Complete one-loop self-energies of the linear sigma model coupled to quarks at finite temperature and in a magnetic field

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Outline

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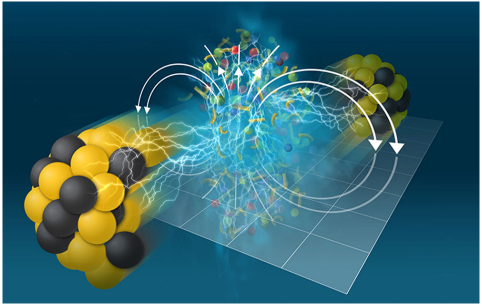
Introduction

Background / Motivation

- QCD exhibits **chiral symmetry breaking** in vacuum.
- Extreme environments (finite T, B) modify hadron properties.
- Self-energy encodes **medium effects**.
- Goal: Compute one-loop calculation in LSMq at finite temperature and external magnetic field and provide the neccesary ingredients fot the study of the QCD phase diagram beyond the mean field approximation



Linear sigma model with quarks

The Lagrangian density is given by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi_0)^2 + D_\mu \pi_- D^\mu \pi_+ + \frac{a^2}{2}(\sigma^2 + \pi_0^2 + 2\pi_-\pi_+) - \frac{\lambda}{4}\left(\sigma^2 + \pi_0^2 + 2\pi_-\pi_+\right) + i\bar{\psi}\partial\!\!\!/\psi - g\bar{\psi}(\sigma + i\gamma^5 \vec{\tau}\cdot\vec{\pi})\psi, \quad (1)$$

Free parameters

$$\lambda,\ g,\ a^2 > 0$$

Covariant derivative

$$D_\mu = \partial_\mu + iqA_\mu$$

$$A^\mu = \frac{B}{2}(0, -y, x, 0)$$

Spontaneous symmetry breaking

$$\sigma \to \sigma + v$$

- Describe QCD in the low-energy regime
- Renormalizable
- Exhibit spontaneously symmetry breaking
- Degrees of freedom: scalar and pseudoscalar mesons, and the two lightest quark flavors

Feynman Rules

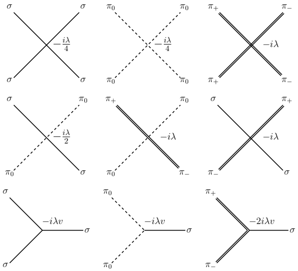


Figure: Feynman diagrams corresponding to the nine interaction vertices among the mesonic fields in the LSMq.

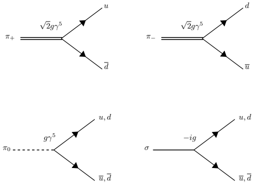
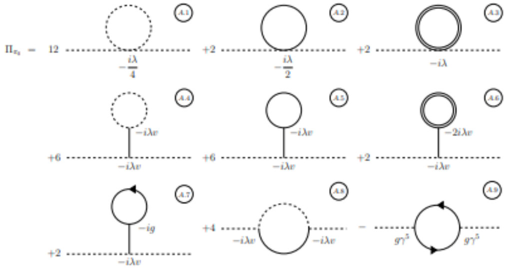


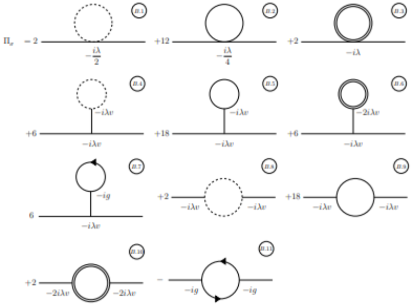
Figure: Feynman diagrams corresponding to the four interaction vertices between mesons and quarks fields in the LSMq.

Complete One-Loop Self-Energies

Neutral Pion



Sigma Meson



Complete set of one-loop self-energy diagrams computed in the LSMq.

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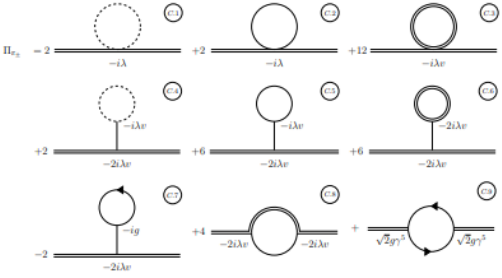
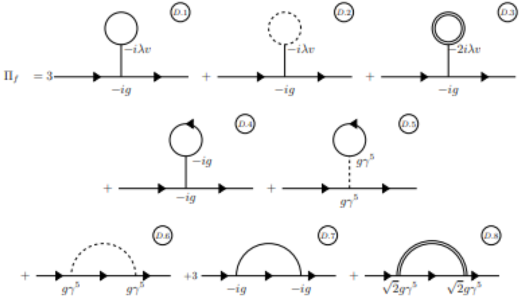
Charged Pion



Quark



Complete set of one-loop self-energy diagrams computed in the LSMq.

Methodology

- Complete One-Loop Self-Energies
  - Thermal (Matsubara)
  - Thermomagnetic (Matsubara + Schwinger PT)
    - Schwinger phases cancel
    - Schwinger phases do not cancel
      - Ritus Basis

Methodology: Thermal Contributions

$$T\sum_n \int \frac{d^3q}{(2\pi)^3} D(i\omega_n, \mathbf{q}) \qquad (2)$$

$$\lambda^2\, T\sum_n \int \frac{d^3q}{(2\pi)^3} D(q)D(k-q) \qquad (3)$$

We move to the imaginary-time formalism. After performing the sum over Matsubara frequencies, we obtain

$$\int \frac{d^3k}{(2\pi)^3}\frac{1}{2E_q}\left(\underbrace{1}_{\text{vacuum}} + \underbrace{2n_B(Eq)}_{\text{matter}}\right) \qquad (4)$$

where $E_q = \sqrt{q^2 + m^2}$

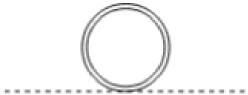
and

$$\int \frac{d^3q}{(2\pi)^3}\frac{1}{E_1E_2}\Bigg[\Big(1+n_B(E_1)+n_B(E_2)\Big)\left(\frac{1}{i\omega+E_1+E_2}-\frac{1}{i\omega-E_1-E_2}\right) + \Big(n_B(E_1)-n_B(E_2)\Big)\left(\frac{1}{i\omega-E_1+E_2}-\frac{1}{i\omega+E_1-E_2}\right)\Bigg], \quad (5)$$

where $E_1 = \sqrt{(\vec{k}-\vec{p})^2+m_1^2}$ and $E_2 = \sqrt{\vec{q}^2+m_2^2}$, and $n_B(X)$ is the B-E distribution function.
We identify two types of contributions inside the brackets:

- vacuum contribution: regularization and renormalized
- thermal contribution: finite UV and encodes medium effects

T-eB Contributions



$$T\sum_n \int \frac{d^3q}{(2\pi)^3} D^B(i\omega_n, \mathbf{q}) \tag{6}$$

- The propagators are modified by the presence of an external magnetic field and are expressed in Schwinger's proper-time representation:

$$D^B(i\omega_n, q_3, q_\perp) = \int_0^\infty \frac{ds}{\cos(|qB|s)} e^{is\left((i\omega_n)^2 - q_3^2 - q_\perp^2 \frac{\tan(|qB|s)}{|qB|s} - m^2\right)} \tag{7}$$

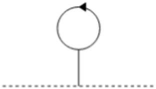
- Matsubara sums can be expressed in terms of Jacobi theta function:

$$\sum_n e^{-\tau\omega_n^2} = \vartheta_3(z, x) = 1 + 2\sum_{n=1}^{\infty} x^{n^2}\cos(2nz) \tag{8}$$

- General result

$$\frac{\lambda|eB|}{8\pi^2}\int_0^\infty \frac{d\tau}{\sinh(|eB|\tau)} \frac{e^{-\tau m_\pi^2}}{\tau}\left(1 + 2\sum_{n=1}^{\infty} e^{-\frac{n^2}{4T^2\tau}}\right).$$

T-eB Contributions



$$\lambda g D(0) N_c \sum_f T \sum_n \int \frac{d^3 q}{(2\pi)^3} \mathrm{Tr}[S^B(q)]$$

- Fermion propagator in PT formalism

$$S^B(i\tilde{\omega}_n, q_3, q_\perp) = \int_0^\infty \frac{ds}{\cos(|q_f B|s)} e^{is\left((i\tilde{\omega}_n)^2 - q_3^2 - q_\perp^2 \frac{\tan(|q_f B|s)}{|q_f B|s} - m_f^2\right)}$$
$$\times \left[\left(\cos(|q_f B|s) + \mathrm{sgn}(q_f B)\gamma^1\gamma^2 \sin(|q_f B|s)\right)\left(m_f - \gamma^0\tilde{\omega}_n - \gamma^3 q_3\right) - \frac{\not{q}_\perp}{\cos(|q_f B|s)} \right]$$

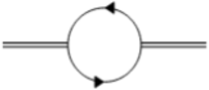
- Jacobi theta function

$$\sum_n e^{-\tau\tilde{\omega}_n^2} = \vartheta_4(z,x) = 1 + 2\sum_{n=1}^\infty (-1)^n x^{n^2} \cos(2nz)$$

- General result

$$\frac{\lambda v g}{2\pi^2 m^2} |q_f B| N_c \sum_f m_f \int_0^\infty \frac{d\tau}{\tanh(|q_f B|\tau)} \frac{e^{-\tau m_f^2}}{\tau} \left(1 + 2\sum_{n=1}^\infty (-1)^n e^{-\frac{n^2}{4T^2\tau}}\right).$$

$$\int d^4x \int d^4y\, F(x,k_1) D^B(x-y) D(y-x) F(y,k_2) \tag{9}$$



$$N_c \sum_f \int d^4x \int d^4y\, F(x,k_1) \mathrm{Tr}\left[S^B(x-y) S^B(y-x)\right] F(y,k_2) \tag{10}$$

- In these cases, the Schwinger phases associated with the propagators do not cancel
- The lack of cancelation of the Schwinger phases breaks translational invariance at the propagator level and requires a separate treatment.
- The computation must be carried out in coordinate space

Schwinger Phase

- In the presence of an external magnetic field, charged propagators carry a Schwinger phase factor that restores gauge invariance but breaks translational invariance.

$$D^B(x-y) = e^{i\phi(x,y)} \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} D^B(p) \tag{11}$$

$$S^B(x-y) = e^{i\phi(x,y)} \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} S^B(k) \tag{12}$$

with

$$\phi(x,y) = q_i \int_x^y d\xi_\mu \left[A^\mu(\xi) + \frac{1}{2} F^{\mu\nu}(\xi-y)_\nu \right] \tag{13}$$

- Schwinger phase represents the effect of the external electromagnetic field accumulated along the path between two space-time points

Ritus functions

The Ritus functions allow us

- Provide the appropriate basis for charged particles in the presence of an external magnetic field.
- Restore a diagonal momentum-space representation of the charged-pion self-energy despite the loss of translational invariance induced by the Schwinger phase.

For scalar fields, the Ritus functions $F(x,k)$ are

$$F(x,k) = \frac{1}{\sqrt{2\pi}} e^{-i(k^0 x^0 - k^3 x^3)} e^{-i\,\mathrm{sgn}(qB)(k^1 - k^2)\theta} R_{k^1,k^2}(r) \tag{14}$$

The Ritus functions for fermions are

$$U(y,k_2) = E(y,k_2)u(k_2), \qquad \bar{U}(x,k_1) = U^\dagger(x,k_1)\gamma^0 \tag{15}$$

with

$$E(x,k_1) = \sum_{\lambda=\pm} \Gamma^\lambda F(x,k_{1\lambda}) \tag{16}$$

[Ritus, Ann. Phys. 69 (1972) 555–582]

$$g^2 \int d^4x \int d^4y\, \bar{U}(x,k_1)\gamma^5 S^B(x-y)\gamma^5 D(y-x)U(y,k_2) \tag{17}$$

$$g^2 \int d^4x \int d^4y\, \bar{U}(x,k_1)\gamma^5 S^B(x-y)\gamma^5 D^B(y-x)U(y,k_2) \tag{18}$$

The general Dirac structure of the one-loop quark self-energy in Euclidean space can be written as

$$\Pi_f(i\omega,\vec{k}) = A(i\omega,\vec{k})i\gamma^0 - B(i\omega,\vec{k})\vec{\gamma}\cdot\vec{k} + C(i\omega,\vec{k}) \tag{19}$$

where the scalar functions A, B and C encode the thermal and magnetic corrections to the fermion propagator

Main Results

- Complete one-loop self-energies were obtained for
  - Neutral pion (π^0)
  - Charged pion ($\pi^\pm$)
  - Sigma meson (σ)
  - Quarks (u, d)
- Finite-temperature effects were incorporated using the Matsubara formalism.
- Magnetic-field effects were included through Schwinger's proper-time representation.
- Charged external states were consistently treated using the Ritus formalism.
- Vacuum, thermal, magnetic and thermomagnetic contributions were explicitly separated.

Main Results

$$\Pi_{\pi^0} = \sum_{i=1}^{9} \Pi_{\pi^0}^{(A.i)} = \Pi_{\text{vac}} + \Pi_T + \Pi_{eB} + \Pi_{T,eB},$$

$$\Pi_{\sigma} = \sum_{i=1}^{11} \Pi_{\sigma}^{(B.i)} = \Pi_{\text{vac}} + \Pi_T + \Pi_{eB} + \Pi_{T,eB},$$

$$\Pi_{\pi^\pm} = \sum_{i=1}^{9} \Pi_{\pi^\pm}^{(C.i)} = \Pi_{\text{vac}} + \Pi_T + \Pi_{eB} + \Pi_{T,eB},$$

$$\Sigma_q = \Sigma_{\text{vac}} + \Sigma_T + \Sigma_{eB} + \Sigma_{T,eB}.$$

Key Features

- Valid for arbitrary temperature and magnetic-field strength.

Preprint available at:
https://arxiv.org/abs/2605.14372

Summary and outlook

- We computed the complete one-loop self-energies in the linear sigma model coupled to quarks at finite temperature and magnetic field.
- This framework allows the extraction of **thermomagnetic effects** over the masses.
- The formalism can be used to compute the dynamics **beyond the mean field approximation of QCD-like matter** and provide relevant information.

Thank You

Acknowledgements

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Back up: Full LSMq Lagrangian

In order to allow for spontaneous symmetry breaking, the σ field develops a vacuum expectation value v, namely

$$\sigma \to \sigma + v. \tag{20}$$

After performing this shift, the Lagrangian can be written as

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2}\partial_\mu \pi_0 \partial^\mu \pi_0 + D_\mu \pi_- D^\mu \pi_+ \\ &- \frac{1}{2}m_\sigma^2 \sigma^2 - \frac{1}{2}m_\pi^2 \pi_0^2 - m_\pi^2 \pi_- \pi_+ + i\bar{\psi}\partial\!\!\!/ \psi \\ &- m_f \bar{\psi}\psi + \frac{a^2}{2}v^2 - \frac{\lambda}{4}v^4 + \mathcal{L}_{int}.\end{aligned} \tag{21}$$

From Eq. (21) we identify the masses of the fields as

$$m_\sigma^2 = 3\lambda v^2 - a^2, \qquad m_\pi^2 = \lambda v^2 - a^2, \qquad m_f = gv. \tag{22}$$

The $\mathcal{L}_{int}$ contains the interaction terms among the fields and is defined as

$$\begin{aligned}\mathcal{L}_{int} =& -\frac{\lambda}{4}\sigma^4 - \lambda v \sigma^3 - \lambda v^3 \sigma - \lambda \sigma^2 \pi_- \pi_+ - 2\lambda v \sigma \pi_- \pi_+ \\ &- \frac{\lambda}{2}\sigma^2 \pi_0^2 - \lambda v \sigma \pi_0^2 - \lambda \pi_-^2 \pi_+^2 - \lambda \pi_- \pi_+ \pi_0^2 - \frac{\lambda}{4}\pi_0^4 \\ &+ a^2 v \sigma - g\bar{\psi}\psi\sigma - ig\gamma^5 \bar{\psi}\left(\tau_+ \pi_+ + \tau_- \pi_- + \tau_3 \pi_0\right)\psi.\end{aligned} \tag{23}$$

The interaction terms contained in $\mathcal{L}_{int}$ determine the Feynman rules required for the perturbative expansion of the theory.

Some results...

$$
\begin{aligned}
-i\Pi^{(A.9)}_{\pi_0} = {} & \frac{ig^2}{(2\pi)^2} N_c \sum_f \int_0^\infty du \int_0^1 dv \frac{|q_f B|}{\sinh(|q_f B| u)} e^{-uv(1-v)k_3^2} e^{-m_f^2 u} e^{(i\omega)^2 uv(1-v)} \\
& \times e^{-\frac{k_\perp^2}{|q_f B|} \frac{\sinh(|q_f B| u(1-v)) \sinh(|q_f B| uv)}{\sinh(|q_f B| u)}} \Bigg\{ \cosh(|q_f B| u(1-2v)) \Bigg[\Bigg(1 + 2\sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2}{4T^2 u}} \\
& \times \cosh\left(\frac{(1-v)(i\omega)n}{T} \right) \Bigg) \left(m_f^2 + \frac{1}{u} \left(uv(1-v)k_3^2 - \frac{1}{2} \right) \right) \\
& - i\omega \left[(1-v)(i\omega) + 2\sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2}{4T^2 u}} \left((1-v)(i\omega) \cosh\left(\frac{(1-v)(i\omega)n}{T} \right) - \frac{n}{2Tu} \sinh\left(\frac{(1-v)(i\omega)n}{T} \right) \right) \right] \\
& - \Bigg(\frac{1}{2u} - (1-v)^2 (i\omega)^2 + 2\sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2}{4T^2 u}} \Bigg[\left(\frac{1}{2u} - (1-v)^2 (i\omega)^2 - \frac{n^2}{4T^2 u^2} \right) \cosh\left(\frac{(1-v)(i\omega)n}{T} \right) \\
& + \frac{n(1-v)(i\omega)}{Tu} \sinh\left(\frac{(1-v)(i\omega)n}{T} \right) \Bigg] \Bigg) \Bigg] \\
& + \frac{1}{\sinh^2(|q_f B| u)} \left[k_\perp^2 \sinh(|q_f B| u(1-v)) \sinh(|q_f B| uv) - |q_f B| \sinh(|q_f B| u) \right]
\end{aligned}
$$

Some results....

$$
\begin{aligned}
-i\Pi_f^{(D.8)} = &-\frac{ig^2}{4\pi^2}\frac{1}{M}\int_0^\infty du\int_0^1 dv\frac{1}{\cosh(|q_fB|uv)}\frac{1}{\cosh(|eB|u(1-v))}e^{-u(1-v)m_\pi^2}e^{-uvm_f^2}e^{-uv(1-v)(i\omega)^2}\\
&\times\frac{|eB||q_fB||q_{f'}B|}{2|eB|\tanh(|q_fB|uv)+2|q_fB|\tanh(|eB|u(1-v))+|q_{f'}B|\tanh(|eB|u(1-v))\tanh(|q_fB|uv)}\\
&\times\Big(\cosh(|q_fB|uv)-is_f\gamma^1\gamma^2\sinh(|q_fB|uv)\Big)\\
&\times\Bigg[\Big(m_f-i\gamma^0(i\omega)\Big)\Bigg[1+2\sum_{n=1}^\infty e^{-\frac{n^2}{4uT^2}}\cosh\left(\frac{v(i\omega)n}{T}\right)\Bigg]+\gamma^0\Bigg\{iv(i\omega)\\
&+2\sum_{n=1}^\infty(-1)^ne^{-\frac{n^2}{4uT^2}}\left[iv(i\omega)\cosh\left(\frac{v(i\omega)n}{T}\right)-\frac{in}{2uT}\sinh\left(\frac{v(i\omega)n}{T}\right)\right]\Bigg\}\Bigg].
\end{aligned}
\tag{25}
$$