'GHOSTS' BEYOND QUANTUM MECHANICS

- WHY DO 'GHOSTS' EXIST?

Equilibration and Locality

Marek Gazdzicki, Mark Gorenstein, Ivan Pidhurskyi, Oleh Savchuk, Leonardo Tinti (Jun 2, 2022)

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e-Print: 2605.03104 [quant-ph]

WHY DO 'GHOSTS' EXIST ?

DEVELOPERS AND PLAYERS OF THE RANDOM BOARD GAME, THE TOY-UNIVERSE, SUGGESTING THE ANSWER TO THE QUESTION

THE COVID TIMES

Equilibration and Locality

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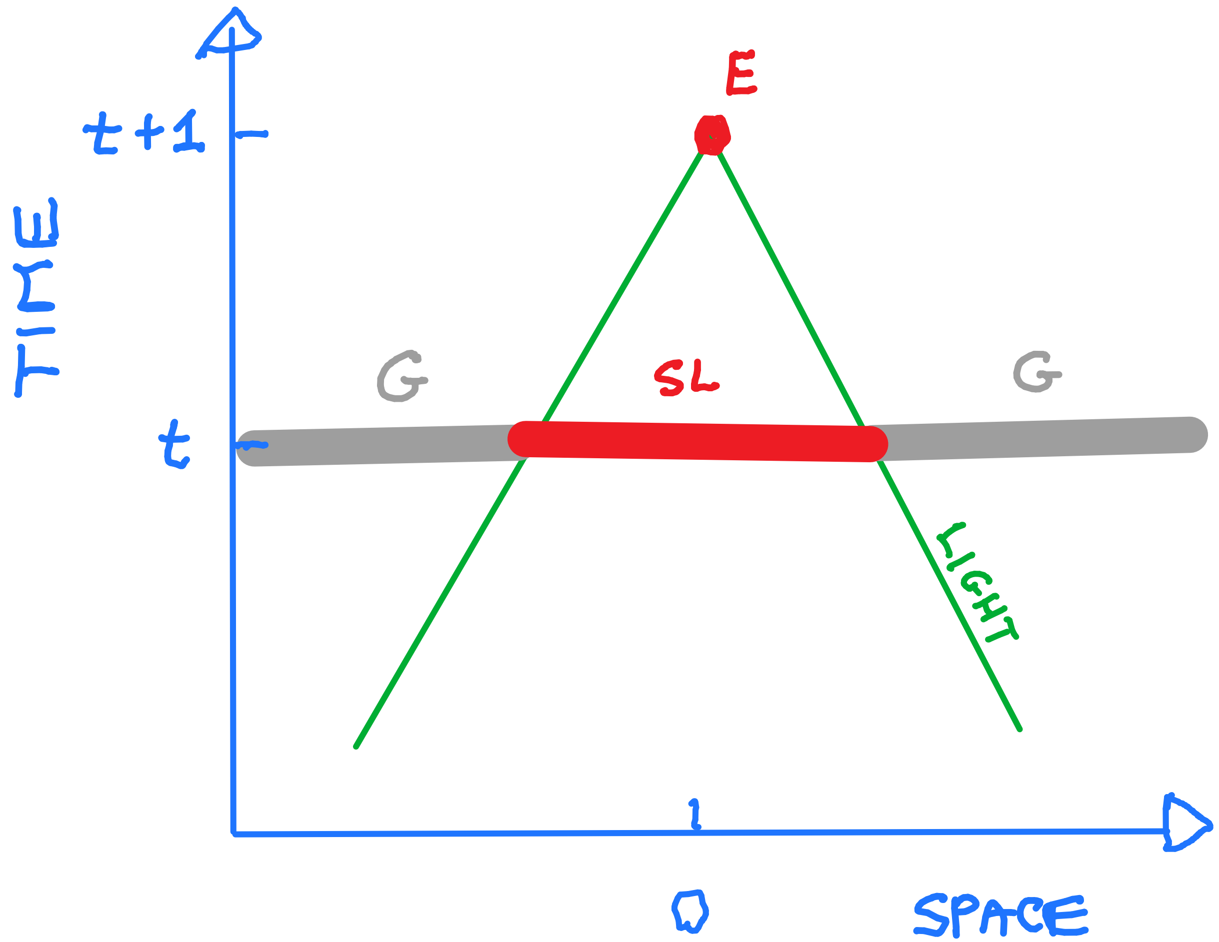
'NO-GHOSTS' - DEFINITION:



PROBABILITY OF AN EVENT E AT t+1, IN GENERAL, COULD DEPEND ON ALL EVENTS AT t, SL AND G,
P(E | (SL, G)).

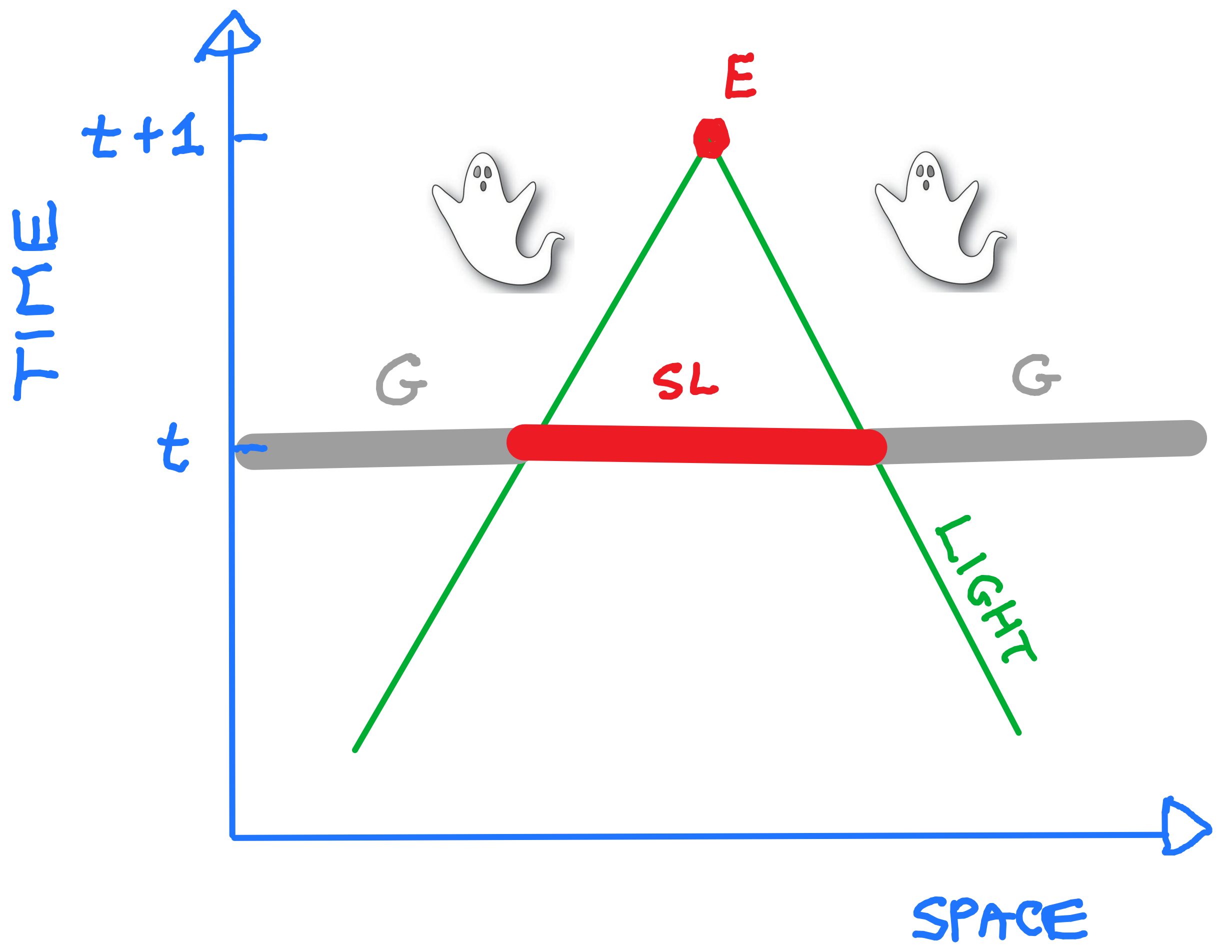
BUT HAVING SPEED OF LIGHT AS MAXIMUM TRANSPORT AND SIGNALING VELOCITY ONE (EINSTEIN ET AL.) EXPECTS THAT THE DEPENDENCE ON SL AND G REDUCES TO THE DEPENDENCE ON SL,

P(E | (SL, G)) = P(E | SL).

$\equiv$

THE STRONG LOCALITY HYPOTHESIS 'NO GHOSTS' IN NATURE

'GHOSTS' – DEFINITION:



IF $P(E \mid (SL, G)) \neq P(E \mid SL)$

- NON-TRIVIAL DEPENDENCE BETWEEN REMOTE EVENTS, 'GHOSTS' IN NATURE (VIOLATION OF STRONG LOCALITY)

HISTORY

≈ 1900: EXPERIMENTS CONTRADICTING CLASSICAL PHYSICS

$\Downarrow$

≈ 1900: QUANTUM MECHANICS

$\Downarrow$

≈ 1950: EINSTEIN-PODOLSKY-ROSEN PARADOX (1935)
BELL'S INEQULITIES (1964)

$\Downarrow$

≈ 2000: MEASURMENTS OF 'GHOSTS': ASPECT ET AL. (1982)
CLAUSER ET AL. (1969)
ZEILINGER ET AL. (1998)
NOBEL 2022
...
HANSEN ET AL. (2015)

'GHOSTS' ARE BUILT IN QUANTUM MECHANICS

HERE WE RELATE 'GHOSTS'
TO SYMMETRIES

NEXT SLIDES:

(THE GAME INSTRUCTION...)

- TOY-UNIVERSE
  (THE GAME: BOARD, PAWNS, MOVEMENTS, DICE)

- POPULAR SYMMETRIES
  (POPULAR MOVEMENT RULES)

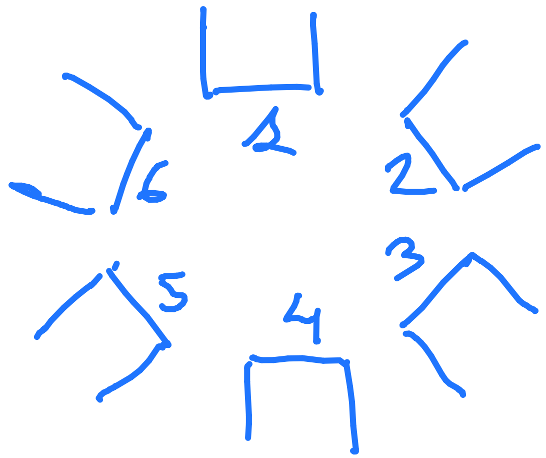
- MICROSTATE-SYMMETRY
  (ADVANECED MOVEMENT RULE)

- RESULTS
  - TOY-UNIVERSE + POPULAR SYMMETRIES
  - TOY-UNIVERSE + MICROSTATE SYMMETRY
    → 'GHOSTS' FOLLOW FROM SYMMETRIES

SPACE:

(THE GAME BOARD)

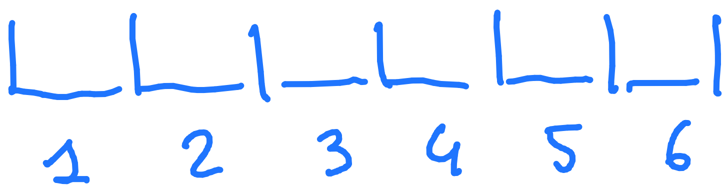
SIX CELLS ARRANGED IN A CIRCLE...



PERIODIC BOUNDARY CONDITIONS

...WHICH ARE FOR SIMPLICITY DRAWN IN A ROW

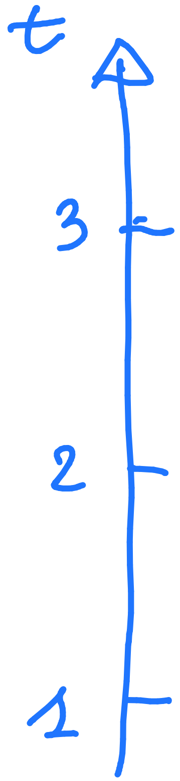
TOY-UNIVERSE



MINIMUM PHASE-SPACE CELL – $\hbar$

TIME:

(GAME MOVES BY ROLLING THE DICE)



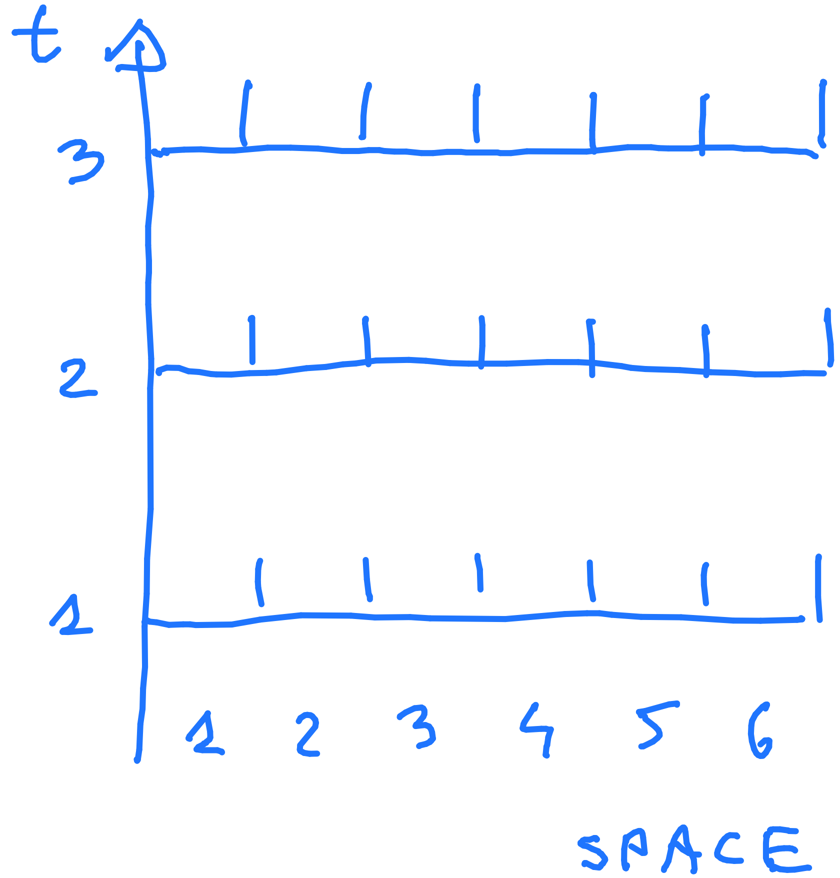
TIME STEPS.

IT IS MEASURED IN STEPS.

THERE IS NO LIMIT TO THE NUMBER OF STEPS IN TIME

EASY TO INTRODUCE INDETERMINISM

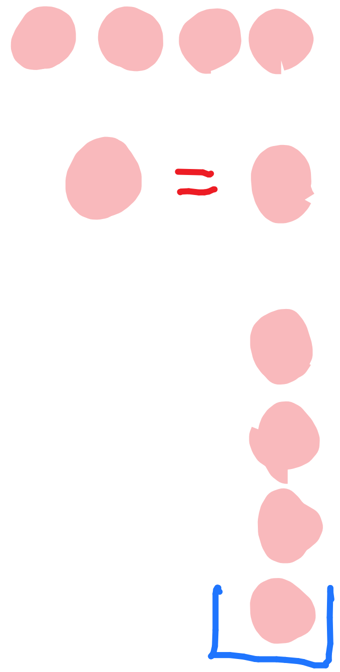
SPACE - TIME :



TOY - UNIVERSE

PARTICLES:

(PAWNS IN THE GAME)



FOUR BALLS AT ALL TIMES

BALLS ARE IDENTICAL

NO LIMIT ON NUMBER OF BALLS IN A CELL

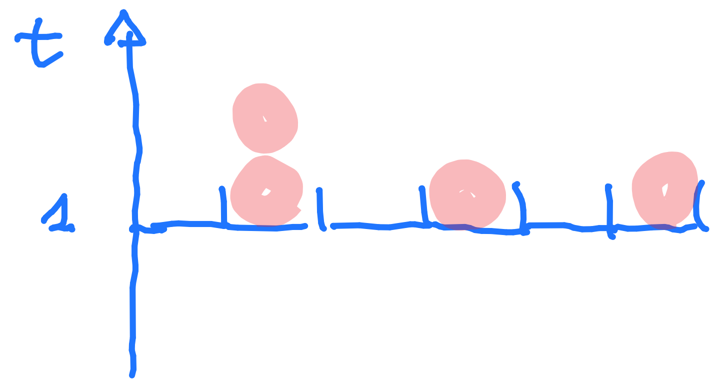
CONSERVATION LAWS

INDISTINGUISHABLE SPIN-ZERO BOSONS

TOY-UNIVERSE

PARTICLES IN SPACE-TIME:

MICROSTATE – ANY ARRANGEMENT OF FOUR BALLS IN SIX CELLS AT A GIVEN TIME (DENOTED AS X, Y, ..)



TOTAL NUMBER OF MICROSTATES:

$$W_{IND}(4,6) = \frac{(4+6-1)!}{4!\,(6-1)!} = 126$$

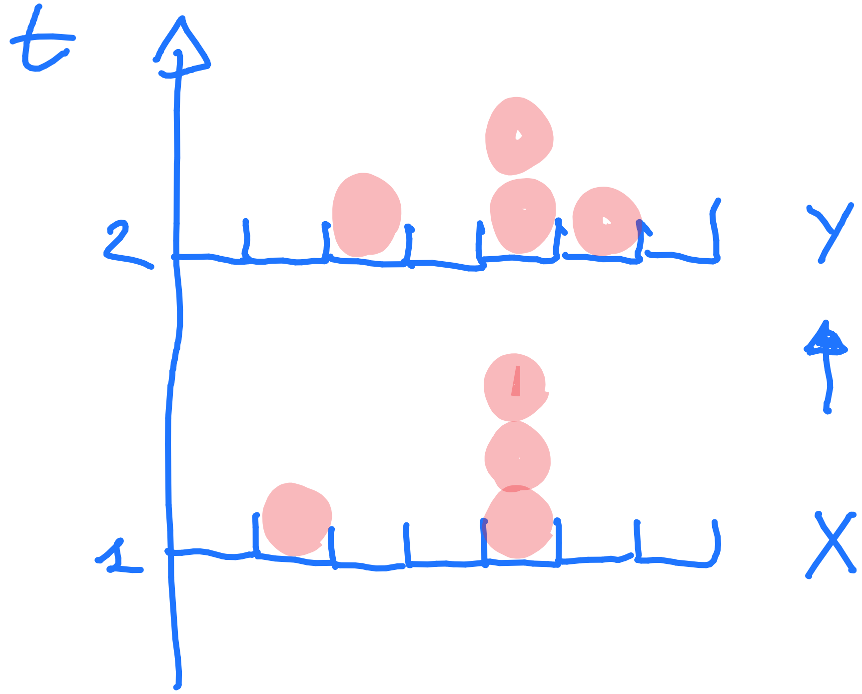
$$\left(W_{DIST}(4,6) = 6^4 = 1296 \right)$$

TOY-UNIVERSE

TRANSITIONS:

(MOVES IN THE GAME BY THROWING A DICE)

AT EACH TIME STEP ARRANGMENT OF BALLS CHANGES — A TRANSITION FROM X TO Y TAKES PLACE WITH TRANSITION PROBABILITY $B(X \to Y \mid X)$ DEPENDING ONLY ON X, Y



$$\sum_Y B(X \to Y \mid X) = 1$$

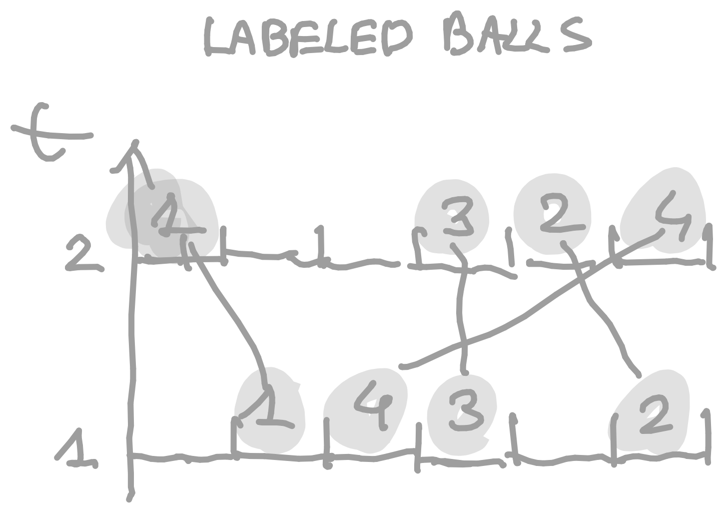
TOY-UNIVERSE → MARKOV CHAIN

INDETERMINISM OF INTERACTIONS AND MOVEMENTS

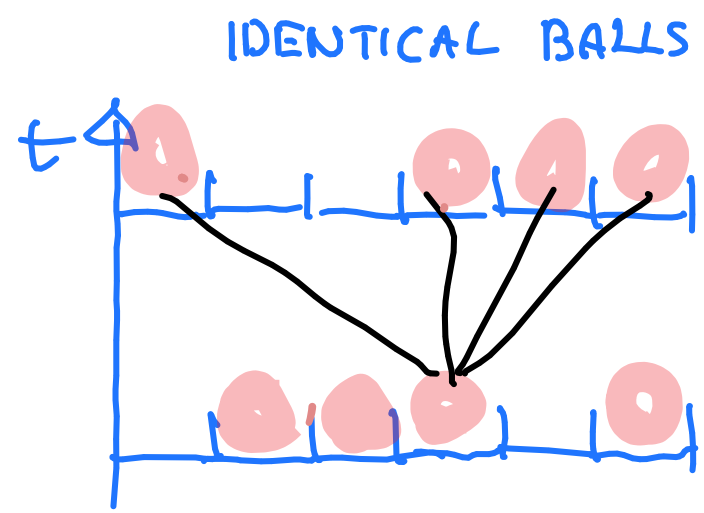
TRANSITIONS:

COMMENT ON LABELED (DIST.) AND IDENTICAL (INDIST.) BALLS:

BALL TRAJECTORY = BALL CELL NUMBERS BEFORE AND AFTER A TRANSITION



ALL LABELED BALLS HAVE TRAJECTORIES



EACH IDENTICAL BALL HAS FOUR POSSIBLE TRAJECTORIES

⇓

NO UNIQUE BALL TRACECTORIES FOR IDENTICAL BALLS

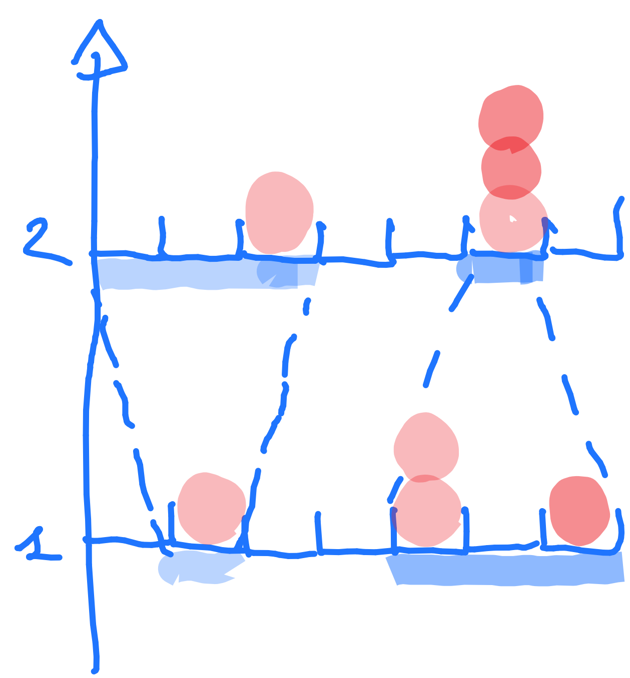
TOY-UNIVERSE

QUANTUM MECHANICS

TRANSPORT LOCALITY: A TRANSITION CAN TRANSPORT NUMBER OF BALLS IN THE CELL BY NO MORE THAN $\Delta = 1$ CELLS.

(SPEED OF LIGHT IN THE GAME)

(FORBIDDEN TELEPORTATION)



LOCALITY CONDITIONS:

$$\sum_{j=i}^{i+k} n_j^X \leq \sum_{l=i-\Delta}^{i+k+\Delta} n_l^Y$$

$$\sum_{l=i}^{i+k} n_l^Y \leq \sum_{j=i-\Delta}^{i+k+\Delta} n_j^X$$

(n – CELL PARTICLE NUMBER)

'MAXIMUM VELOCITY' $v \leq c = 1$
FOR IDENTICAL BALLS

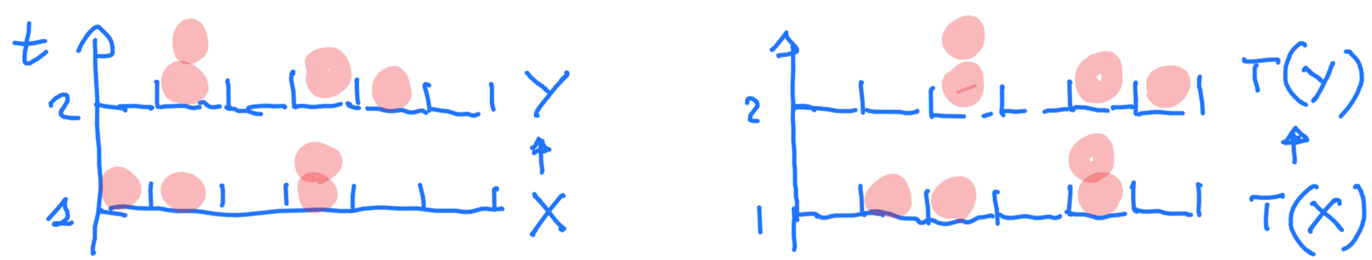
SPECIAL RELATIVITY + QUANTUM MECHANICS

TOY-UNIVERSE

POPULAR SYMMETRIES:

- SPACE TRANSLATION
- SPACE REVERSAL
- TIME TRANSLATION
- TIME REVERSAL (KOLMOGOROV CYCLING COND.)
- TRANSITION MATRIX SYMMETRY (+ ERODICITY $\to$ $\to$ STEADY STATE = EQUILIBRIUM)

EXAMPLE: SPACE TRANSLATION SYMMETRY:



$$B(X \to Y \mid X) = B(T(X) \to T(Y) \mid T(X))$$

RESULT (1)

TOY-UNIVERSE + POPULAR SYMMETRIES

$\Downarrow$

IT IS POSSIBLE TO CONSTRUCT MODELS FOR INDISTINGUISHABLE PARTICLES ($\equiv$ FIND $B(x \to y \mid x)$) WITHOUT 'GHOSTS'

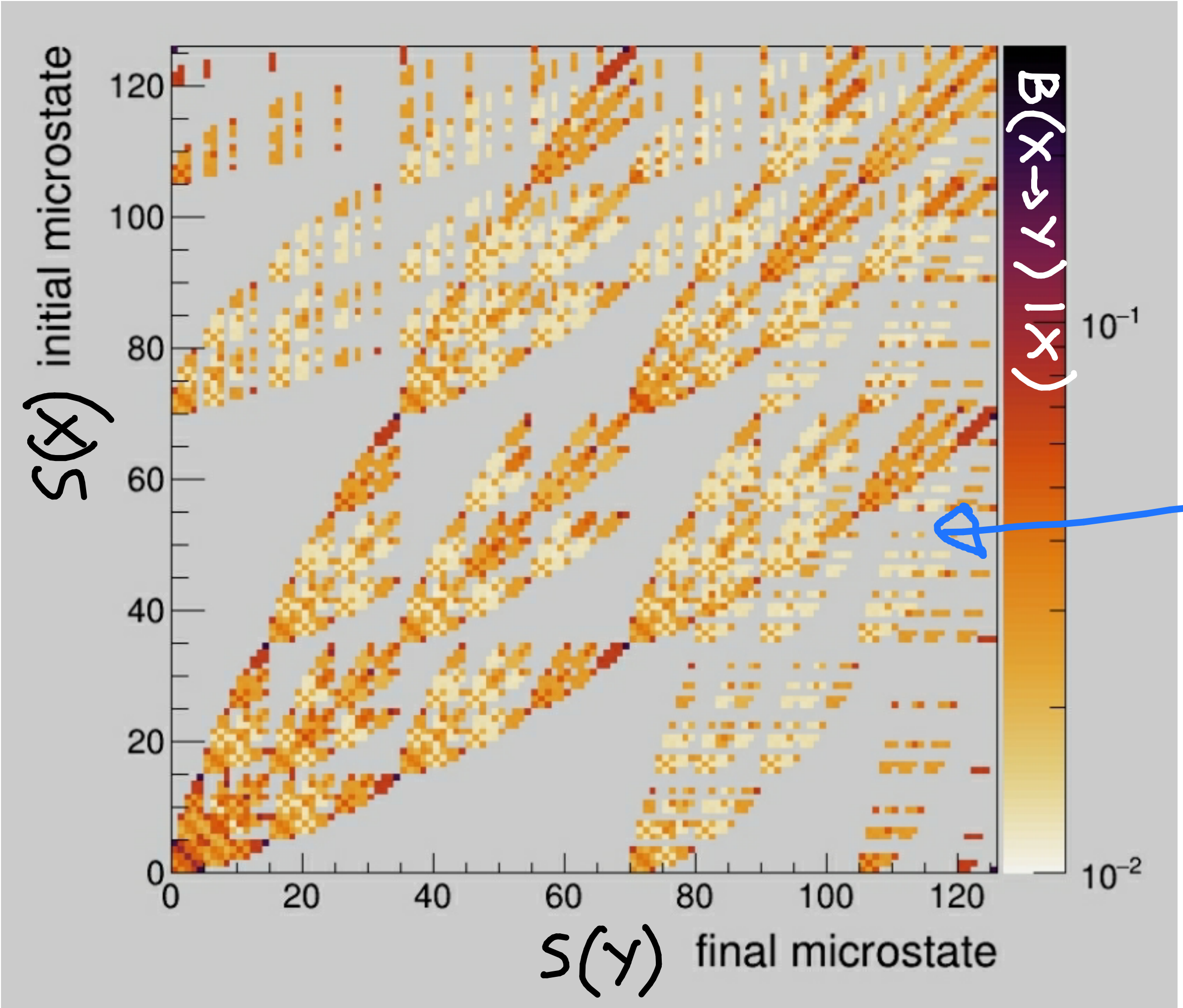
IT IS POSSIBLE, BUT NOT EASY.

IT TOOK US A YEAR OR TWO TO CONSTRUCT THEM.

RESULTS

EXAMPLE: TOY-UNIVERSE + POPULAR SYMMETRIES

THE MODELS WITHOUT 'GHOSTS' ARE RATHER COMPLICATED,



MANY DIFFERENT COLOURS

(GRAY – FORBIDDEN TELEPORTATION)

RESULTS

NOTE, MICROSTATE SEQUENTIAL NUMBERS S(X), S(Y) UNIQUELY RELATE TO MICROSTATES X AND Y, SEE THE PAPER, BUT THE RELATION IS ARBITRARLY SELECTED

MICROSTATE-SYMMETRY: THE STRONGEST POSSIBLE SYMMETRY

LET M BE ARBITRARY TRANSFORMATION THAT PRESERVES THE SET OF ALLOWED TRANSITIONS,

THEN B IS MICROSTATE-SYMMETRIC IF

$$B(X \to Y \mid X) = B(M(X) \to M(Y) \mid M(X)) \quad \forall M$$

$$\Downarrow$$

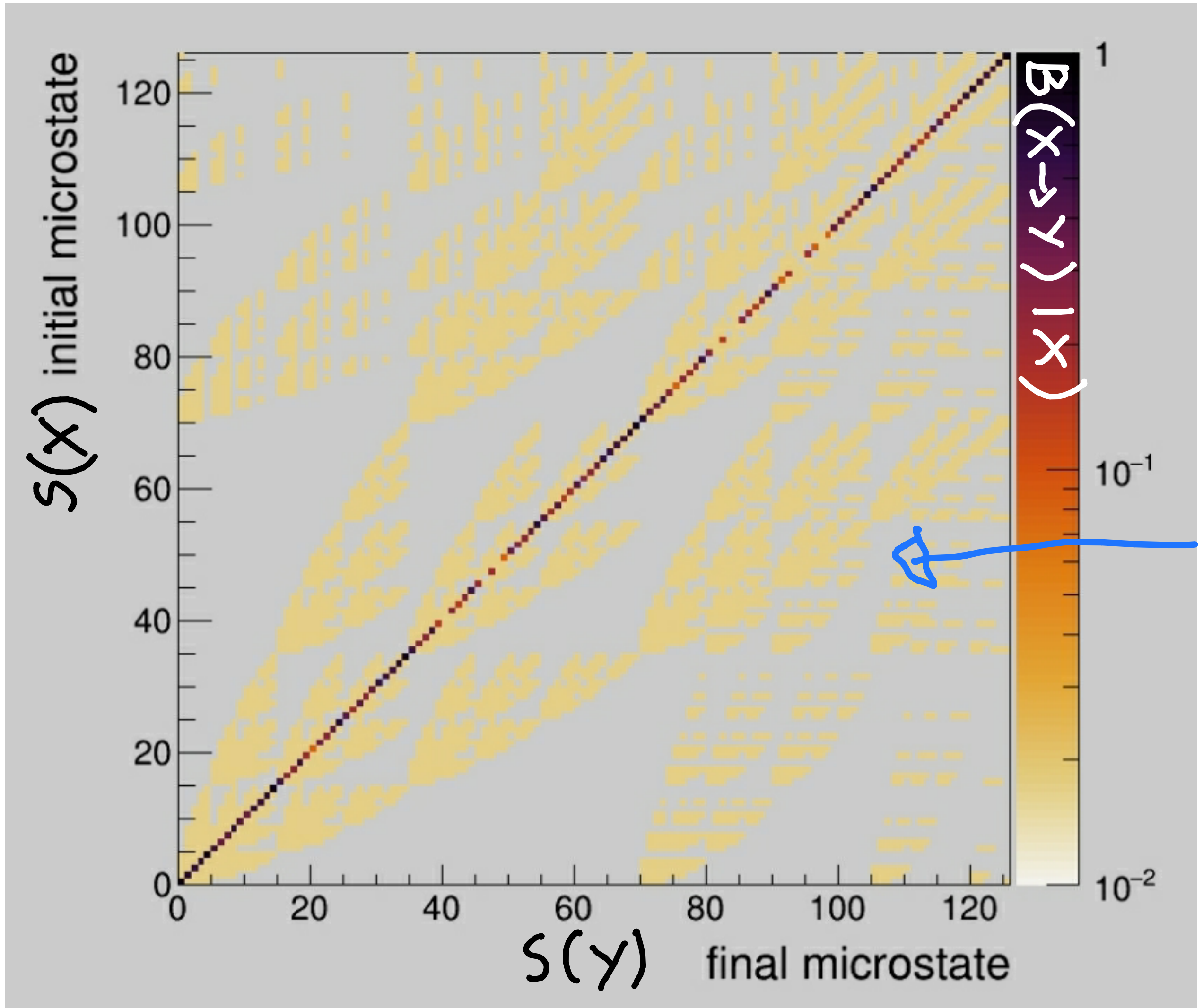
$$B(X \to Y \mid X) = \mathrm{CONST}(X, Y)$$

IT INCLUDES ALL POPULAR SYMMETRIES

MICROSTATE-SYMMETRY EQUILIBRIUM

HEAVY-ION COLLISIONS $\longleftrightarrow$ THE FASTEST APPROACH TO EQUILIBRIUM

RESULT (II): TOY UNIVERSE + MICROSTATE-SYMMETRY, BUT DIAGONAL, ALL MODELS HAVE 'GHOSTS' AND THEY ARE SIMPLE,



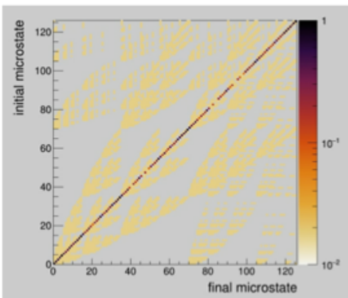
ONLY TWO NON-DIAGONAL COLOURS (GRAY - FORBIDDEN TELEPORTATION TRANSITIONS)

HI THERE ♡

RESULTS

WHY DO 'GHOSTS' EXIST ?

HI THERE !



THE SIMPLEST TOY-UNIVERSE (TELEPORTATION IS FORBIDDEN) REQUIRES 'GHOSTS'.

⇓

HYPOTHESIS:

'GHOSTS' EXIST BECAUSE OF SYMMETRIES.

THEY ARE NOT SPECIFIC TO QUANTUM MECHANICS

⇓

HUNTING FOR 'GHOSTS' BEYOND QUANTUM MECHANICS

HUNTING FOR 'GHOSTS' BEYOND QUANTUM MECHANICS

Strong Locality as a Tetrahedron: A Symmetry-Reduced Geometric Representation of the (3,3,2,2) Bell Scenario

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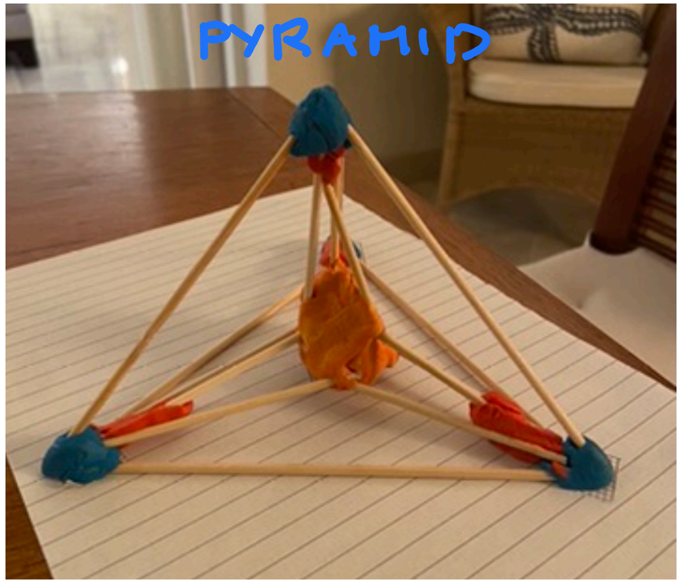
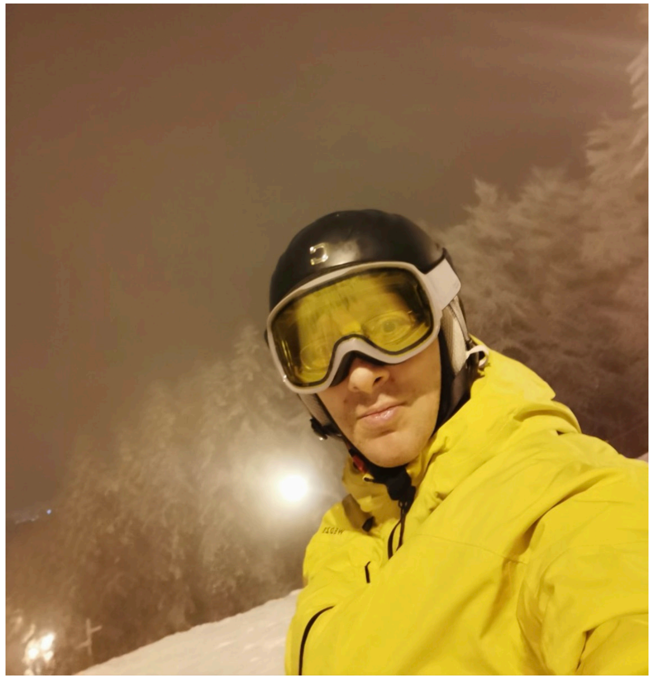


HAUNTERS:

PAWEŁ



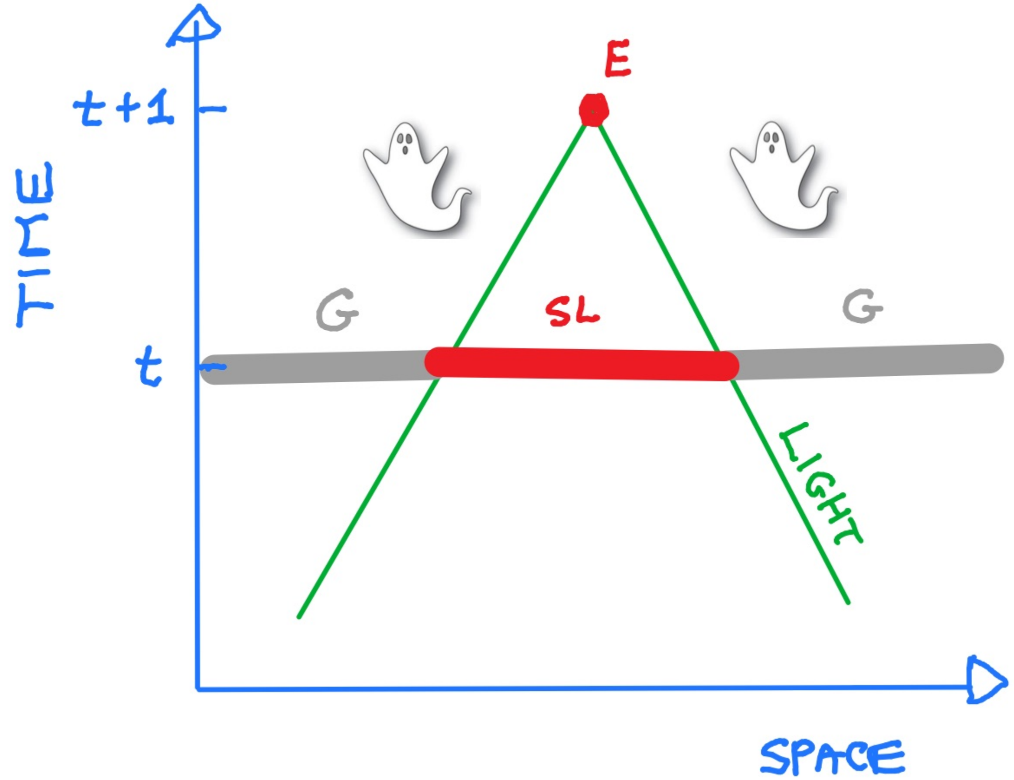
FRANCESCO

THE POST-COVID WINTER VACATIONS

MAREK

STRONG LOCALITY

SCREENING-OFF CRITERION

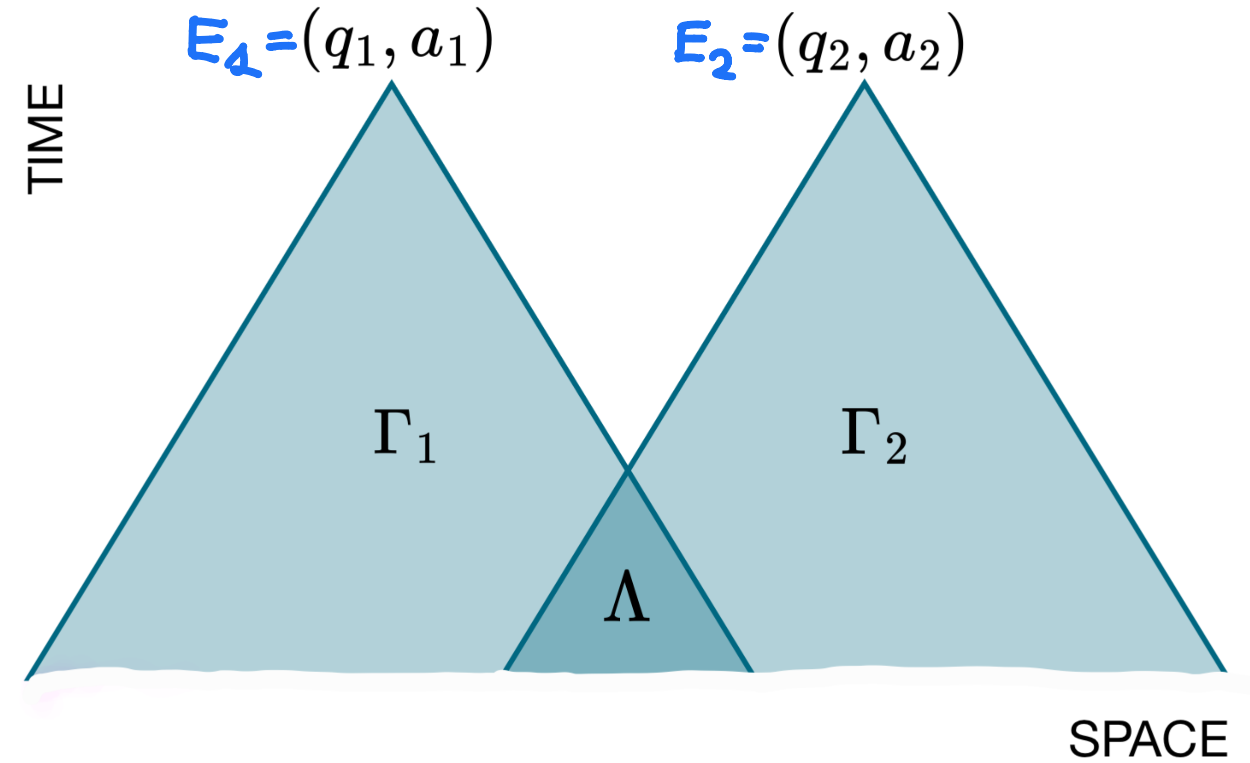


$P(E \mid SL, G) = P(E \mid SL)$

~ IMPRACTICAL IN THE REAL UNIVERSE

$\Rightarrow$

BELL'S PRACTICAL CRITERION

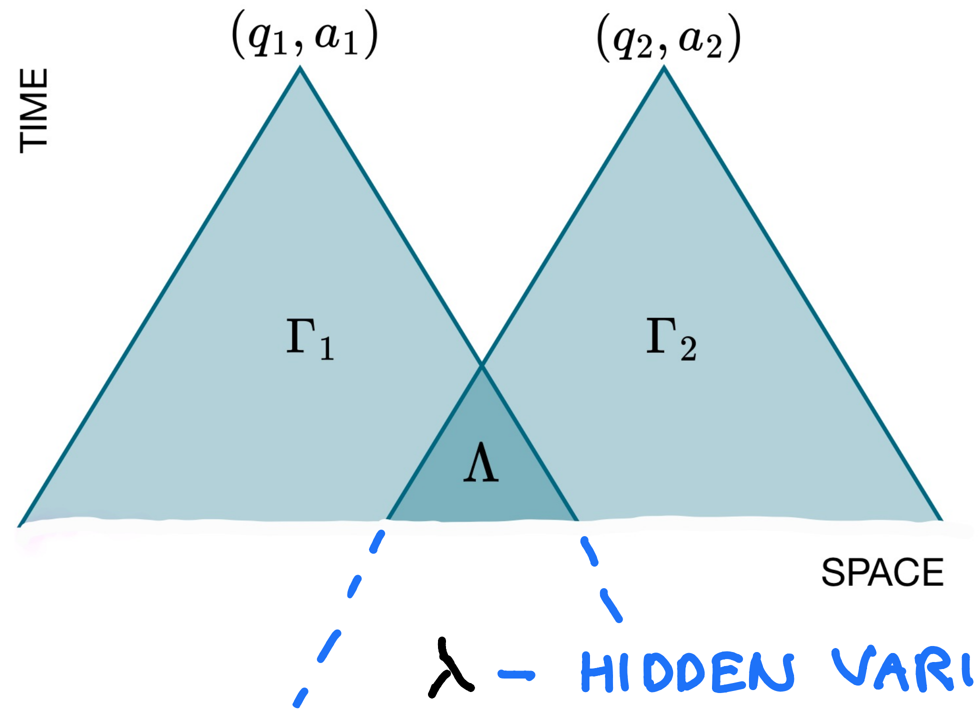


BELL INEQUALITIES WITH E_1, E_2

PRACTICAL, BUT WITH SUPERDETERMINISTC LOOPHOLE (SEE BELOW)

J. S. Bell. On the Einstein-Podolsky-Rosen paradox. *Physics Physique Fizika*, 1:195–200, 1964.

HIDDEN VARIABLE



λ – HIDDEN VARIABLE WHOSE VALUE IS SET IN THE COMMON PAST Λ OF E_1 AND E_2.

FREE-WILL POSTULATE:

$$h(q_1, q_2 | \lambda) = h(q_1, q_2)$$

$$\Downarrow$$

$$f(\lambda | q_1, q_2) = f(\lambda)$$

$\downarrow$

MAY NOT BE TRUE:
SUPERDETERMINISTIC LOOPHOLE

G. 't Hooft. The Free-Will Postulate in Quantum Mechanics. 1 2007.

$P(a_1, a_2 | q_1, q_2)$

IN SL MODELS

BELL'S DEFINITION OF STRONGLY LOCAL MODELS IN TERMS OF HIDDEN DISTRIBUTIONS:

$$p(a_1, a_2 | q_1, q_2, \lambda) = p(a_1 | q_1, \lambda) \cdot p(a_2 | q_2, \lambda)$$

(INDISTINGUISHABLE SITES)

$\downarrow$ AVERAGING OVER HIDDEN VARIABLE λ

THE OBSERVED DISTRIBUTION:

$$P(a_1, a_2 | q_1, q_2) = \sum_{\lambda} p(a_1 | q_1, \lambda) \, p(a_2 | q_2, \lambda) \, f(\lambda)$$

CHSH MEASURE S

CLAUSER
HORNE
SHIMONY
HOLT

THE (2,2,2,2) BELL'S CASE:

$q_1, q_2 \in \{0, 1\}$ — MEASUREMENT SETTINGS (QUESTIONS)

$a_1, a_2 \in \{-1, 1\}$ — MEASUREMENT OUTCOMES (ANSWERS)

OBSERVED DISTRIBUTION $P(a_1, a_2 \mid q_1, q_2)$ AND ITS MOMENTS

FOR EXAMPLE $M_{q_1 q_2} \equiv \langle a_1 a_2 \rangle_{q_1 q_2}$

$$S \equiv M_{00} + M_{01} + M_{10} - M_{11}$$

ALGEBRAIC LIMIT $|S| \leqslant 4$

IT IS 'EASY' TO SHOW THAT $|S| \leqslant 2$
IN STRONGLY LOCAL MODELS

J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt. Proposed experiment to test local hidden-variable theories. *Physical Review Letters*, 23:880–884, 1969.

S IN QUANTUM AND NON-SIGNALLING MODELS

QUANTUM MODELS:

$|S| \leq \sqrt{2} \cdot 2$ ← TSIRELSON BOUND

B. S. Tsirelson. Quantum generalizations of Bell's inequality. *Letters in Mathematical Physics*, 4:93–100, 1980.

NON-SIGNALING MODELS:

(MARGINAL DISTRIBUTION AT ONE LOCATION DOES NOT DEPEND ON SETTING AT OTHER LOCATION)

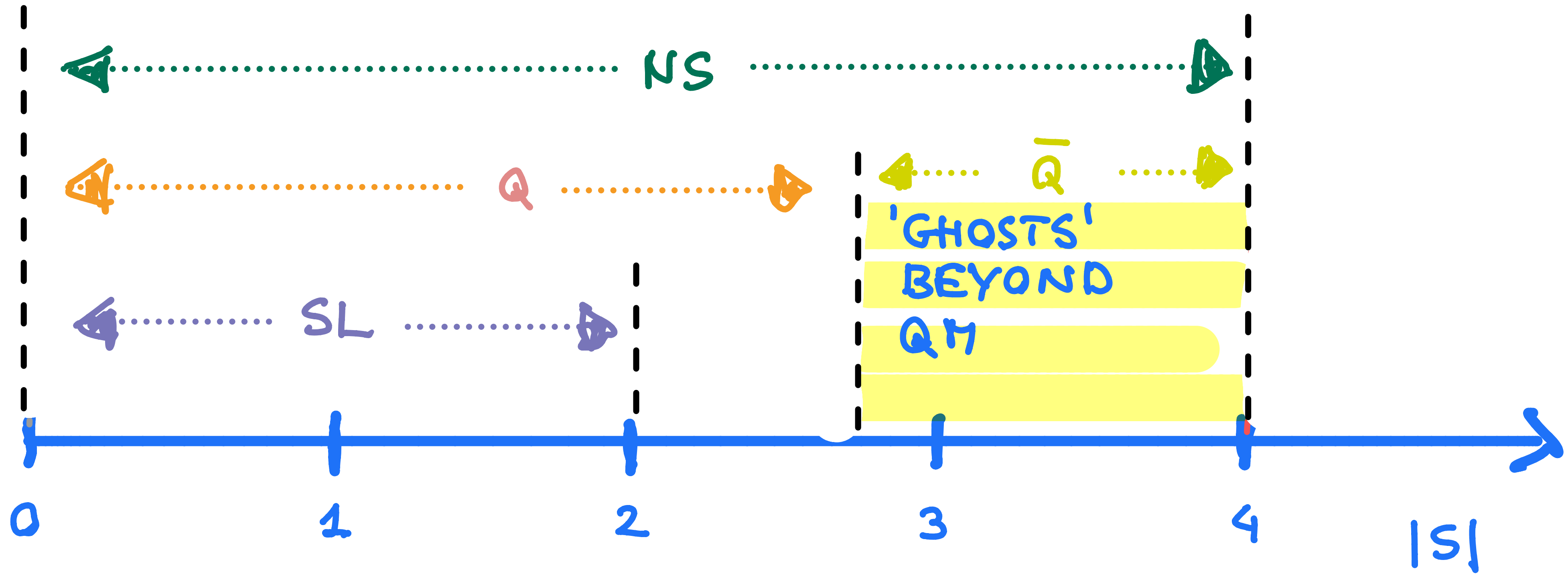
$|S| \leq 4$ ← ALGEBRAIC MAXIMUM

S. Popescu and D. Rohrlich. Quantum nonlocality as an axiom. *Found. Phys.*, 24(3):379–385, 1994.

THE GENERAL CONDITION FOR INDISTINGUISHABLE SITES:

$$P(a_1, a_2 | q_1, q_2) = P(a_2, a_1 | q_2, q_1)$$

S IN DIFFERENT MODELS



SL – STRONGLY LOCAL

Q – QUANTUM

NS – NON-SIGNALING
($P(\alpha_1 | q_1)$ INDEPENDENT OF $q_2, \ldots$)

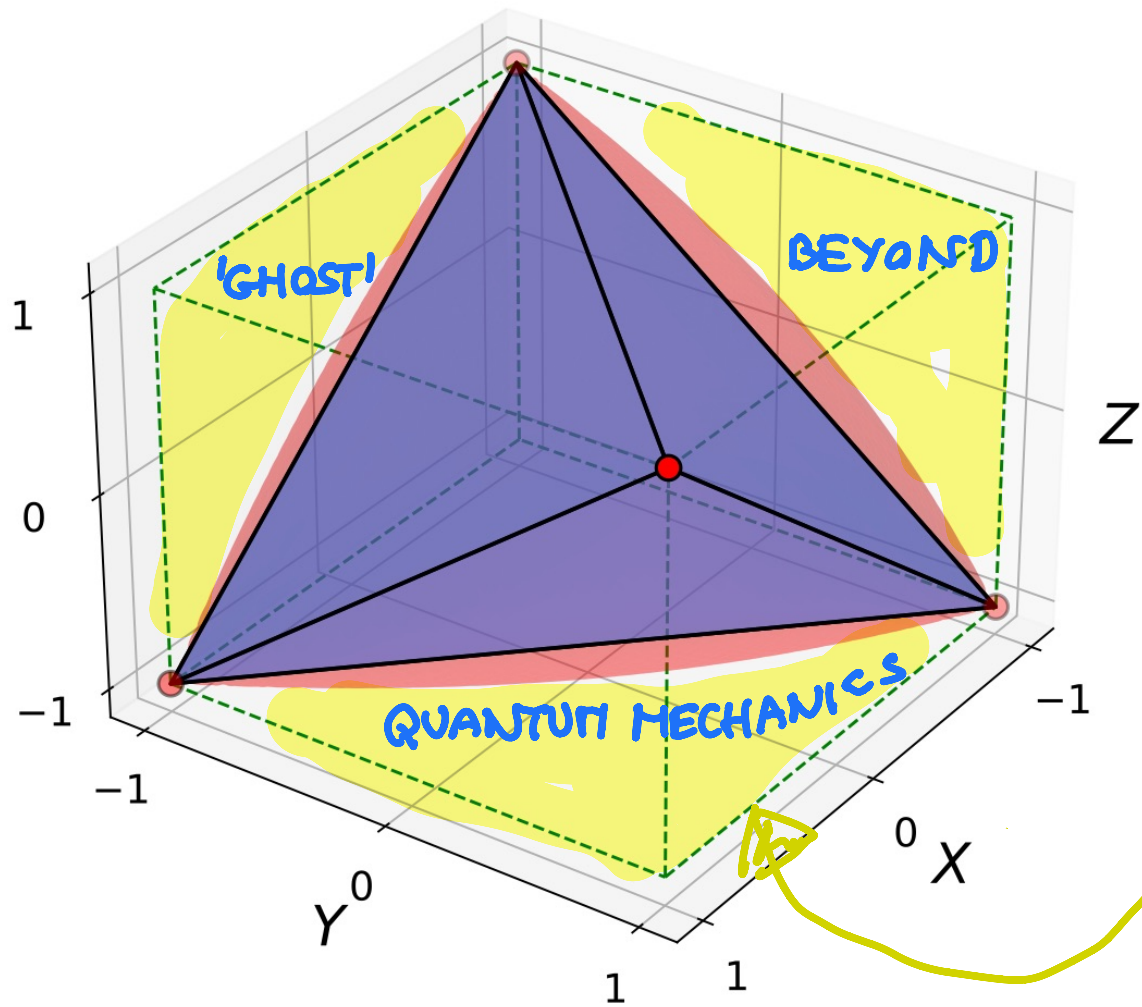
$\bar{Q}$ – POST-QUANTUM

PYRAMID METHOD

THE (3,3,2,2) BELL'S CASE
(THREE QUESTIONS, TWO ANSWERS)

ONLY OFF-DIAGONAL MIXED MOMENTS:

$X = M_{01}\,,\quad Y = M_{02}\,,\quad Z = M_{12}$



IT IS 'EASY' TO SHOW THAT:

SL – TETRAHEDRON

Q – ELLIPTOPE

$(\det(G) = 1 + 2XYZ - X^2 - Y^2 - Z^2 \geq 0)$

NS – NON-SIGNALING CUBE.

POST-QUANTUM $(\bar{Q})$

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PYRAMID METHOD

THE (3,3,2,2) BELL'S CASE
(THREE QUESTIONS, TWO ANSWERS)

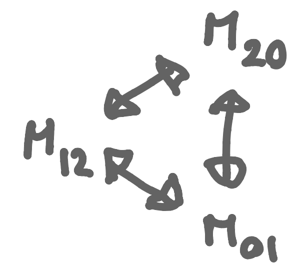
THE FULL POLYTOPE OF STRONGLY LOCAL MODELS LIVES IN 36 D SPACE OF CONDITONAL PROBABILITIES AND HAS 684 FACET-DEFINING (BELL) INEQUALITIES.

THE PYRAMID METHOD (NORMALISATION, SYMMETRY AND X-Y-Z TRICK) REDUCES THE POLYTOPE TO TETRAHEDRON (3D SPACE) AND NUMBER OF (BELL) INEQUALITIES TO TREE:

36 D ⟶ 3D

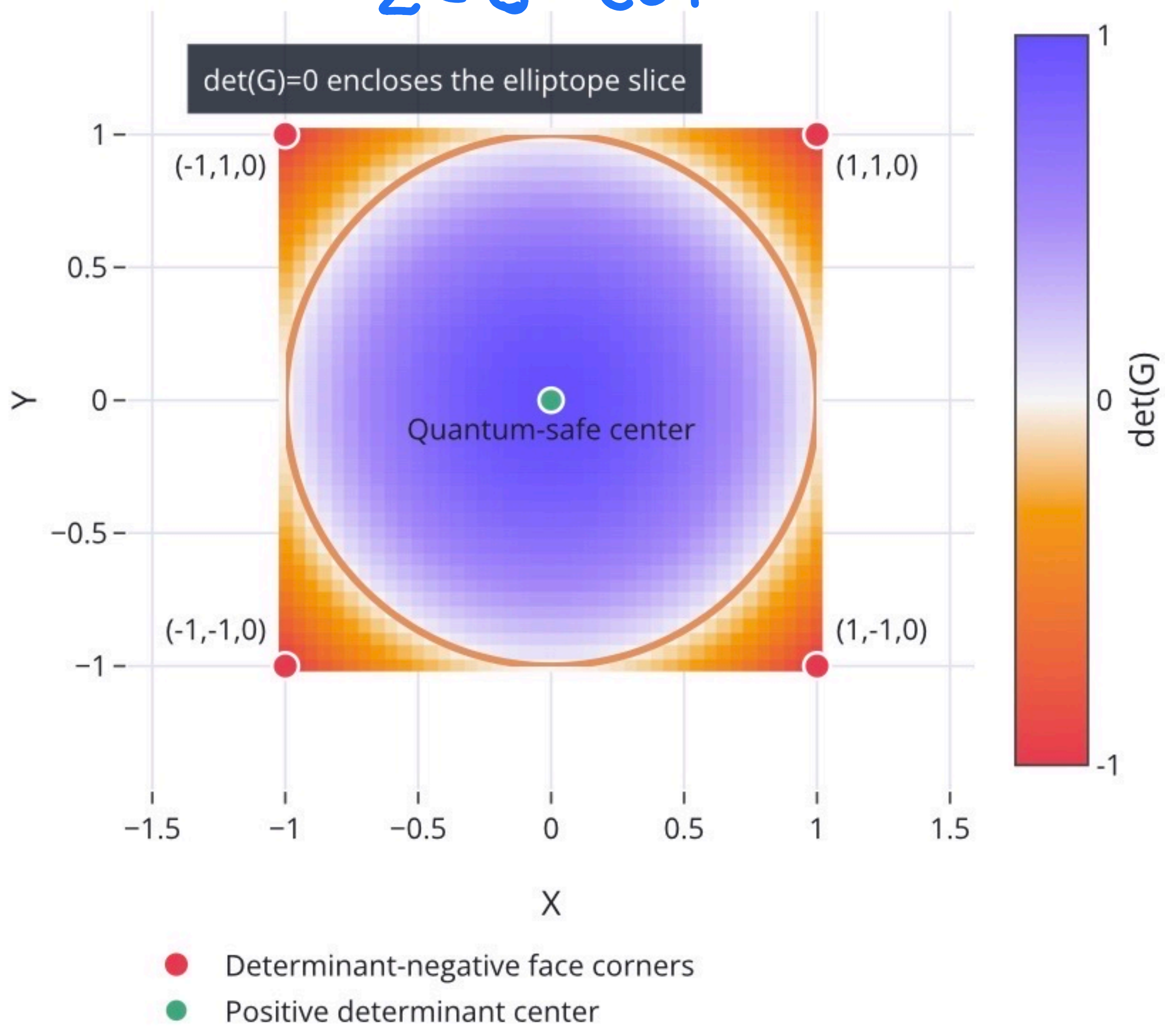
684 INEQ. ⟶ 3 INEQ.

CHSH vs PYRAMID

	CHSH	PYRAMID
# DIMENSIONS	1D	3D
TESTING CORRELATION TRANSITIVITY	−	+ 
DIAGONAL-FREE TESTING	−	+
SENSITIVITY TO $\overline{SL}$: $1 - V_{SL}/V_{NS}$	50% −	66% +
SENSITIVITY TO $\overline{Q}$ $1 - V_{Q}/V_{NS}$	30% −	38% +

PYRAMID ADVANTAGES (1) IN POST-QUANTUM SEARCH

Z = 0 CUT



$(\det(G) = 1 + 2XYZ - X^2 - Y^2 - Z^2 \geq 0)$

THE PYRAMID:

POST-QUANTUM MAXIMALLY VIOLATES TRANSITIVITY, FOR EXAMPLE:

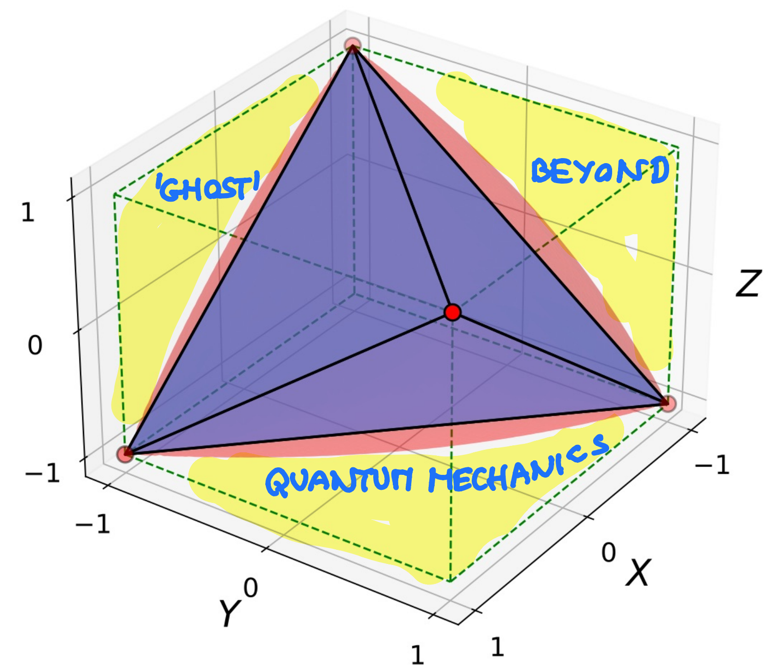
$X = 1$, $Y = 1$ | FULLY CORRELATED OUTCOMES

BUT

$Z = 0$ | UNCORRELATED OUTCOMES

NO TRANSITIVITY TESTING IN THE 1D CHSH CASE

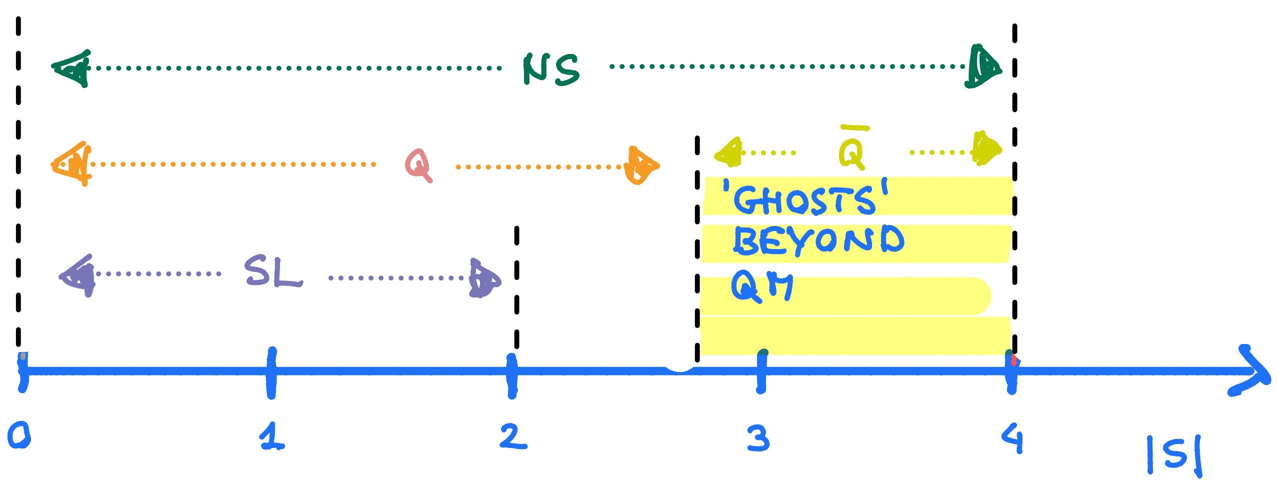
PYRAMID ADVANTAGES (II) IN POST-QUANTUM SEARCH



THE PYRAMID ALLOCATES A LARGE VOLUME FRACTION TO POST-QUANTUM

$\approx 38\%$

THIS COMPARES TO $\approx 30\%$ IN THE CHSH CASE.



SUMMARY

- THE CELL MODEL EXAMPLE SUGGESTS THAT VIOLATION OF STRONG LOCALITY IN NATURE MAY BE DUE TO SYMMETRY OF THE UNIVERSE → SEARCH FOR 'GHOSTS' BEYOND QUANTUM MECHANICS

Equilibration and Locality

Marek Gazdzicki, Mark Gorenstein, Ivan Pidhurskyi, Oleh Savchuk, Leonardo Tinti (Jun 2, 2022)

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- THE PYRAMID METHOD FOR STUDYING VIOLATION OF STRONG-LOCALITY IS WELL SUITED FOR 'GHOSTS' SEARCHING BEYOND QUANTUM MECHANICS.

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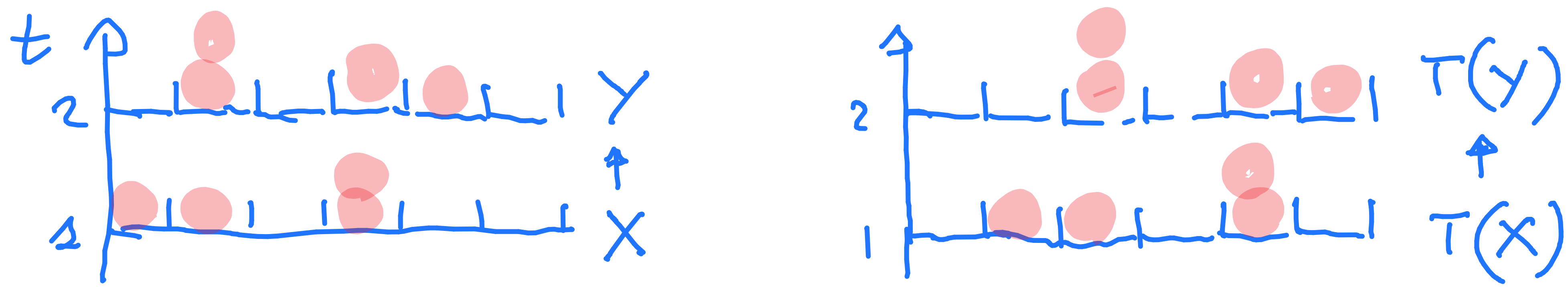
ADDITIONAL

SLIDES

SPACE-TRANSLATION SYMMETRY:

GIVEN A TRANSLATION T OF PARTICLES IN CELLS, TRANSITION PROBABILITY $X \to Y$ IS THE SAME AS PROBABILITY FOR TRANSLATED MICROSTATES $T(X) \to T(Y)$.



$$B(X \to Y \mid X) = B(T(X) \to T(Y) \mid T(X))$$

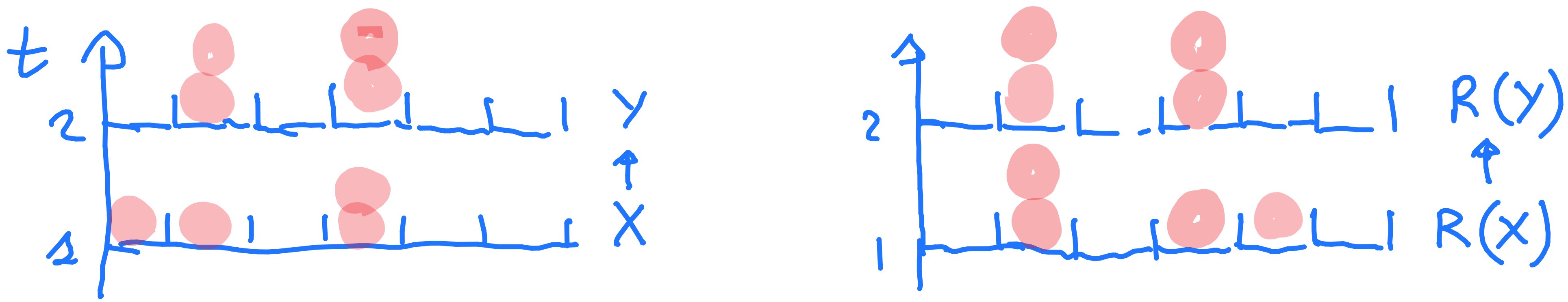
POPULAR SYMMETRIES

SPACE

SPACE-TRANSLATION INVARIANCE

SPACE-REVERSAL SYMMETRY:

GIVEN A REFLECTION R OF PARTICLES WITH RESPECT TO A CELL, TRANSITION PROBABILITY $X \to Y$ IS THE SAME AS PROBABILITY FOR REFLECTED MICROSTATES $R(X) \to R(Y)$



$$B(X \to Y \mid X) = B(R(X) \to R(Y) \mid R(X))$$

POPULAR SYMMETRIES
SPACE

SPACE-ROTATION INVARIANCE

TIME - TRANSLATION SYMMETRY:

THE TRANSITION PROBABILITY $X \to Y$ IS INDEPENDENT OF TIME:

$$B(X \to Y \mid X)(t) = \text{CONST}(t)$$

THE DICE DOES NOT CHANGE WITH TIME

$\boxed{\text{TIME-TRANSLATION INVARIANCE}}$

POPULAR SYMMETRIES

TIME

TIME-REVERSAL SYMMETRY:

FOR EVERY RECURRING TIME-SEQUENCE OF MICROSTATES $(x_1, x_2 \ldots x_m, x_1)$ THE PROBABILITY OF VISITING MICROSTATES IN THE ORIGINAL ORDER OR THE REVERSED ONE IS THE SAME:

$$B(x_1 \to x_2 | x_1) \cdot B(x_2 \to x_3 | x_2) \cdots B(x_{m-1} \to x_m | x_{m-1}) \cdot B(x_m \to x_1 | x_m) =$$

$$= B(x_1 \to x_m | x_1) \cdot B(x_m \to x_{m-1} | x_m) \cdots B(x_3 \to x_2 | x_3) \cdot B(x_2 \to x_1 | x_2)$$

(KOLMOGOROV CYCLING CONDITION FOR ERGODIC SYSTEMS)

POPULAR SYMMETRIES
TIME

TIME-REVERSAL INVARIANCE

TRANSITION-MATRIX SYMMETRY:

$$B(x \to y \mid x) = B(y \to x \mid y)$$

$+$ ERGODICITY $\to$ UNIQUE STEADY STATE

$\Downarrow$

STEADY STATE = EQUILIBRIUM STATE

PROBABILITY TO FIND A MICROSTATE x IN AN ENSEMBLE OF SYSTEMS FOR $t \to \infty$ IS INDEPENDENT OF TIME AND x:

$$\tilde{p}(x,t) = 1/W_{IND}$$

$$= 1/W_{DIST}$$

POPULAR SYMMETRIES

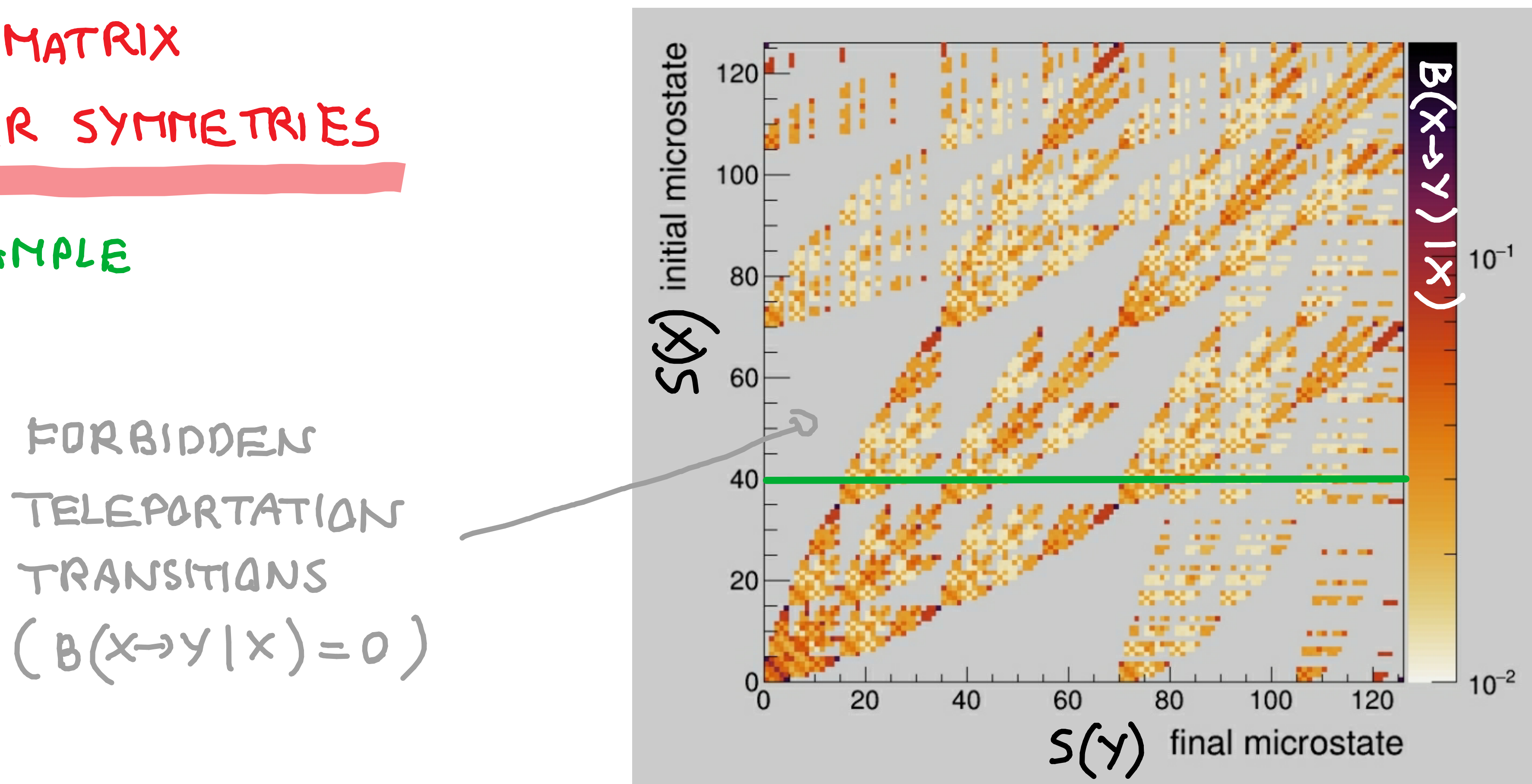
EQUILIBRIUM

EQUILIBRIUM STATES IN NATURE

EXAMPLE OF
TRANSITION MATRIX
WITH POPULAR SYMMETRIES

THE DICE EXAMPLE

THE DICE

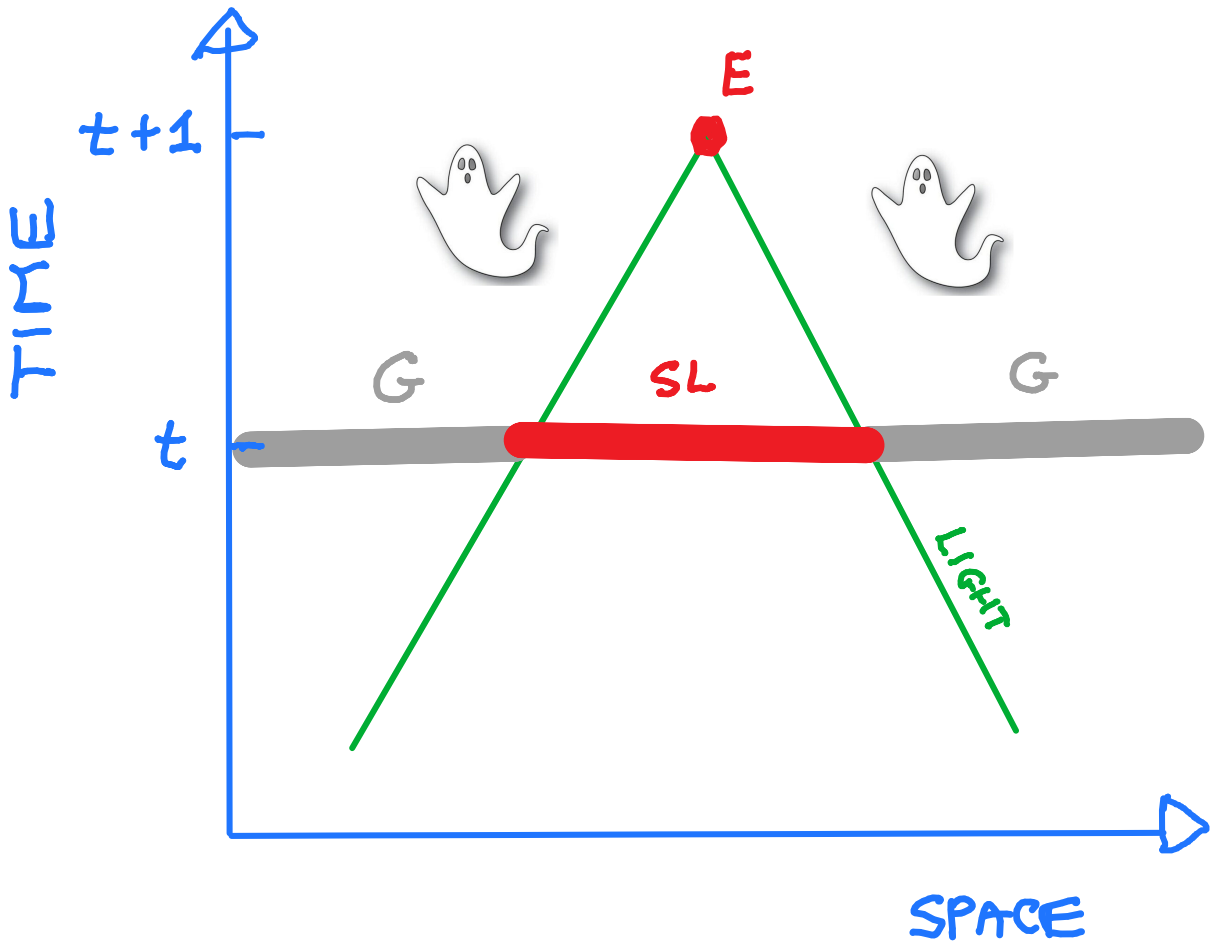


IN SPITE OF ALL SIMPLIFICATIONS,
IT IS RATHER COMPLICATED

NOTE, MICROSTATE SEQUENTIAL NUMBERS S(X), S(Y) UNIQUELY RELATE TO MICROSTATES
X AND Y, SEE THE PAPER, BUT THE RELATION IS ARBITRARLY SELECTED

REMINDER:

GHOSTS – DEFINITION:



IF $P(E \mid (SL, G)) \neq P(E \mid SL)$

- NON-TRIVIAL DEPENDENCE BETWEEN REMOTE EVENTS, 'GHOSTS' IN NATURE (VIOLATION OF STRONG LOCALITY)

RESULTS