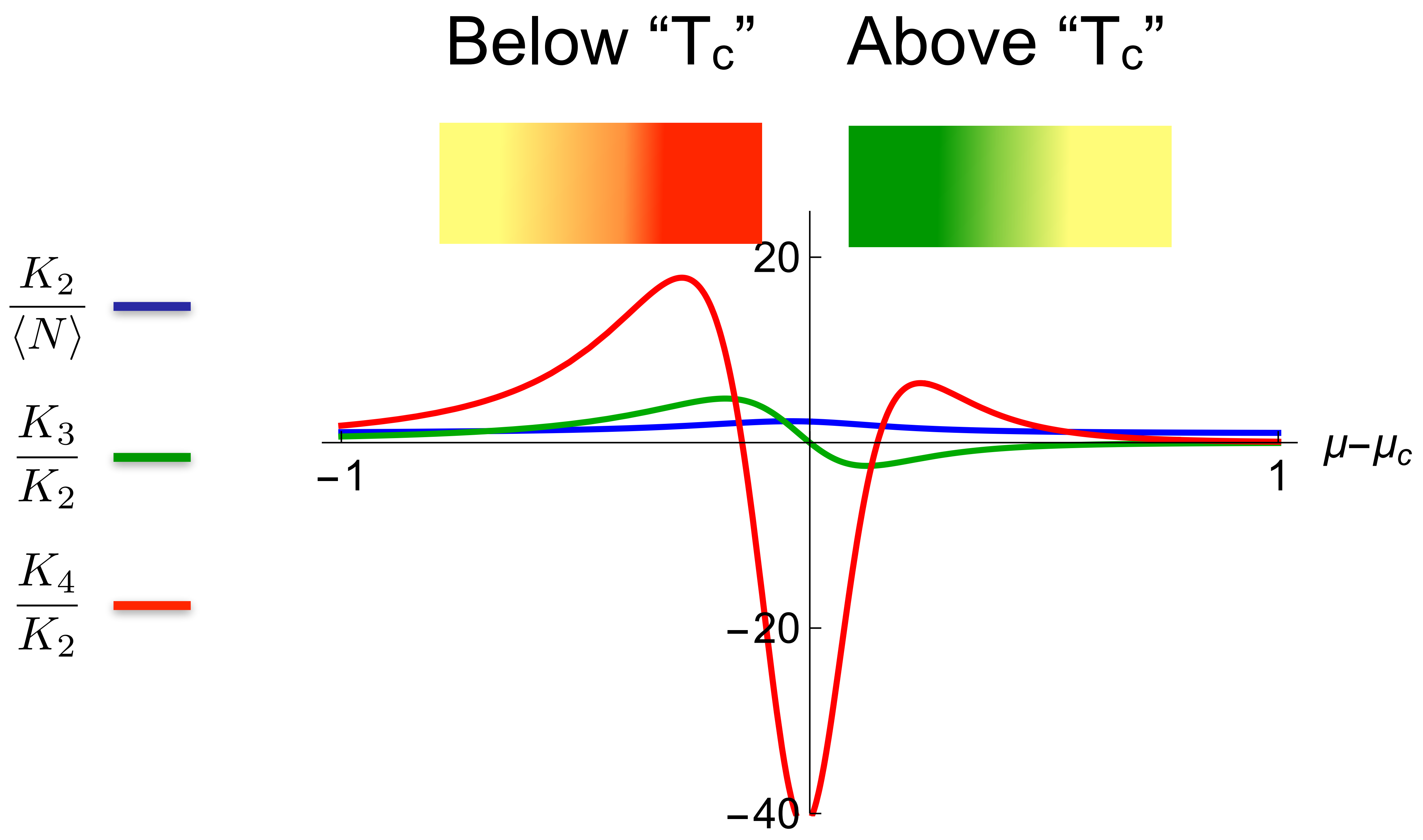


$K_4 = \frac{\partial}{\partial(\mu/T)} K_3$

What to expect?







Stephanov, arXiv:1104.1627

Cumulants and Factorial cumulants

Cumulants:

generating function: $g(t) = \ln\left[\sum_n P(n)e^{nt}\right]$

$$\kappa_n = \frac{\partial^n}{\partial t^n} g(t)\Big|_{t=0}$$

Gaussian: $\kappa_n = 0; \quad n \geq 2$

Factorial Cumulants (no anti-particles)**:**

generating function: $g_F(z) = \ln\left[\sum_n P(n)z^n\right]$

$$FC_n = \frac{\partial^n}{\partial z^n} g_F(z)\Big|_{z=1}$$

Poisson: $FC_n = 0; \quad n \geq 1$

Relations : $g(t) = g_F(e^t)$

$$FC_n = \sum_{k=1}^{n} s(n,k)\kappa_k; \qquad \kappa_n = \sum_{k=1}^{n} S(n,k) FC_k$$

$s(n,k)$; $S(n,k)$ Sterling numbers 1st and 2nd kind

For example: $FC_1 = \kappa_1 = \langle n \rangle; \quad FC_2 = \kappa_2 - \kappa_1; \quad FC_3 = \kappa_3 - 3\kappa_2 + 2\kappa_1;$

Same singularity structure!

Factorial cumulants and correlation functions

$$\rho_1(p) = \frac{dN}{dp}; \quad \rho_2(p_1, p_2) = \frac{d^2N}{dp_1 dp_2}; \ \cdots$$

Two particle density: $\rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + C_2(p_1, p_2)$

Three particle density:
$$\rho_3(p_1, p_2, p_3) = \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) + \rho_1(p_1)C_2(p_2, p_3) + \rho_1(p_2)C_2(p_1, p_3)$$
$$+\rho_1(p_3)C_2(p_1, p_2) + C_3(p_1, p_2, p_3)$$

$C_n(p_1, \cdots, p_n)$ n-particle genuine correlations functions

Factorial cumulants are integrals over correlation functions: $FC_n = \int_{\mathrm{acceptance}} dp_1 \cdots dp_n \, C(p_1, \cdots, p_n)$

Poisson: $FC_n = 0; \quad n \geq 1;$ in contrast with $\kappa_n = \langle n \rangle$

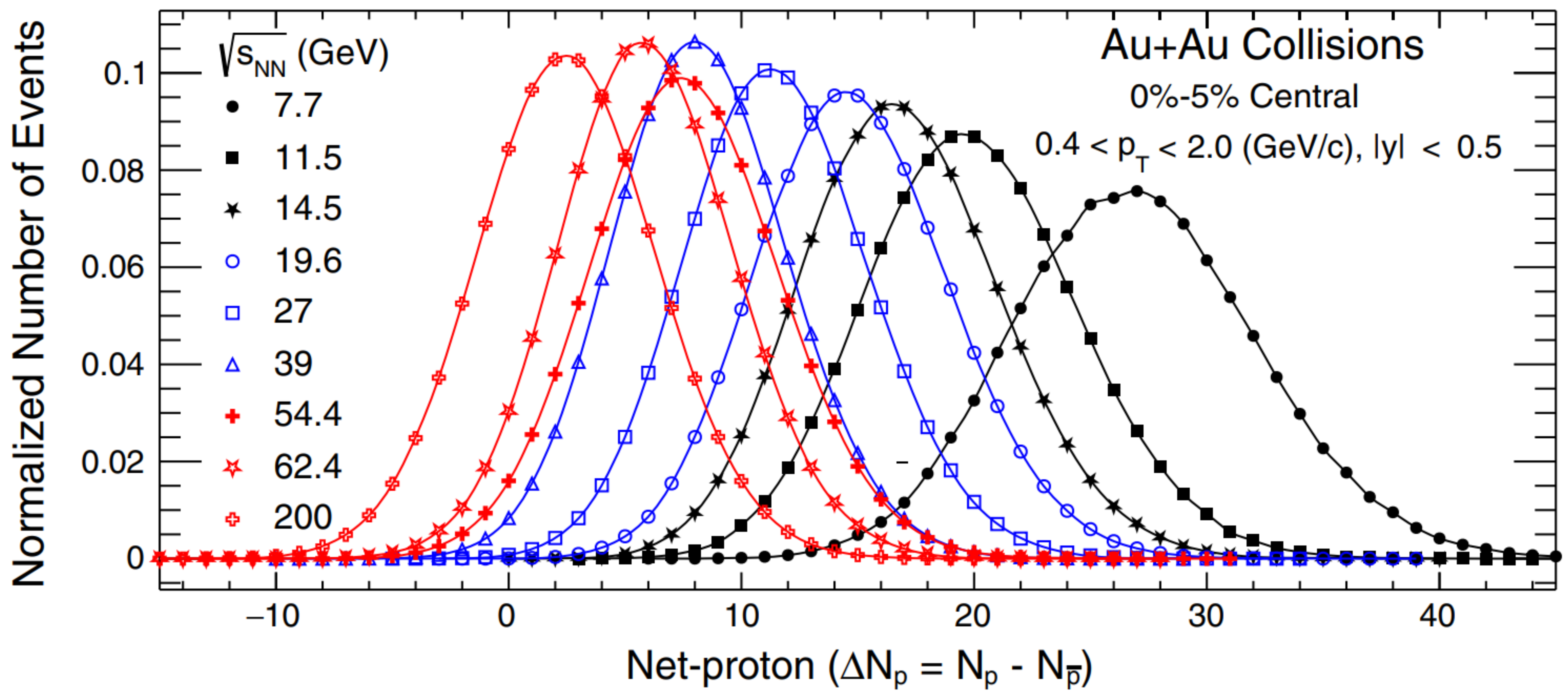
Measuring cumulants (derivatives)

$$K_2 = \langle N - \langle N \rangle \rangle^2 = \sum_N P(N)(N - \langle N \rangle)^2$$

$$K_3 = \langle N - \langle N \rangle \rangle^3 = \sum_N P(N)(N - \langle N \rangle)^3$$

$$P(N) = \frac{N_{events}(N)}{N_{events}(total)}$$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)



Compare Data with Lattice QCD and other field theoretical models

Experiment

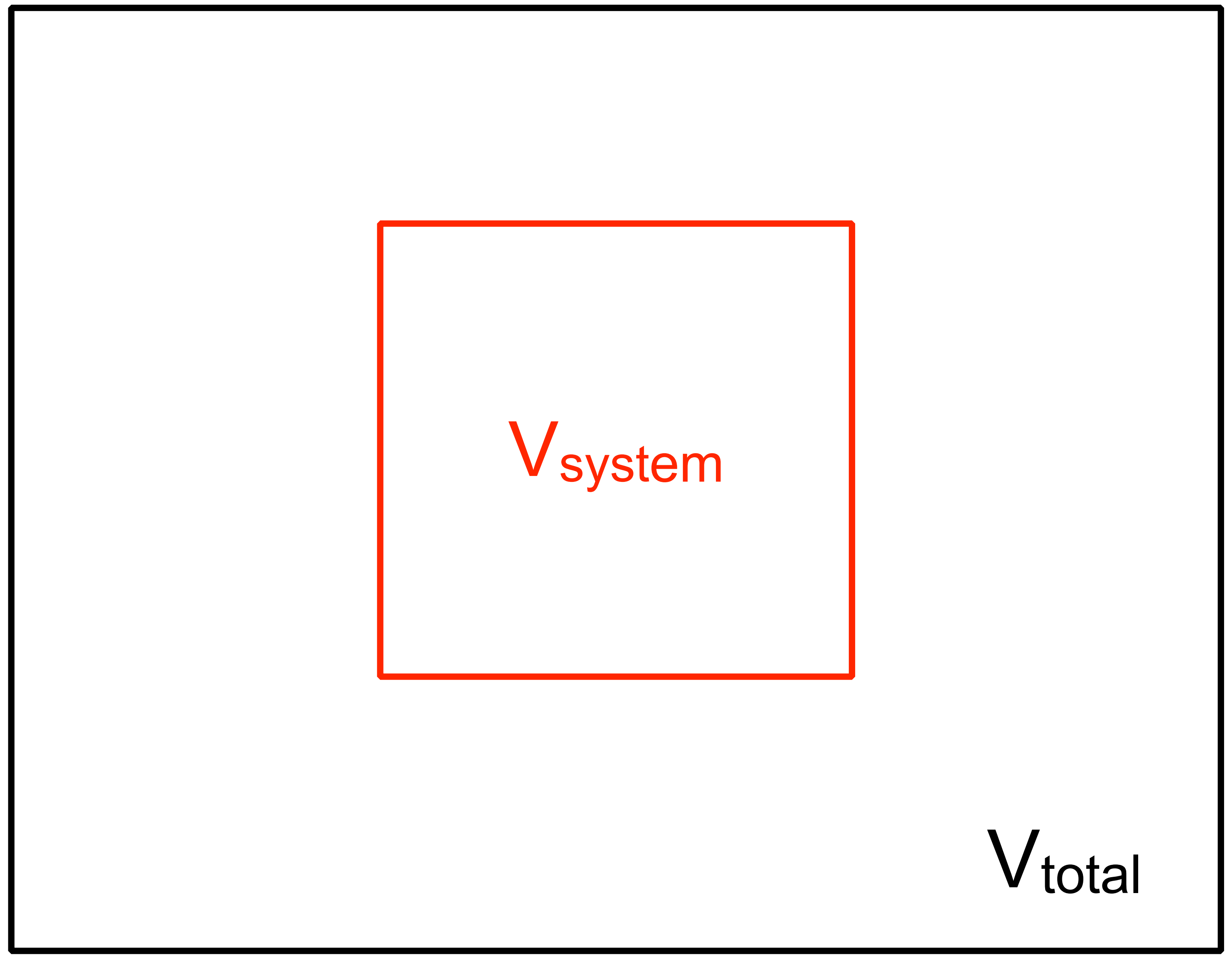
- Baryon number conserved globally
  - Solution (V. Vovchenko et al, arXiv 2003.13905, arXiv:2007.03850)
- Experiment measures protons only
- Volume is not fixed in experiment
  - Possible solution (Rustamov et al, 2211.14849, Holzmann et al, 2403.03598)
- Momentum cuts
- expansion, time evolution
- Detector fluctuates (efficiency etc…)

Lattice, FRG etc

- Baryon number (and other charges) conserved only on average (grand canonical ensemble)
- "measures" ALL baryons
- Volume is fixed
- Includes all momenta
- static system, no expansion

Need a dynamical model

Grand canonical ensemble



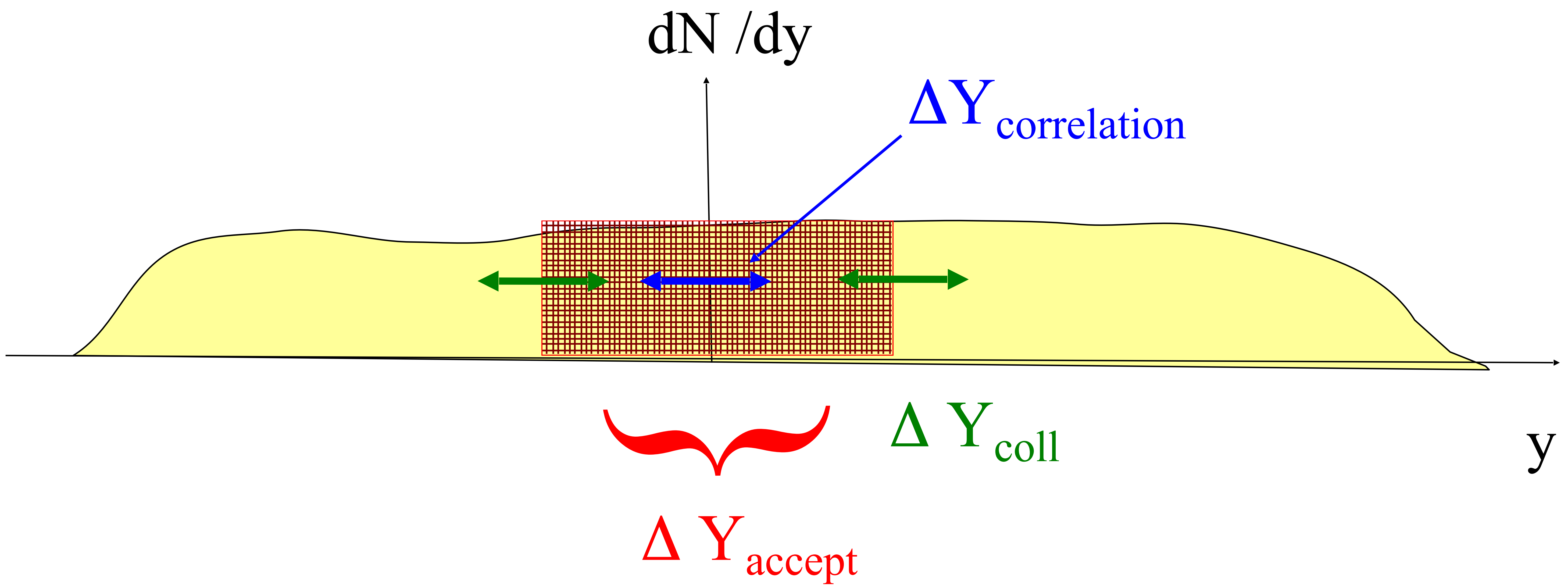
$$V_{total} \to \infty$$
$$V_{system} \to \infty$$

$$\frac{V_{system}}{V_{total}} \to 0$$

In coordinate space!!!!

For finite V_{total} there are corrections $\sim \dfrac{V_{system}}{V_{total}}$

How to make a grand-canonical ensemble in experiment



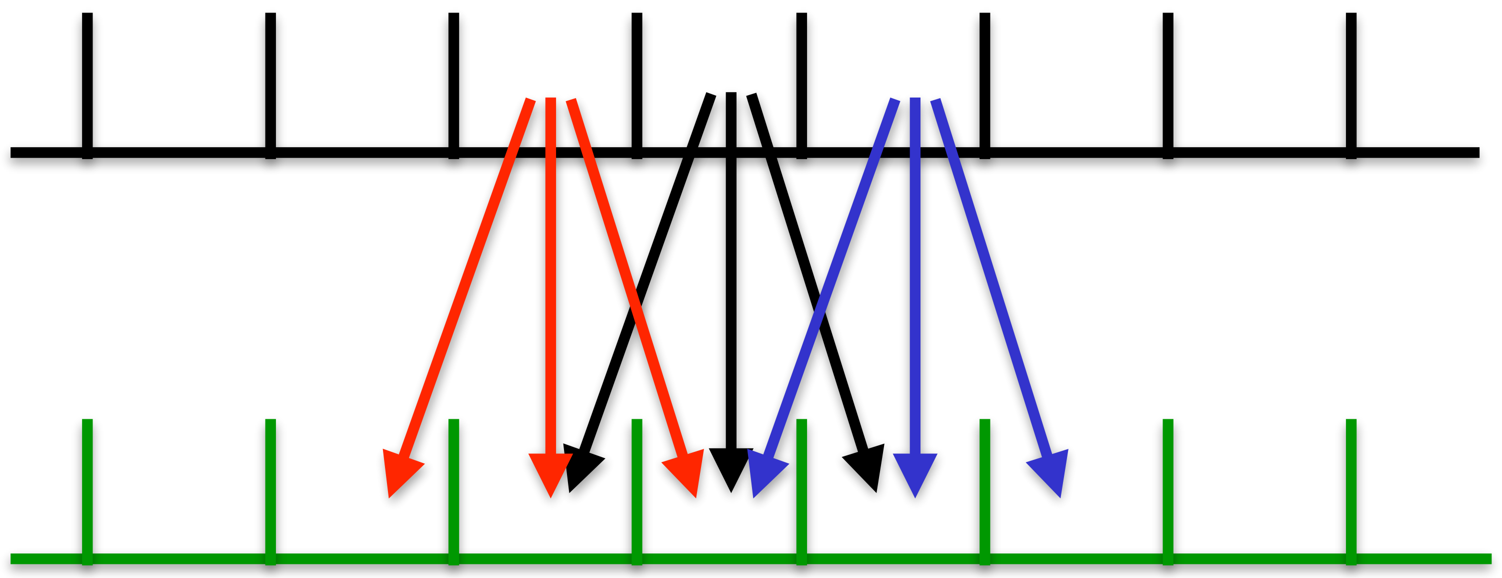
Conditions for "charge" fluctuations:

- $\Delta Y_{corrrelation} \ll \Delta Y_{accept}$ **(catch the physics)**
- $\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{coll}$ **(keep the physics and minimize charge conservation effect)**

Cumulant ratios in "experiment" (μ_B~0)

(without volume fluctuations)

Thermal smearing

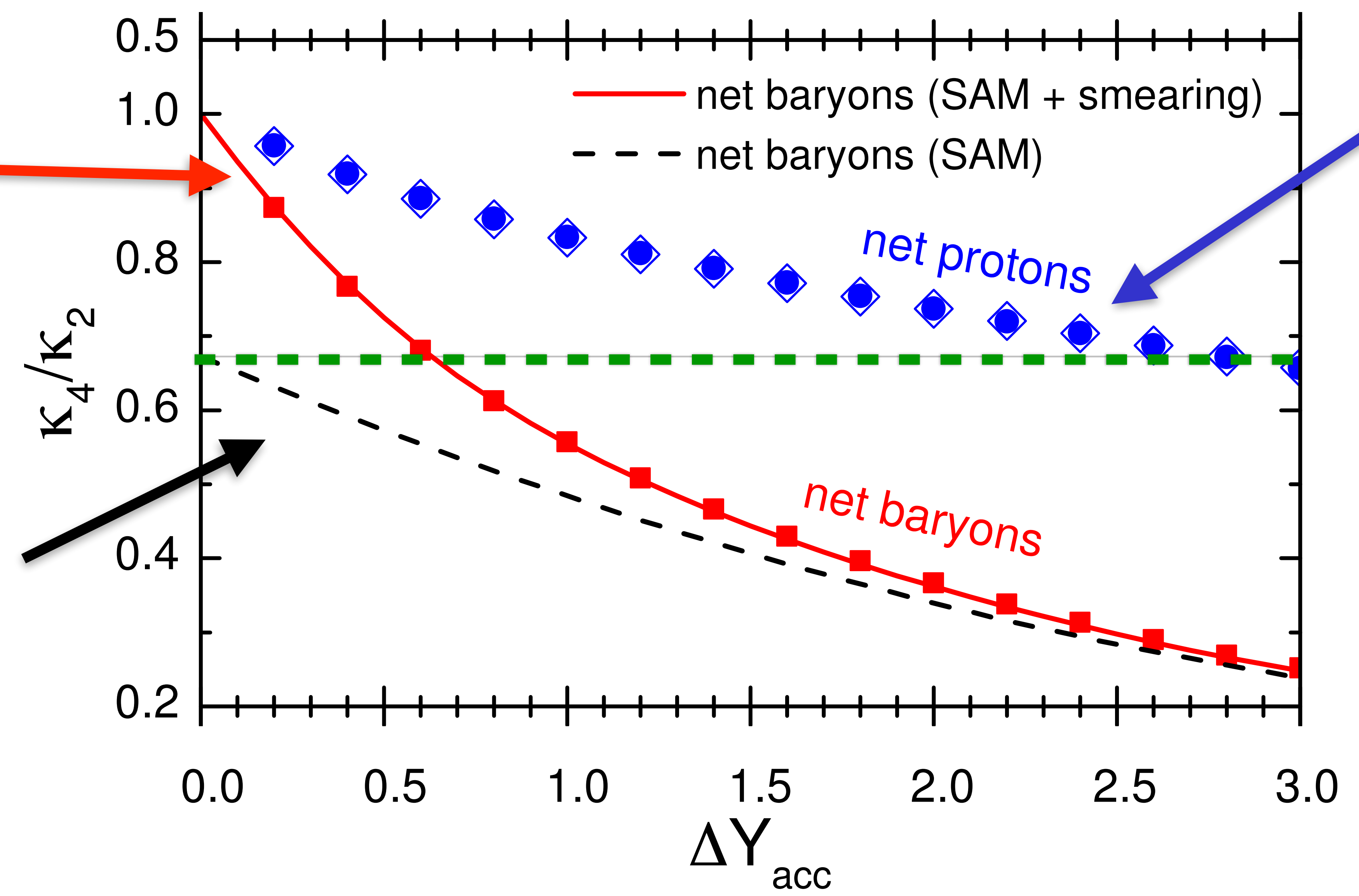


η, spacial rapidity

Y, momentum space rapidity

Lattice + baryon conservation + thermal smearing

Lattice + baryon conservation



What is REALLY measured

- net baryons
- net protons
- net protons (Kitazawa-Asakawa)

Lattice result

Cumulants Lattice vs Experiment
Obvious conclusion!

- Cumulants from Lattice and cumulants from measured (net)-protons SHOULD NOT agree if they study the SAME system

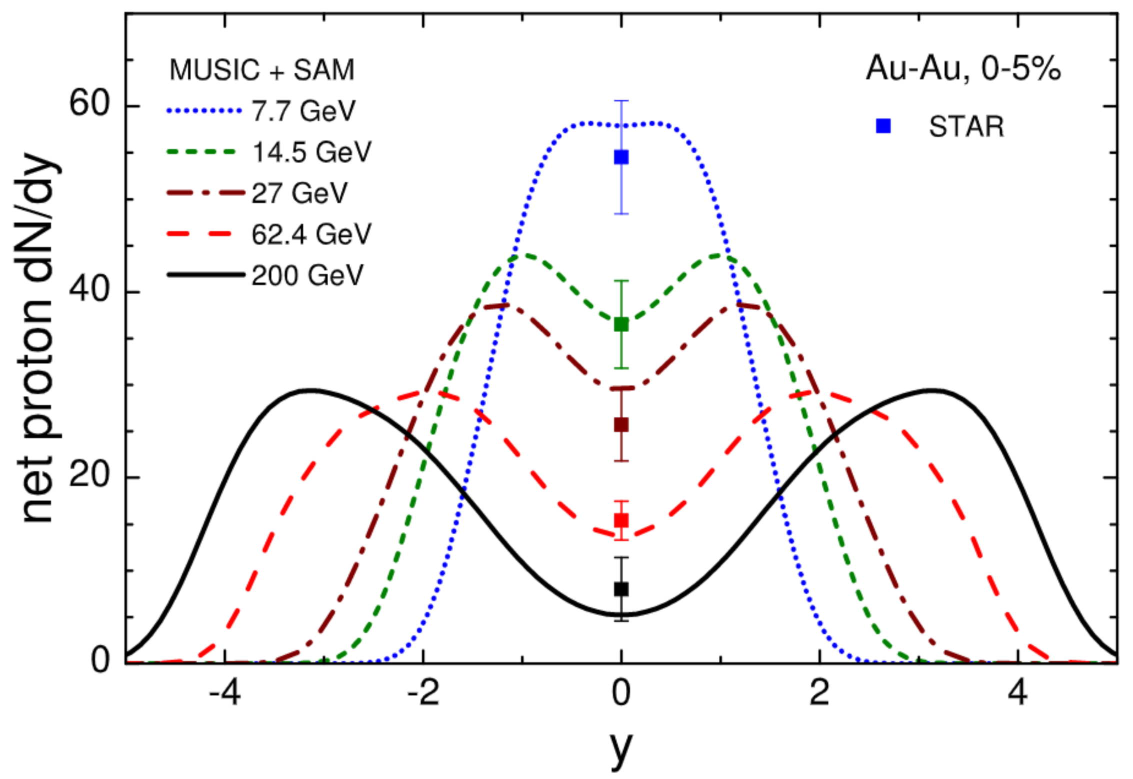
- IF they agree: Experiment and Lattice study DIFFERENT systems!

- Exception: Both Lattice and experiment find Poisson statistics, i.e. system without any correlation a.k.a a boring system. In this case we simply cannot tell

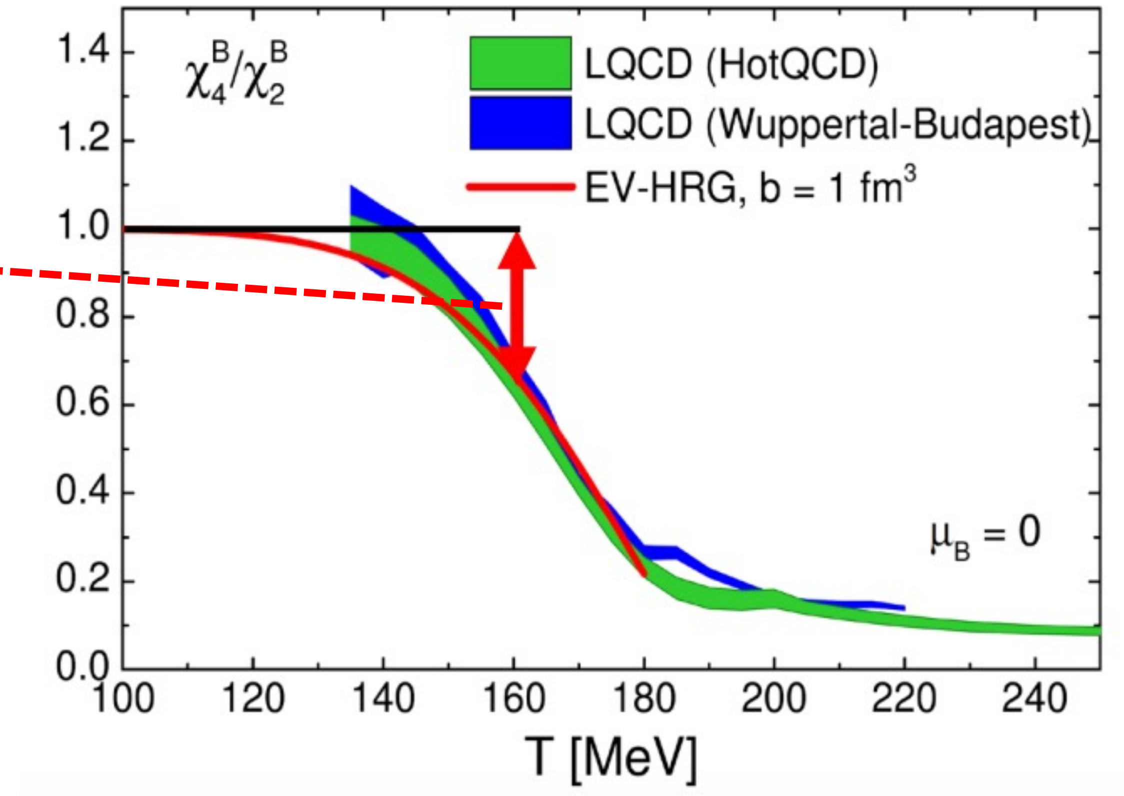
Calculation of non-critical contributions at RHIC-BES

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]
  - Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3



- Non-critical contributions are computed at particlization
  - QCD-like baryon number distribution (χ_n^B) via **excluded volume** b = 1 fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - **Exact global baryon conservation*** (and other charges)
    - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
    - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)] https://github.com/vlvovch/fist-sampler



- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro
Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

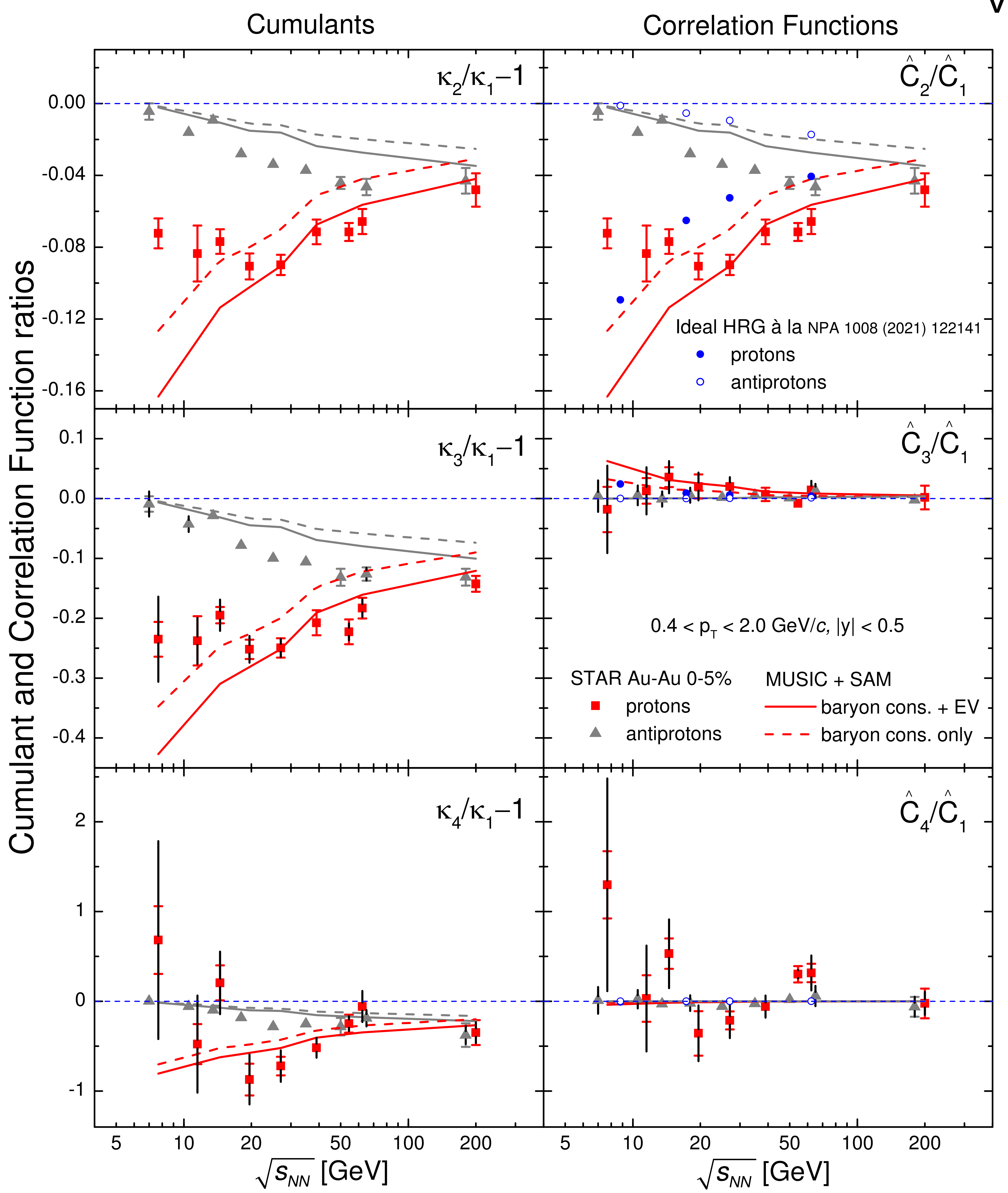
Results (Prediction) for proton cumulants

Vovchenko, Shen, VK, 2107.00163

- Viscous hydro
- EOS tuned to LQCD
- Correct for global charge conservation
- Protons NOT baryons

- Baseline!
  No critical point or phase transition

- No volume fluctuations



STAR BESI data

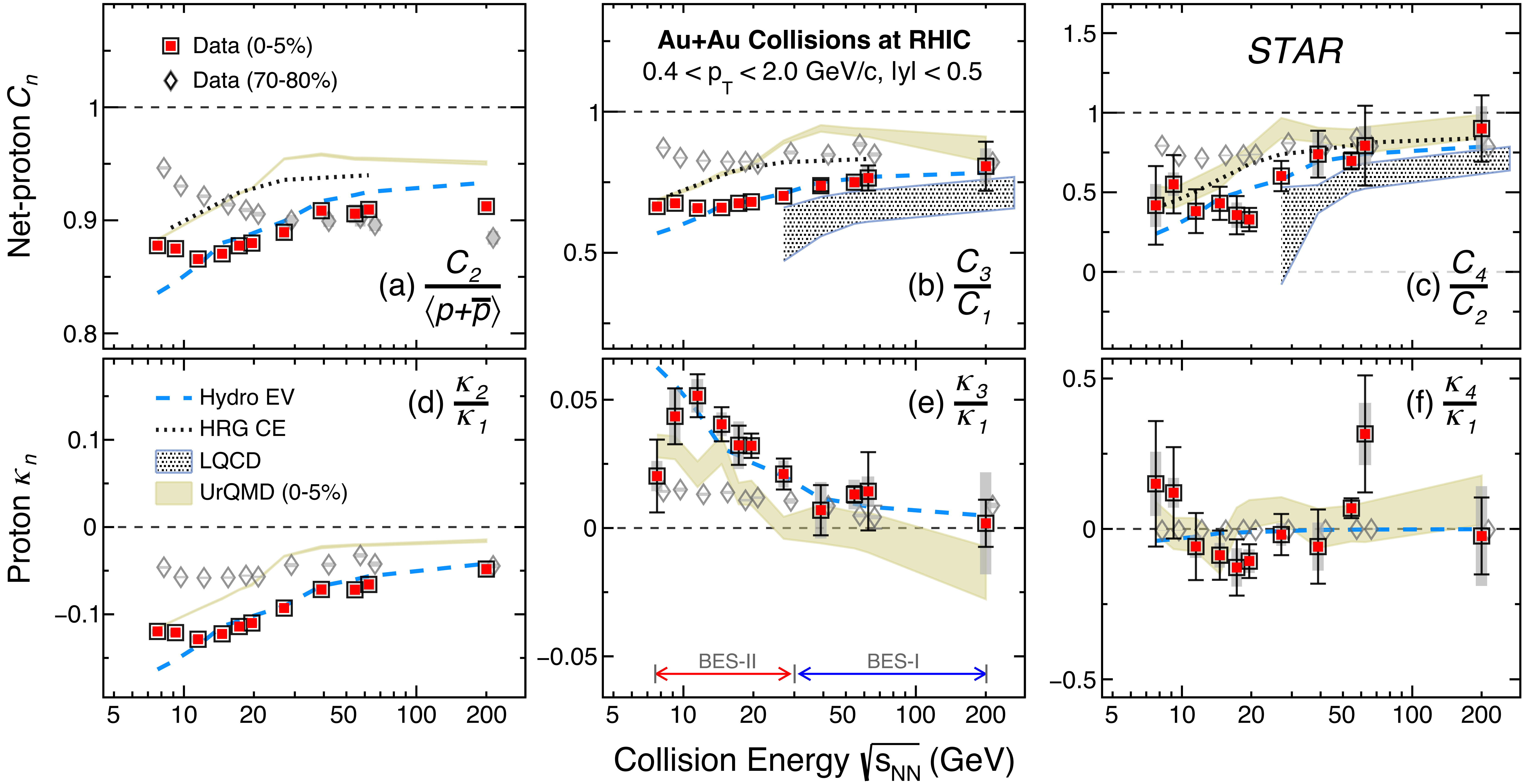
See also: Braun-Munzinger et al, NPA 1008 (2021) 122141

STAR BESII data

arXiv:2504.00817

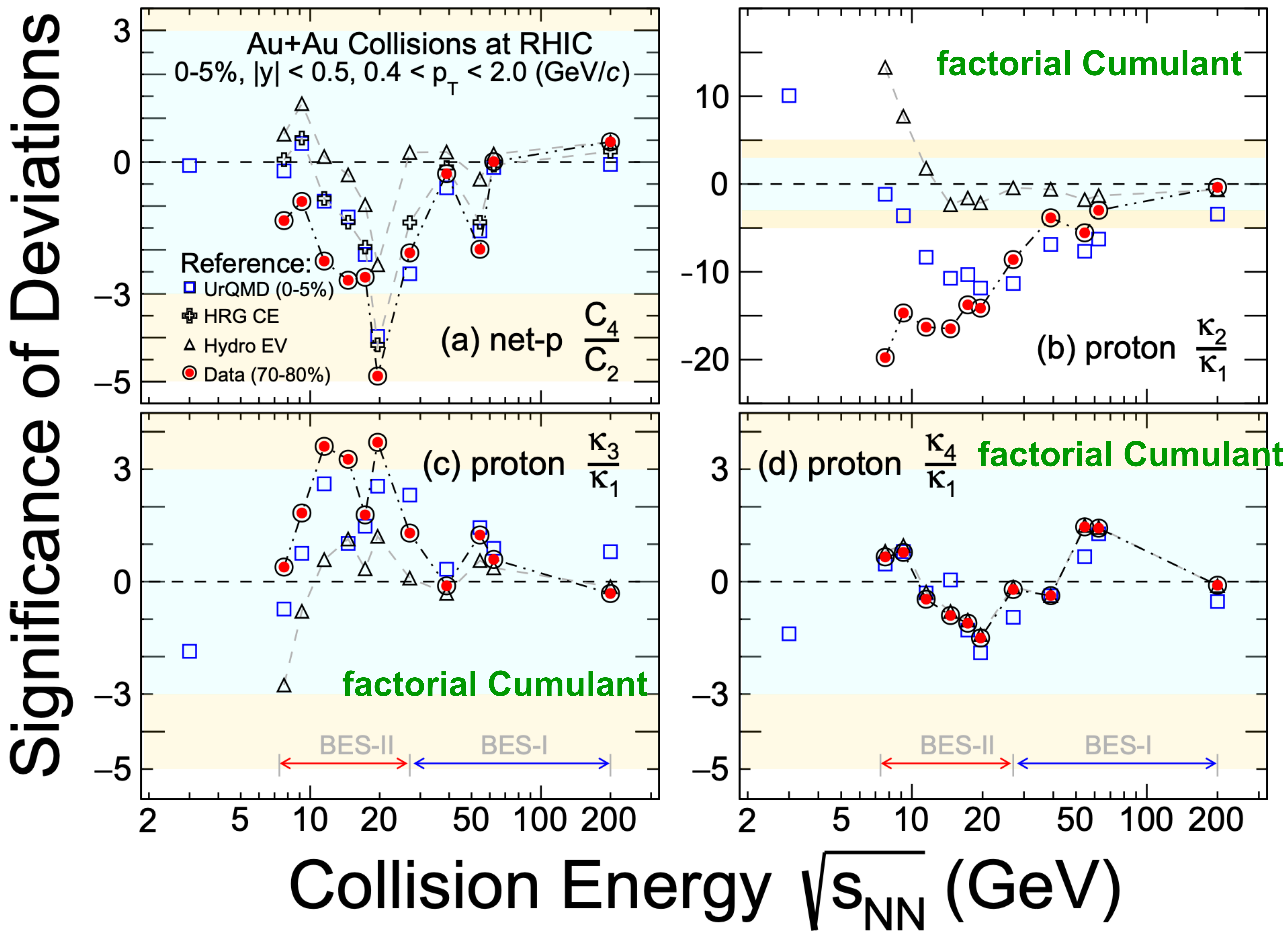
Cumulants

Factorial Cumulants (FC)



Significance

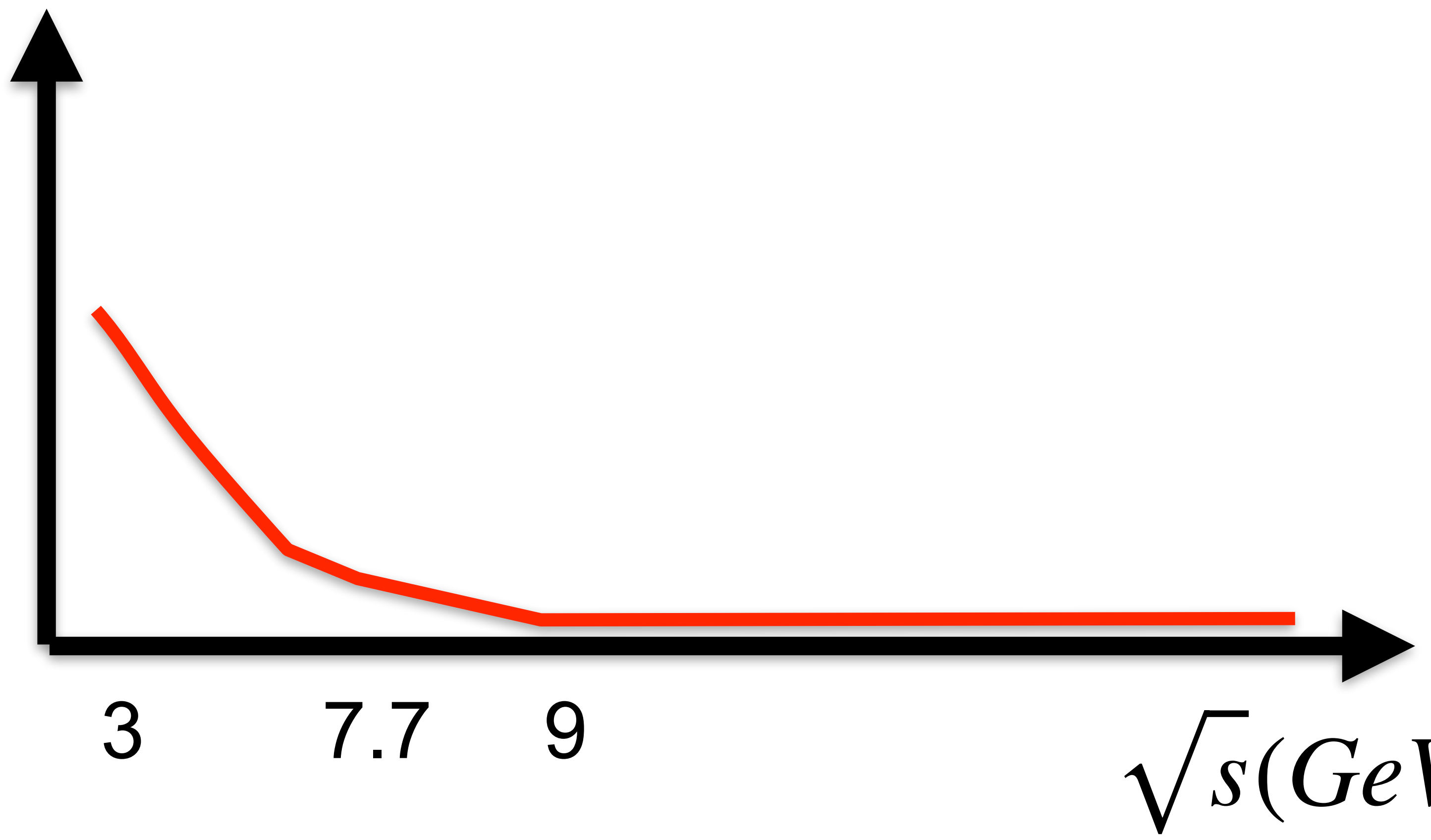
X. Dong, CPOD 2026



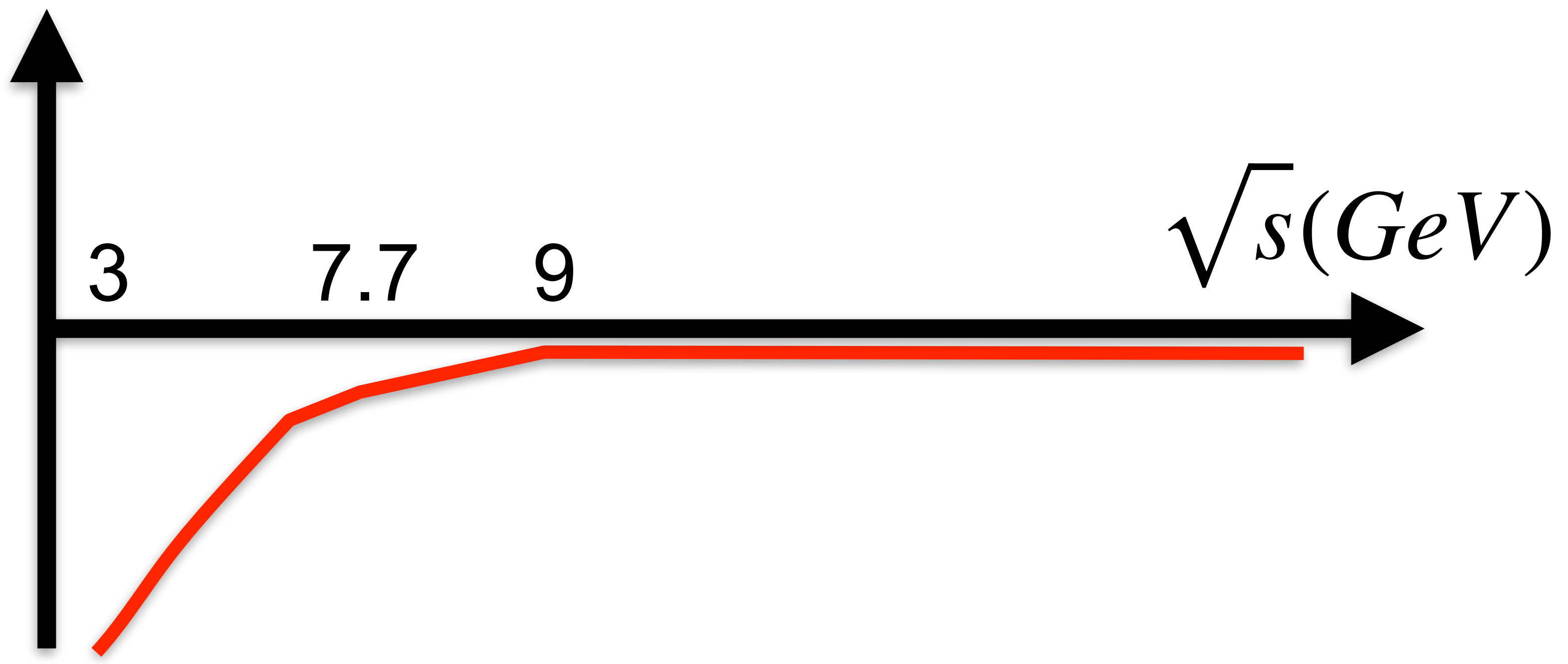
STAR, PRL 135 (2025) 142305

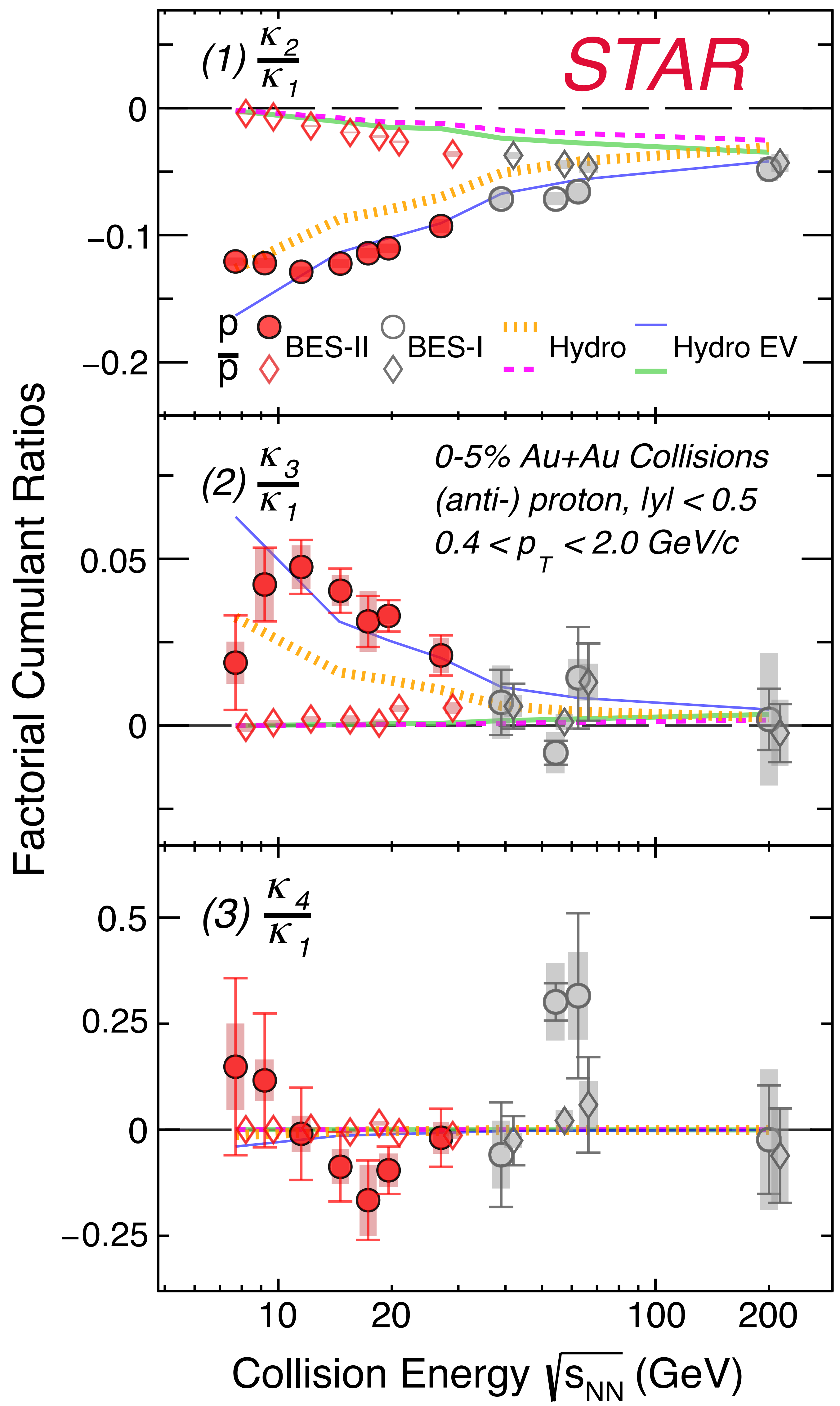
The "signal" (relative to baseline)

$\frac{FC_2}{FC_1}$ - baseline



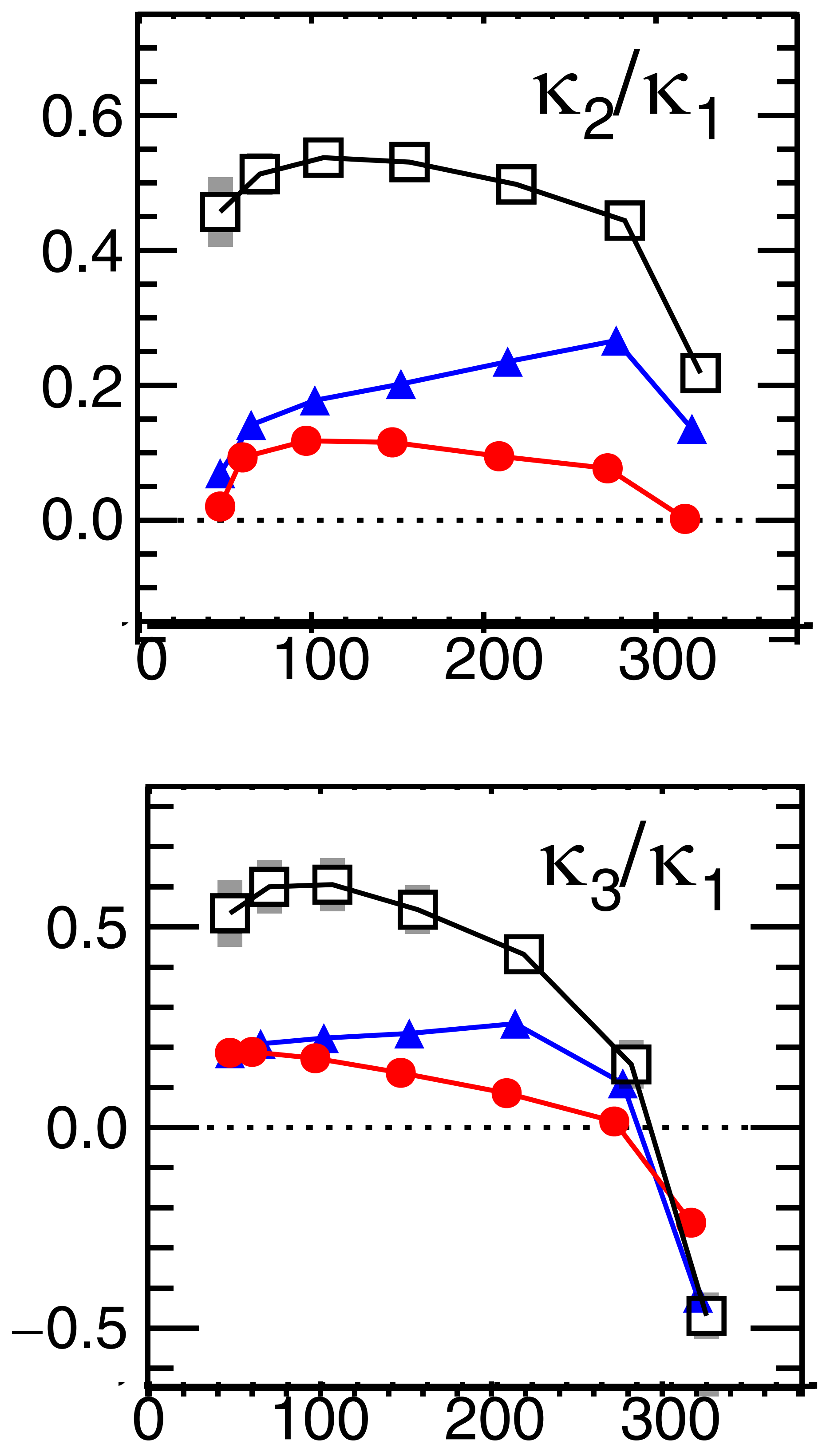
$\frac{FC_3}{FC_1}$ - baseline





STAR 2209.11940

$\sqrt{s} = 3GeV$

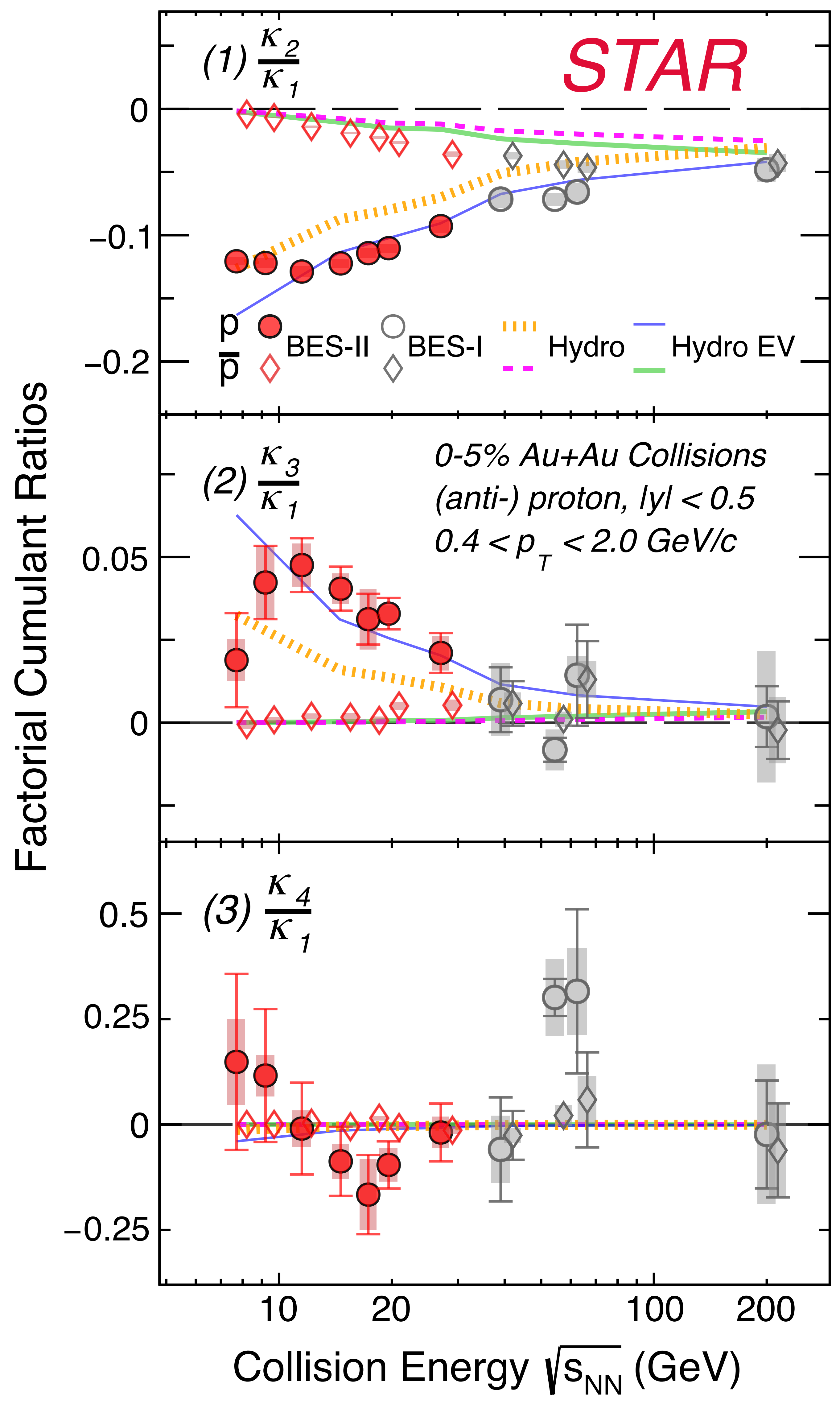


CAVEAT: 3 GeV has acceptance $-0.5 \leq Y \leq 0$

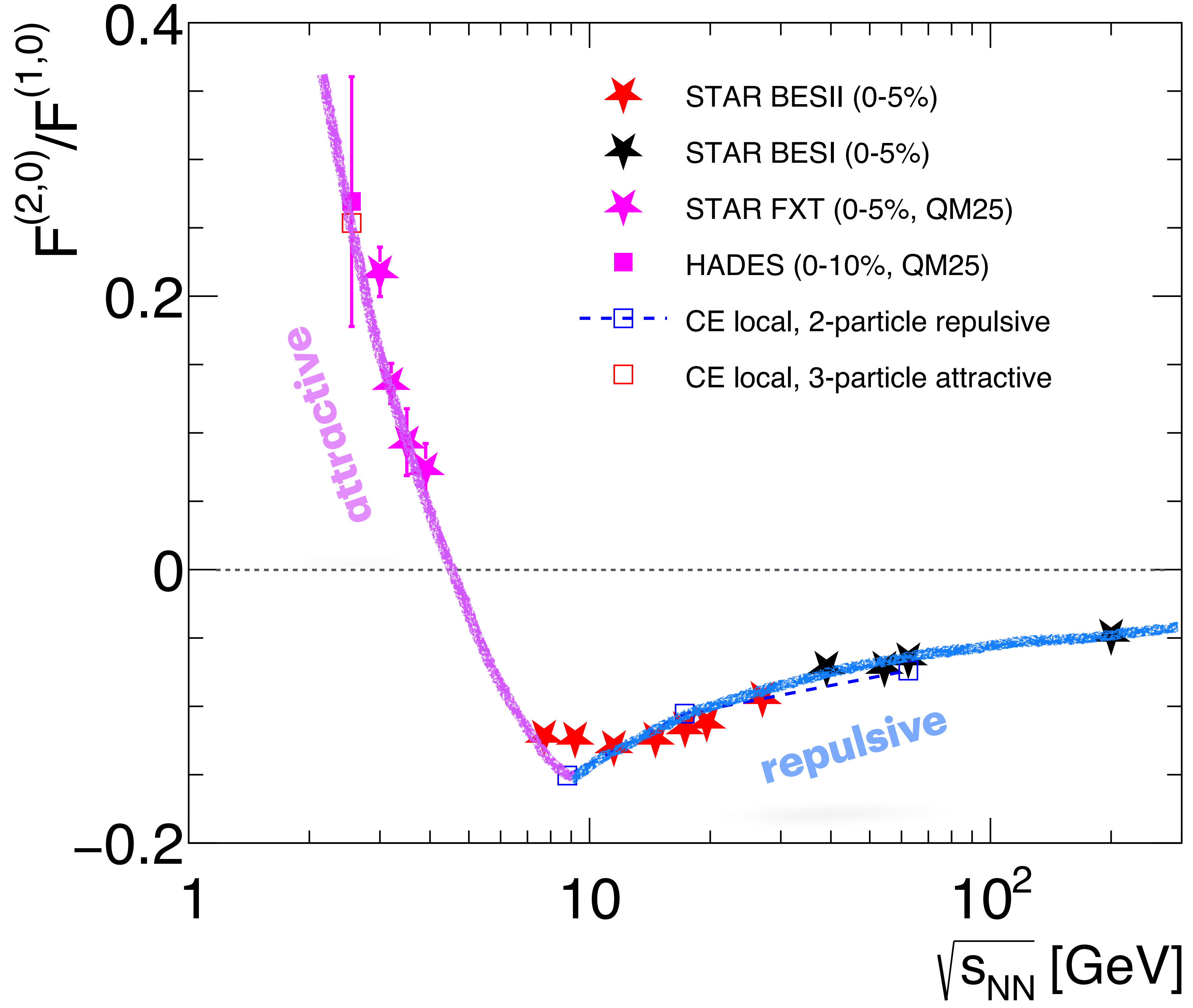
New STAR data (BESII)

Looks like we need some kind of attraction!

Factorial cumulants



Attraction does the trick



Friman, Redlich, Rustamov, arXiv: 2508.18879

Interaction in momentum space
Strength of interaction is adjusted to fit the data

Beth-Uhlenbeck

B. Friman, V.K. in preparation

$$Z_{GC}(\mu_B, T) = \exp\left(e^{\mu_B/T}\, z_1 \;+\; e^{2\,\mu_B/T}\, z_2\right)$$

$$z_1 = V \int \frac{d^3p}{(2\,\pi)^3} e^{-\varepsilon_p/T}$$

$$z_2^0 = V \int \frac{d^3p}{(2\,\pi)^3} \int_{2\,m_N}^{\infty} \frac{d\,M}{\pi} \sum_{j=0\ \text{(even)}}^{\infty} (2j+1) \frac{\partial \delta_{j\,j}^{1\,0}}{\partial M}\, e^{-\sqrt{p^2+M^2}/T}, \qquad (S=0)$$

$\delta_{j,l}$ proton proton phase shift

$M =$ center of mass energy

$$z_2^1 = V \int \frac{d^3p}{(2\,\pi)^3} \int_{2\,m_N}^{\infty} \frac{d\,M}{\pi} \sum_{\ell=1\ \text{(odd)}}^{\infty} \sum_{j=\ell-1}^{\ell+1} (2j+1) \frac{\partial \delta_{j\,\ell}^{1\,1}}{\partial M}\, e^{-\sqrt{p^2+M^2}/T}, \qquad (S=1)$$

Phase shifts are extracted up to $M \simeq 2\,\mathrm{GeV}$ (SAID data base, G.W. university)

Complication: for $T \simeq 150\,\mathrm{MeV}$ inelastic channels are needed

Beth-Uhlenbeck

$$Z_{GC}(\mu_B, T) = \exp\left(e^{\mu_B/T}\, z_1 + e^{2\,\mu_B/T}\, z_2\right)$$

Cumulants:

$$\kappa_1 = \frac{\partial}{\partial(\mu/T)} \ln(Z) = e^{\mu_B/T}\, z_1 + 2e^{2\,\mu_B/T}\, z_2$$

$$\kappa_2 = e^{\mu_B/T}\, z_1 + 4e^{2\,\mu_B/T}\, z_2$$

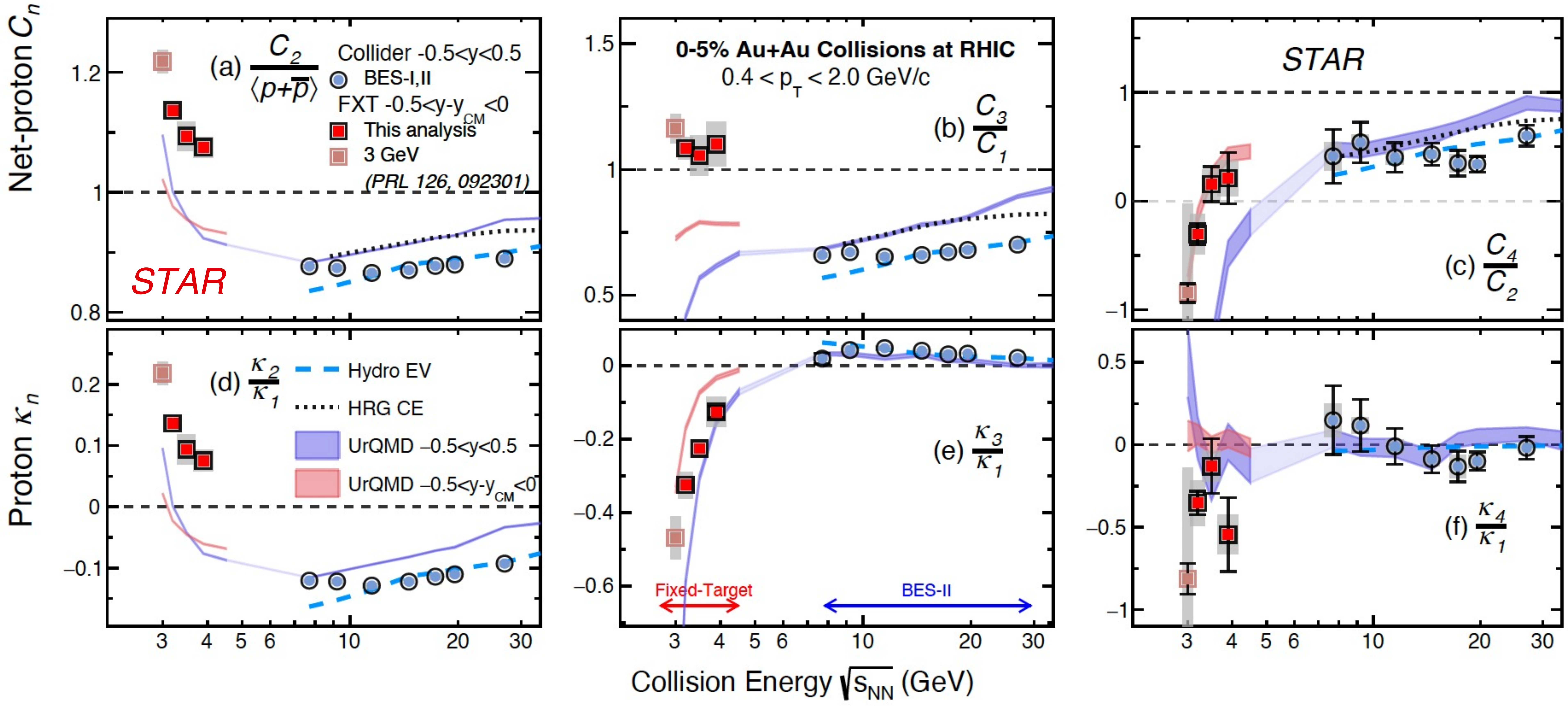
$$FC_2 = \kappa_2 - \kappa_1 = 2\, e^{2\,\mu_B/T}\, z_2$$

Factorial cumulant “measures” the strength of the interaction

However….

Possible Spoiler

Zachary Sweger (for STAR) QM 2025



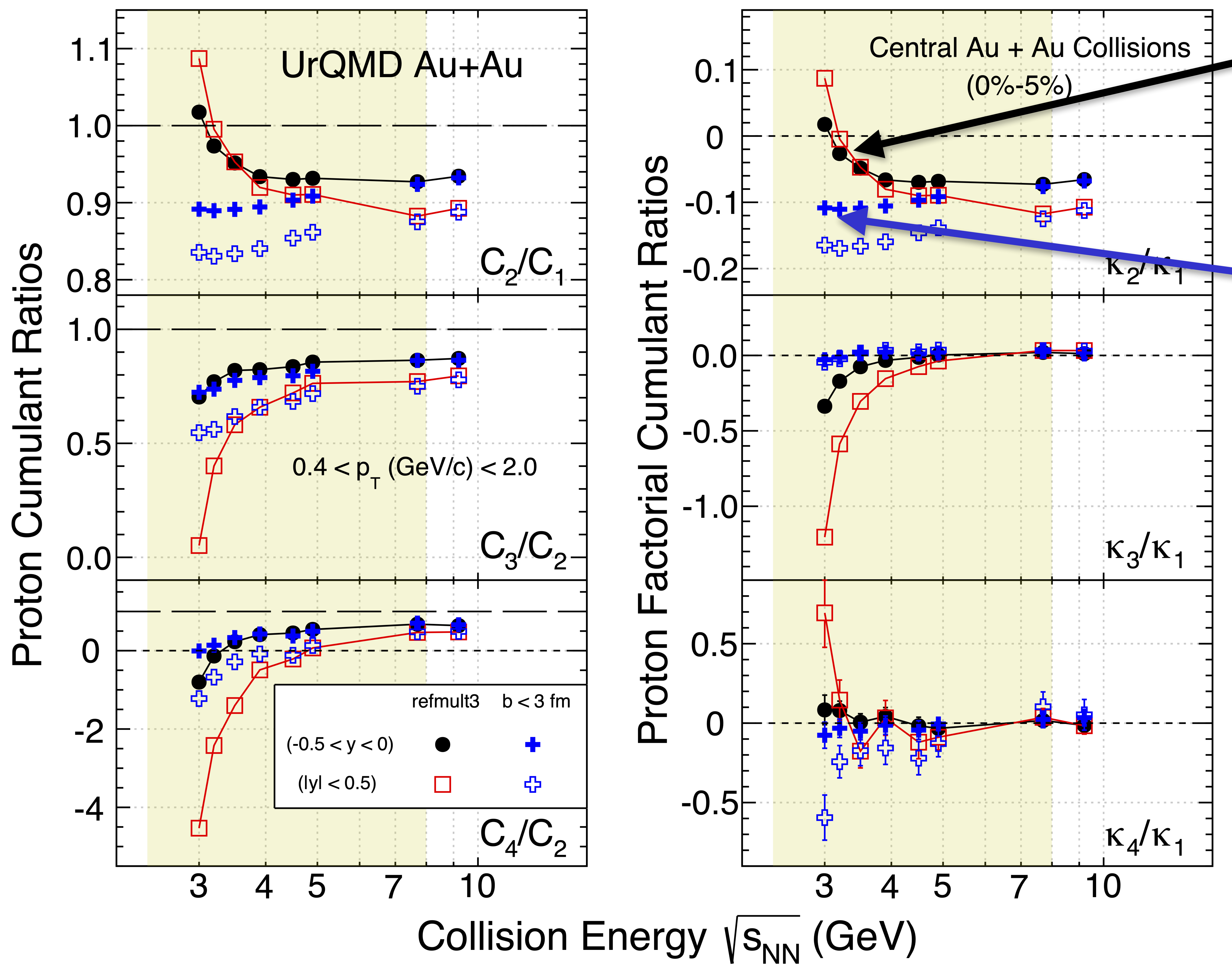
Cumulants

Factorial Cumulants

UrQMD (WITHOUT mean field) get energy dependence qualitatively right!!!

The (possible) culprit

X. Zhang, Y. Zhang, X. Luo, N. Xu, arXiv: 2506.18832



Fluctuating impact parameter
STAR centrality selection

Fixed impact parameter (3=3 fm)
minimal volume fluctuations.

N.B.: Centrality Bin Width Corrections (CBWC) applied to both

Possible culprit:

- volume fluctuations
  - CBWC does not work well at low energies (Friman, VK 2511.11869, NPA in print)
  - New methods (Rustamov et al, 2211.14849; Wang et al, 2505.03666)

Test for Baseline

A. Bzdak, V. Vovchenko, VK, arXiv:2503.16405

Both global charge conservation and volume fluctuations are long range correlations

Factorial cumulant: $C_n = \int_{\Delta_Y} dy_1 \cdots \int_{\Delta_Y} dy_n C(y_1, \cdots, y_n)$

$C(y_1, \cdots, y_n)$: n-particle correlations function

Long range correlations: $C(y_1, \cdots, y_n) = const$ within ΔY

$$\Rightarrow C_n \sim (\Delta Y)^n$$

$\Rightarrow \frac{C_n}{C_1{}^n} = const$ as function of ΔY and $\left.\frac{C_2}{C_1{}^2}\right|_{\text{protons}} = \left.\frac{C_2}{C_1{}^2}\right|_{\text{antiprotons}}$ as function of ΔY

Test of Baseline

Both GLOBAL charge conservation and volume fluctuations introduce only

LONG Range correlations in rapidity (larger than acceptance)

Baryon number conservation

Within acceptance:

$$\alpha = \frac{\langle N \rangle_{\Delta Y}}{\langle N + \bar{N} \rangle_{4\pi}} \qquad \bar{\alpha} = \frac{\langle \bar{N} \rangle_{\Delta Y}}{\langle N + \bar{N} \rangle_{4\pi}}$$

$$P(n, \bar{n}) = \sum_{N,\bar{N}} B(n, N; \alpha) B(\bar{n}, \bar{N}, \bar{\alpha}) P(N, \bar{N})$$

$P(N, \bar{N})$ Distribution of protons (N) and anti-protons subject to global baryon number conservation

$B(n, N, \alpha)$ Binomial distribution with Bernoulli prob α

Factorial cumulants:

$$C_k(n; \Delta Y) = \alpha^k C_k(N, 4\pi)$$

$$C_k(\bar{n}; \Delta Y) = \bar{\alpha}^k C_k(\bar{N}, 4\pi)$$

analogous to “efficiency” corrections

$\Rightarrow \dfrac{C_k}{C_1^{\,k}} = const$ as function of ΔY for both protons and anti protons

Include volume fluctuations

Holzmann et al. 2403.03598

$$C_1[N] = \langle N_w \rangle C_1[n] = \langle N_w \rangle \langle n \rangle = \langle N \rangle ,$$

$$C_2[N] = \bar{C}_2[N] + \langle N \rangle^2 \frac{\kappa_2[N_w]}{\langle N_w \rangle^2},$$

$$C_3[N] = \bar{C}_3[N] + 3 \langle N \rangle \bar{C}_2[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + \langle N \rangle^3 \frac{\kappa_3[N_w]}{\langle N_w \rangle^3},$$

$$C_4[N] = \bar{C}_4[N] + 4 \langle N \rangle \bar{C}_3[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + 3\bar{C}_2^2[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + 6 \langle N \rangle^2 \bar{C}_2[N] \frac{\kappa_3[N_w]}{\langle N_w \rangle^3} + \langle N \rangle^4 \frac{\kappa_4[N_w]}{\langle N_w \rangle^4}.$$

Since $\bar{C}_k \sim \alpha^k \Rightarrow C_k \sim \alpha^k$

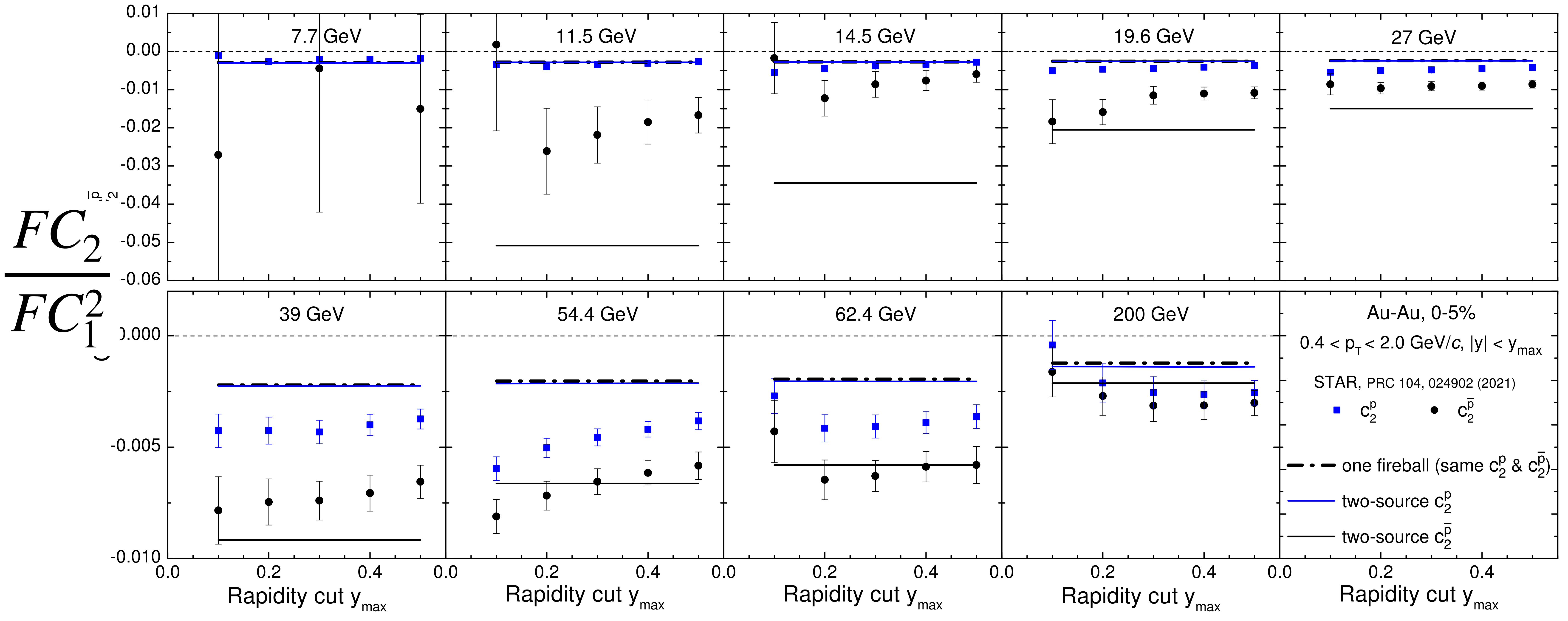
$\bar{C}_k$: Factorial cumulant WITHOUT volume fluctuations

C_k : Factorial cumulant WITH volume fluctuations

If $\dfrac{C_k}{C_1{}^k} \neq const$ as function of ΔY: Some other (short range) physics is at play as well (Example: excluded volume)

Test of baseline
BES-I data

- $\frac{FC_2}{FC_1^2}$ more or less constant
- $\frac{FC_2[p]}{FC_1^2[p]} \neq \frac{FC_2[\bar{p}]}{FC_1^2[\bar{p}]}$
- baseline OK for protons
- No good for anti-protons ??
- How will it look with BES II data?



Two component model

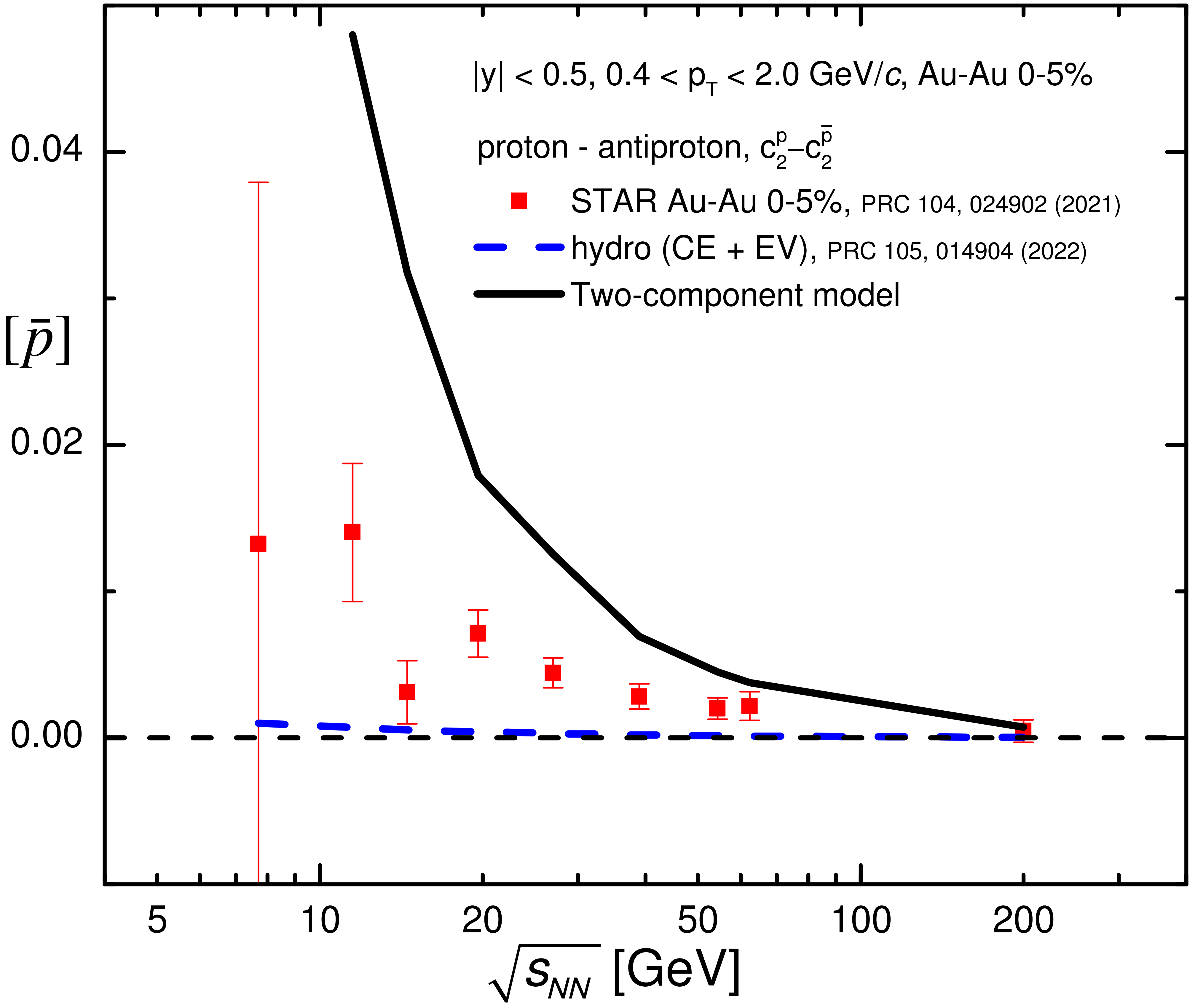
2 sources: stopped and produced particles

- All anti-protons are produced
- protons come from produced and stopped sources

$$N_p(\text{produced}) = \mathrm{N}_{\bar{\mathrm{p}}}$$

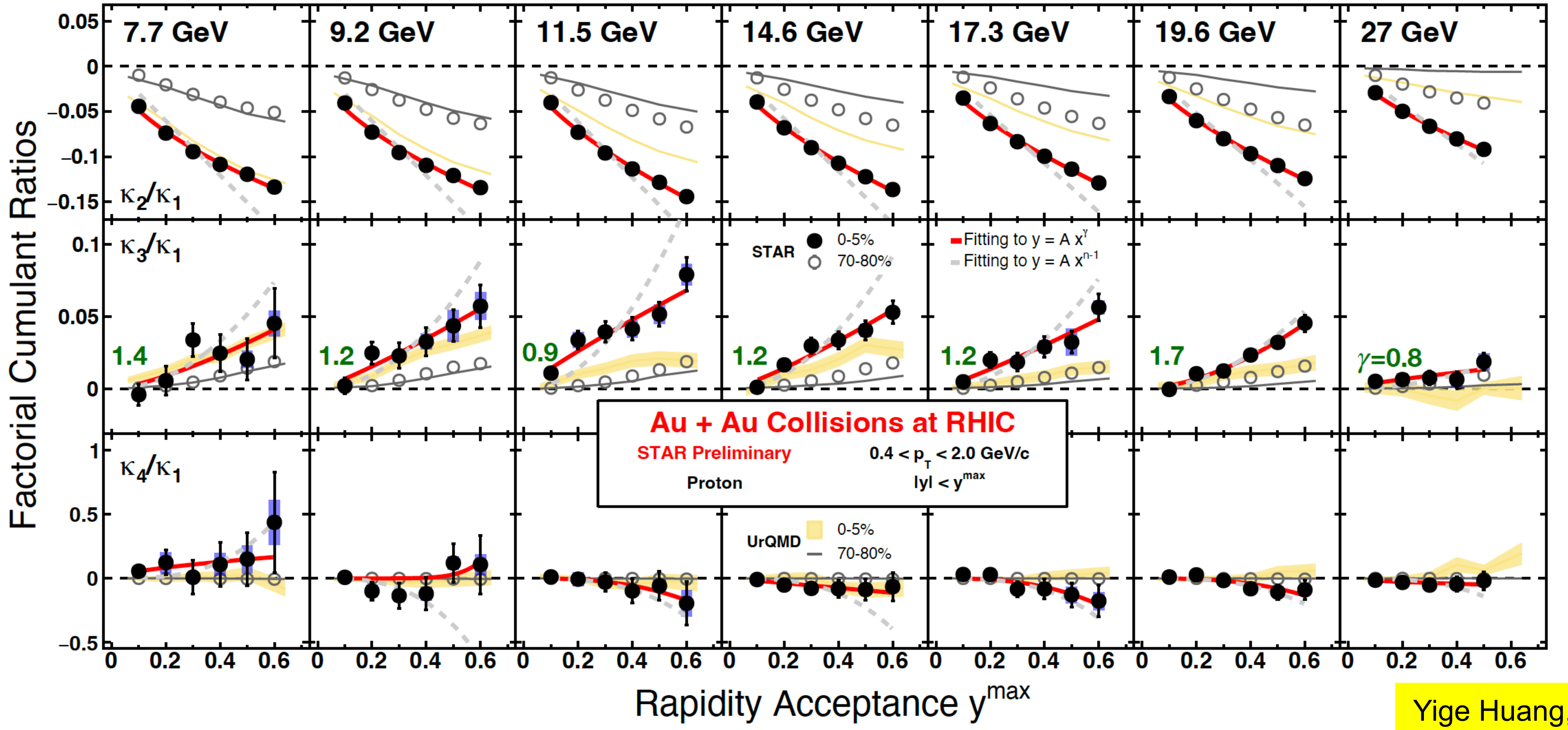
$$N_p = N_p(\text{stopped}) + \mathrm{N}_{\bar{\mathrm{p}}}$$

$$\frac{FC_2}{FC_1^2}[p] - \frac{FC_2}{FC_1^2}[\bar{p}]$$



- Produced source: Thermal with zero net baryon number $\langle B - \bar{B} \rangle = 0$
- Stopped source: Follows binomial distribution

Proton Factorial Cumulant Ratios vs. Rapidity Window



Yige Huang, QM25

Scaling not seen in BESII data

1) Proton κ_n/κ_1 follows a power increase vs. Δy

2) Power constant smaller than simple expectation (n-1)

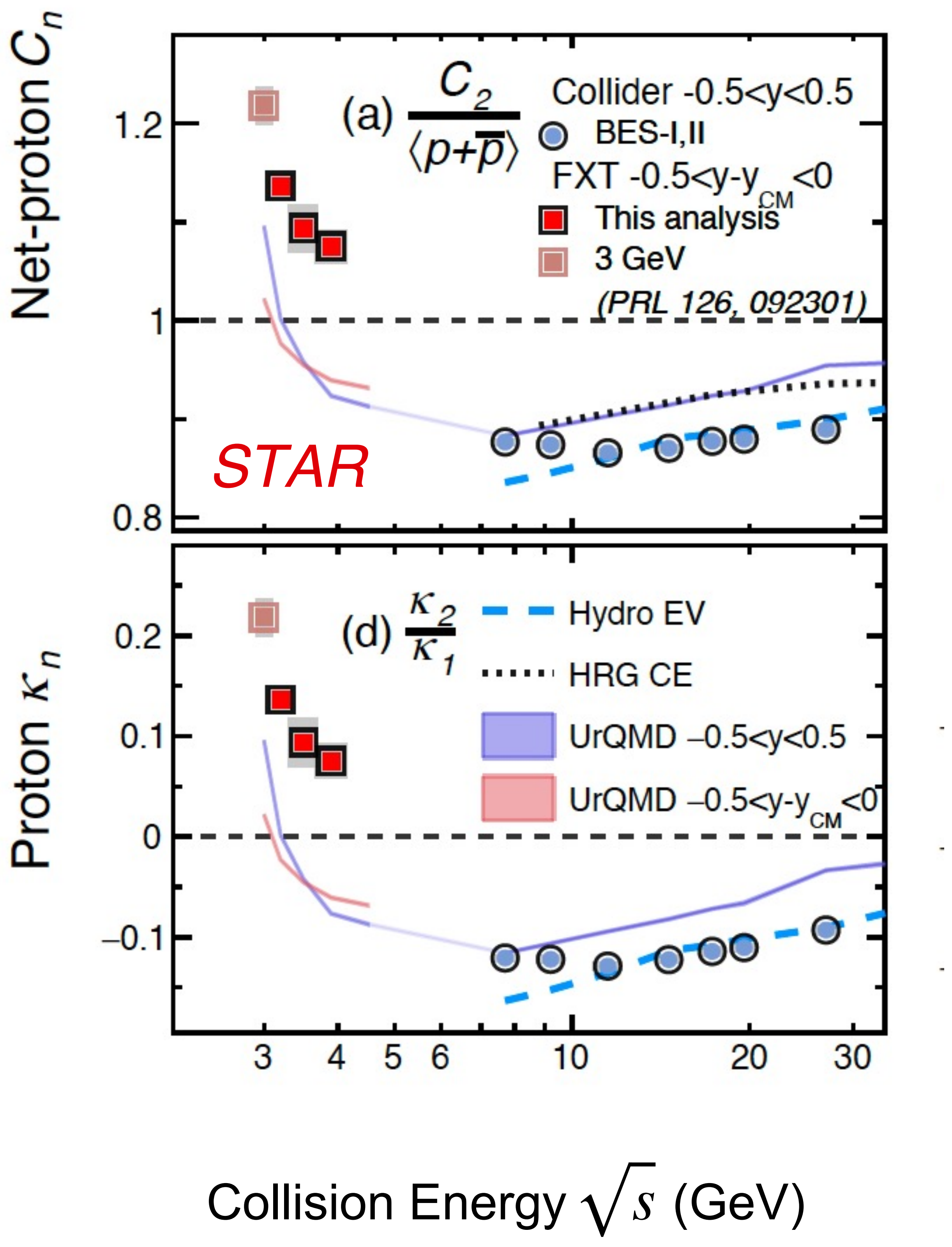
Summary

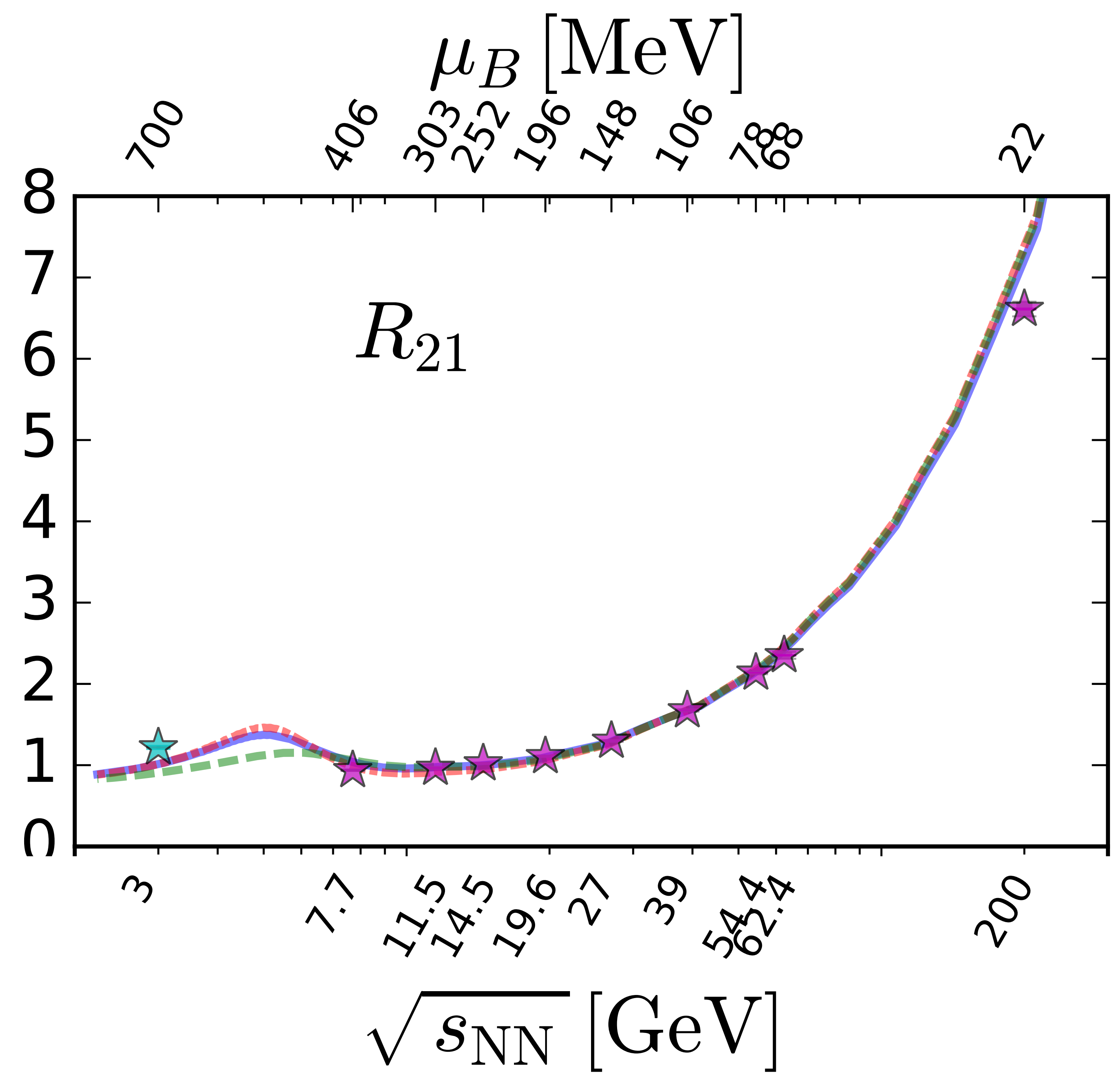
- Net charge fluctuations may finally say something about fractional charges
- STAR has delivered on the BESII data!
  - Cannot hide behind error bars anymore
- Baseline with baryon number conservation and repulsive interaction tuned to LQCD agrees with data down to $\sqrt{s} \sim 10\,\mathrm{GeV}$
- Data below $\sqrt{s} \sim 10\,\mathrm{GeV}$ seem to require some kind of "attraction"
- HOWEVER, UrQMD get the trend in the energy dependence right. Volume fluctuations a low energy?
- Possible test of a baseline involving baryon number conservation and volume fluctuations via ratio of factorial cumulants $\frac{FC_n}{FC_1^N}$
  - Does not seem to scale in BES II data
- Anti protons from BES I are NOT understood. BESII comparison needed. Two source model?.

Happy Birthday, Andrzej!

Backup

Data for symmetric $[-\Delta Y, \Delta Y]$ below $\sqrt{s} < 7.7\,\mathrm{GeV}$ desperately needed!



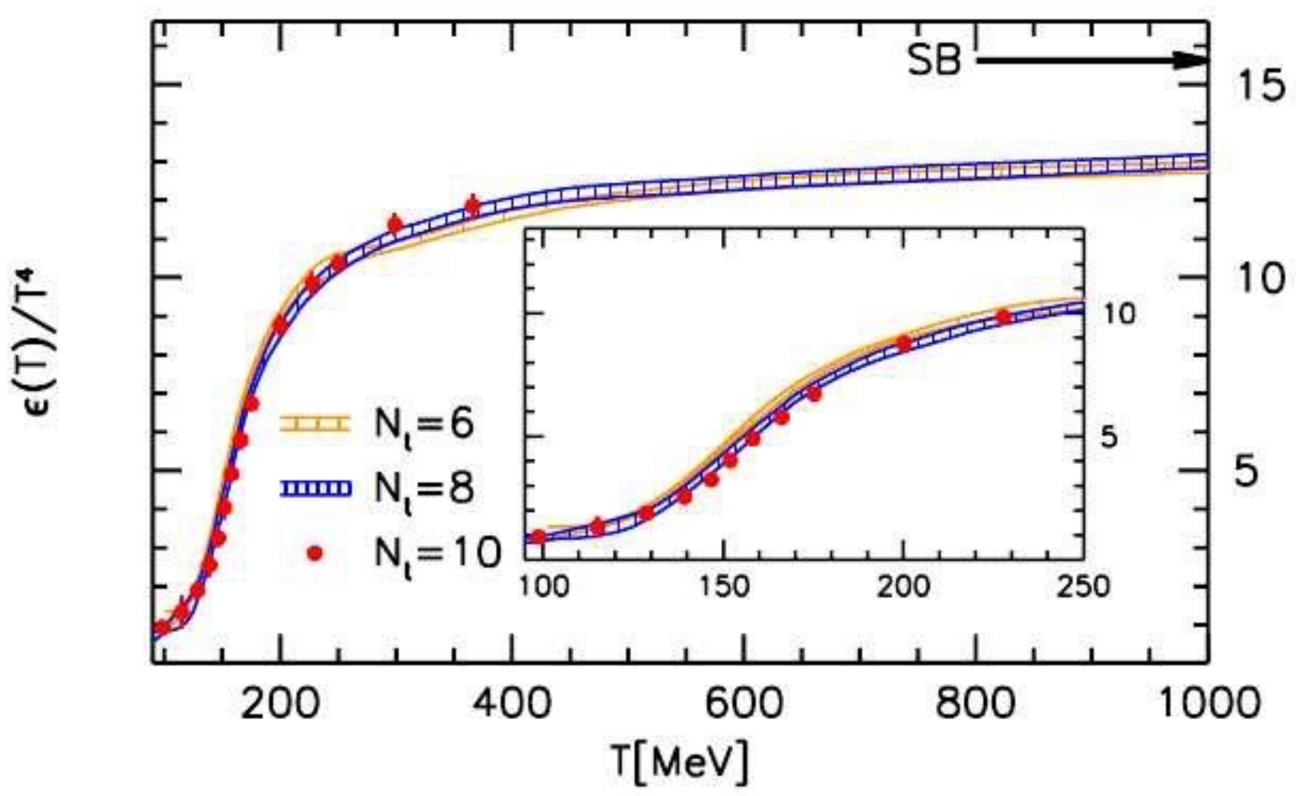


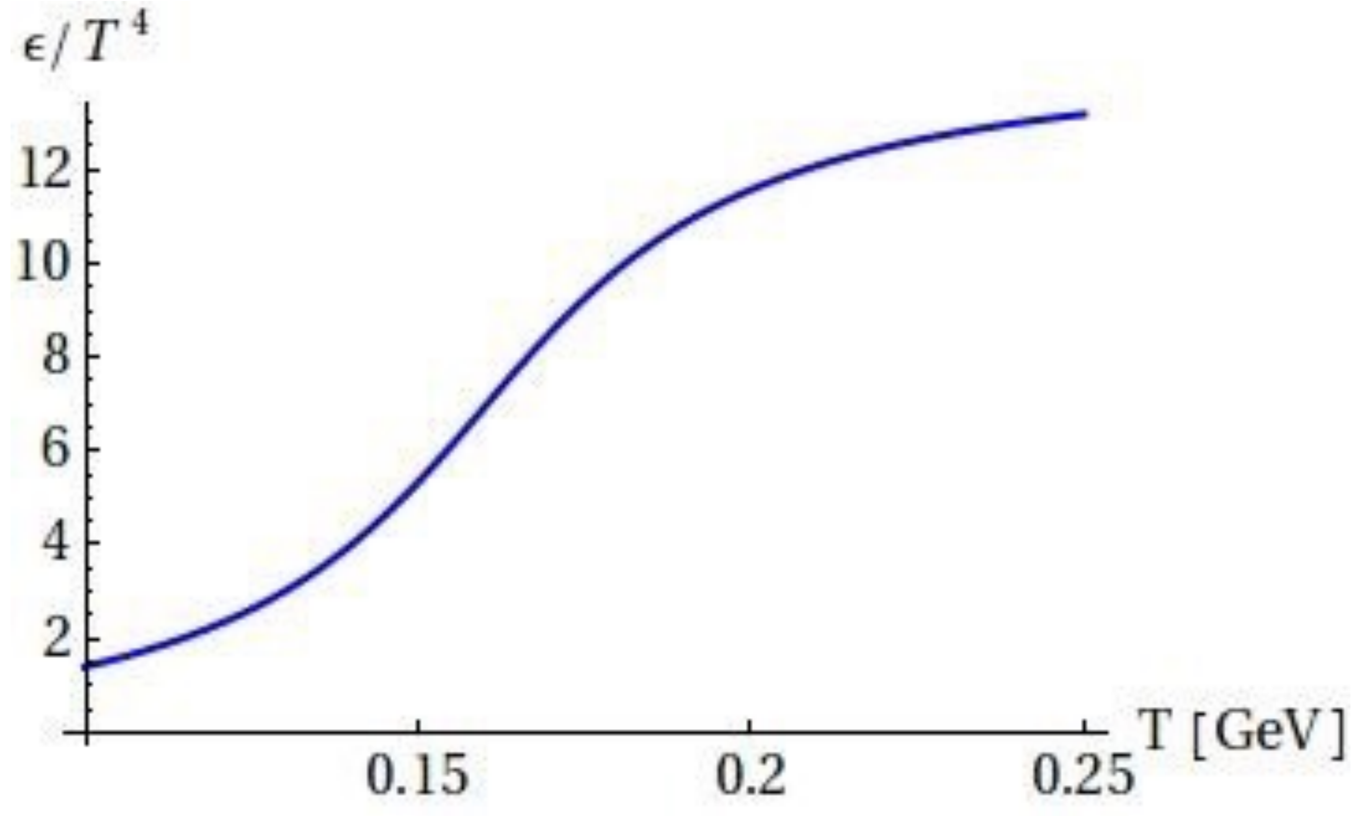
CBM is perfectly positioned to do so

Meanwhile: understand what UrQMD in STAR acceptance does

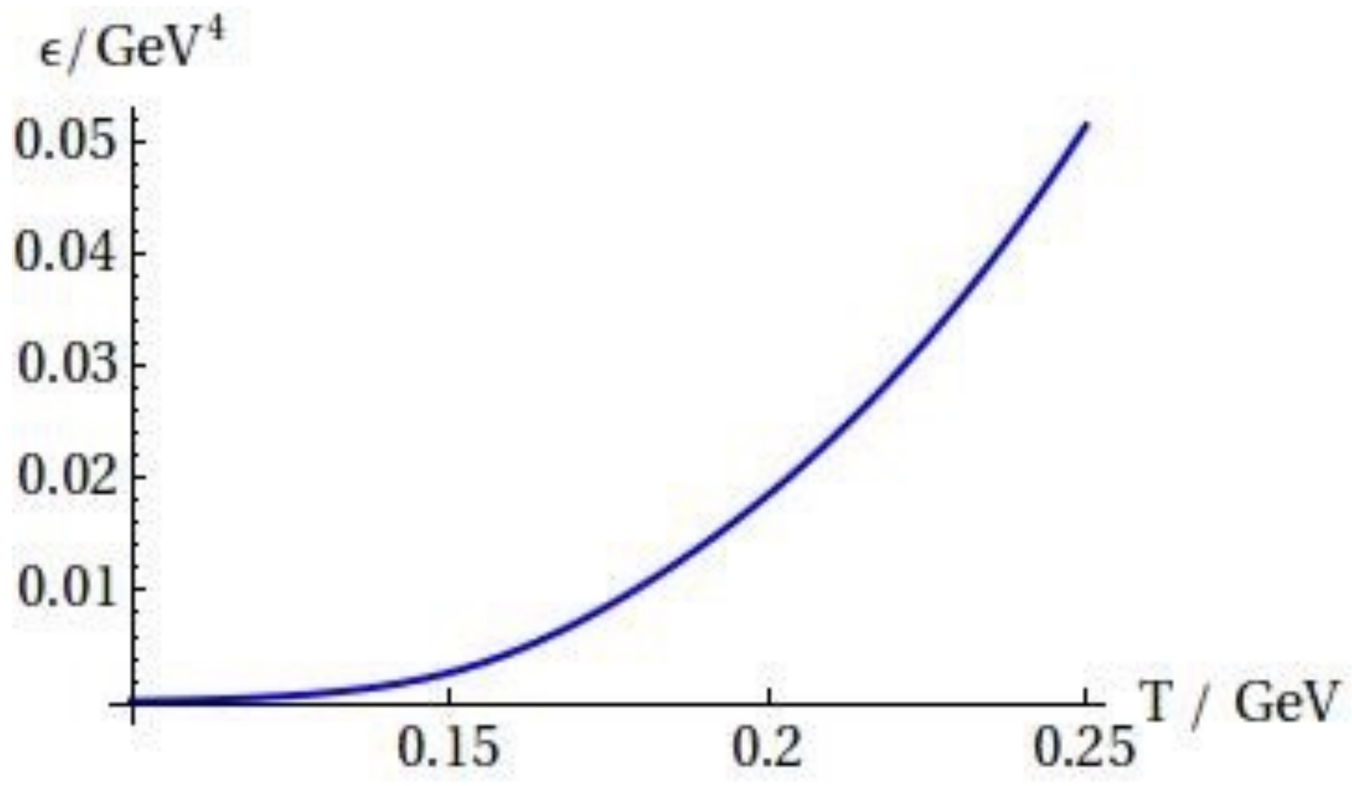
Cumulants and Phase structure

S. Borsanyi et al, JHEP 1011 (2010) 077





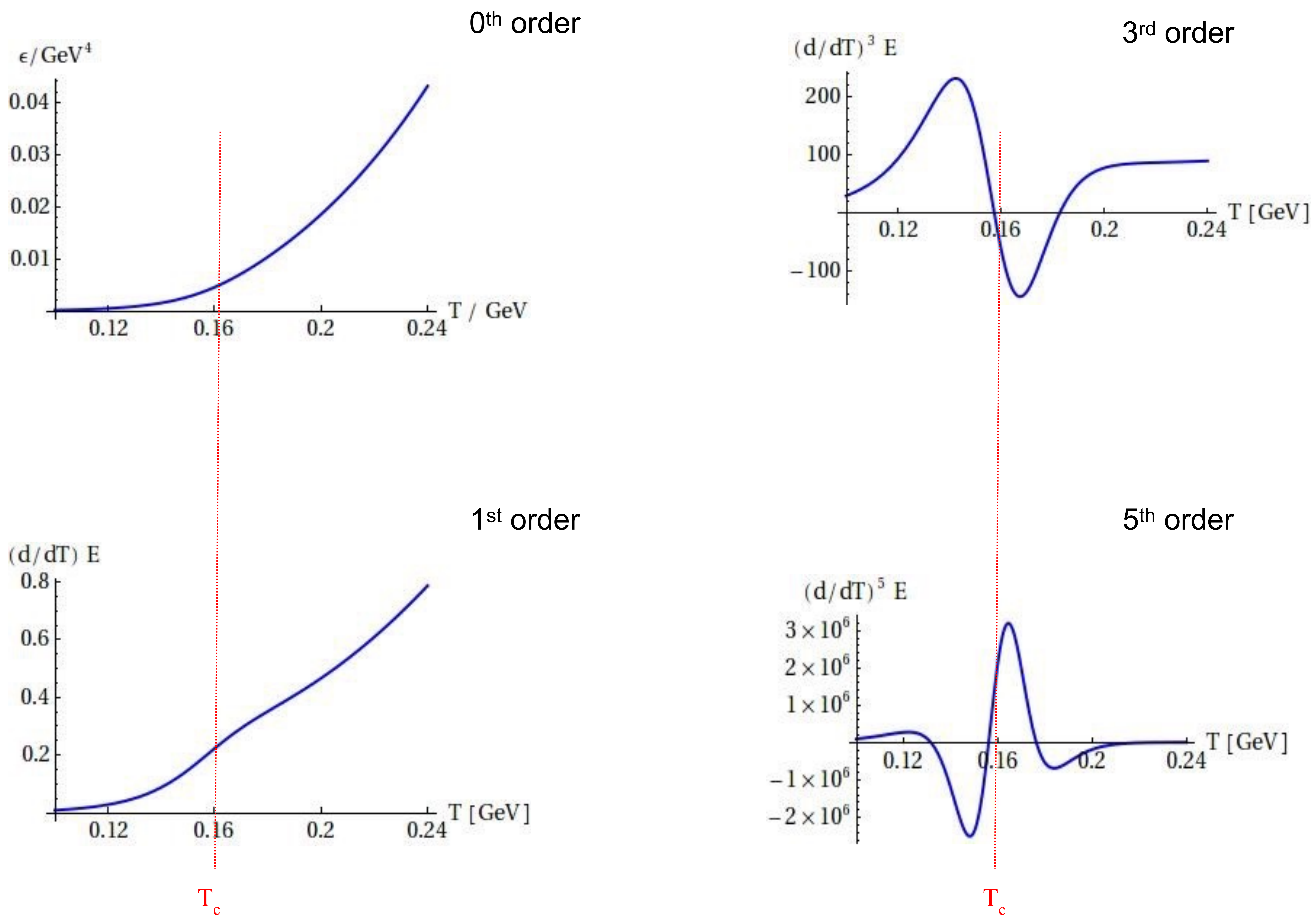
What we always see....



What it really means....

"T_c" ~ 155 MeV

Derivatives



How to measure derivatives

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z} tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

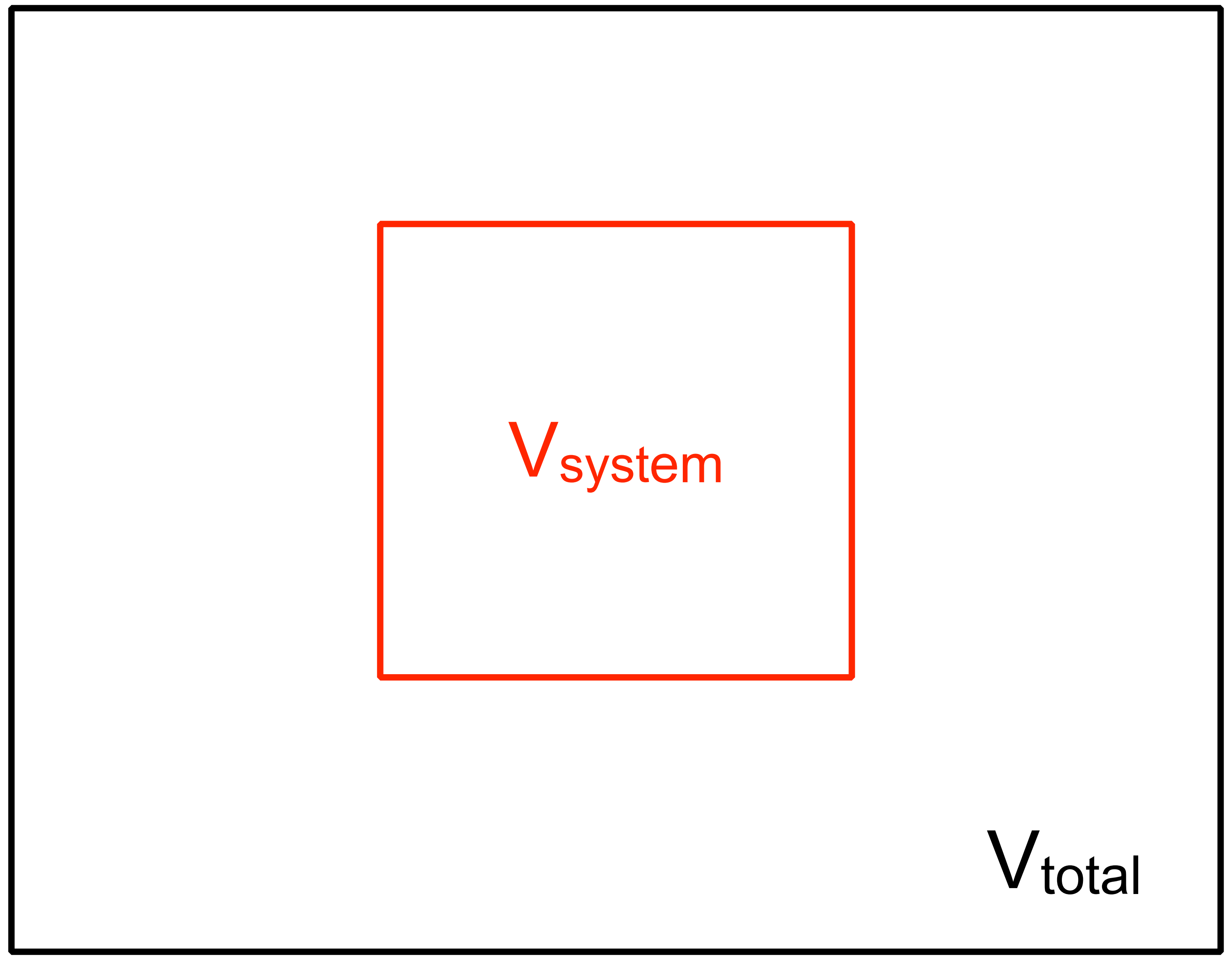
$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

Grand canonical ensemble



$$V_{total} \rightarrow \infty$$
$$V_{system} \rightarrow \infty$$

$$\frac{V_{system}}{V_{total}} \rightarrow 0$$

In coordinate space!!!!

Lattice:

$V_{total} \rightarrow \infty$

grand-canonical ensemble

Coordinate space

Experiment:

V_{total} finite!

$V_{system} \ll V_{total}$ (hopefully)

effect of global charge conservation

Momentum Space