Schwinger Pair Production, Thermal Spectra & Confining Density Functionals

David Blaschke

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CASUS
CENTER FOR ADVANCED SYSTEMS UNDERSTANDING

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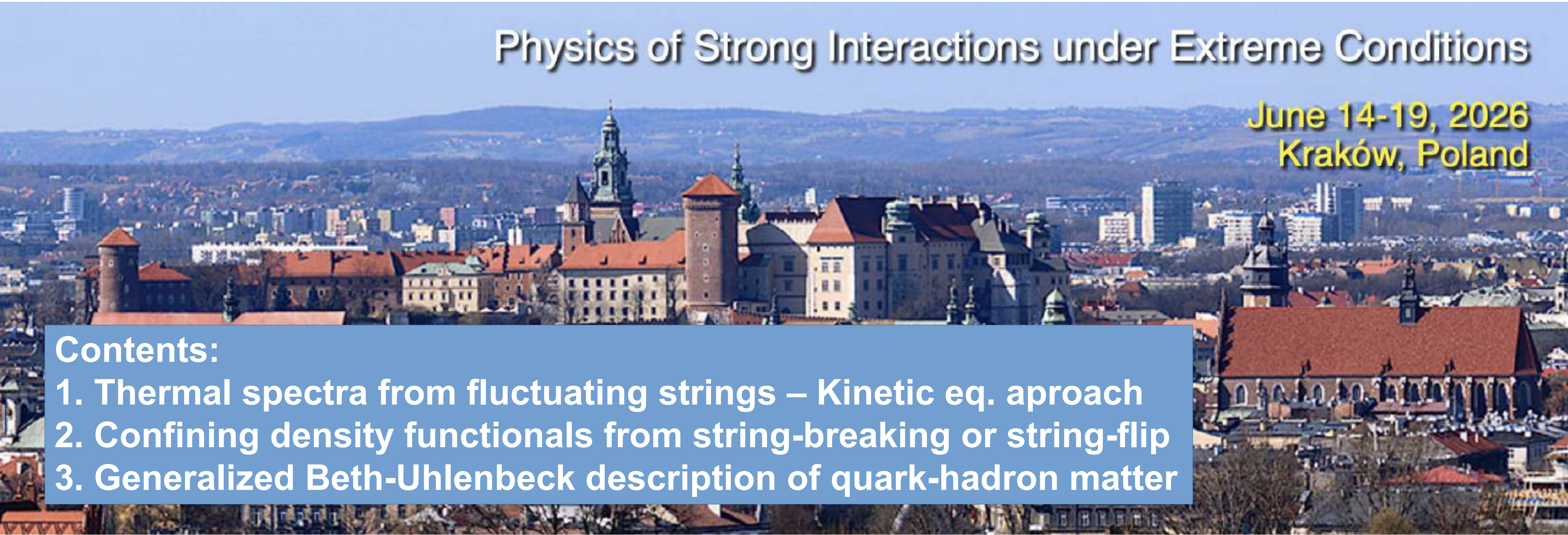


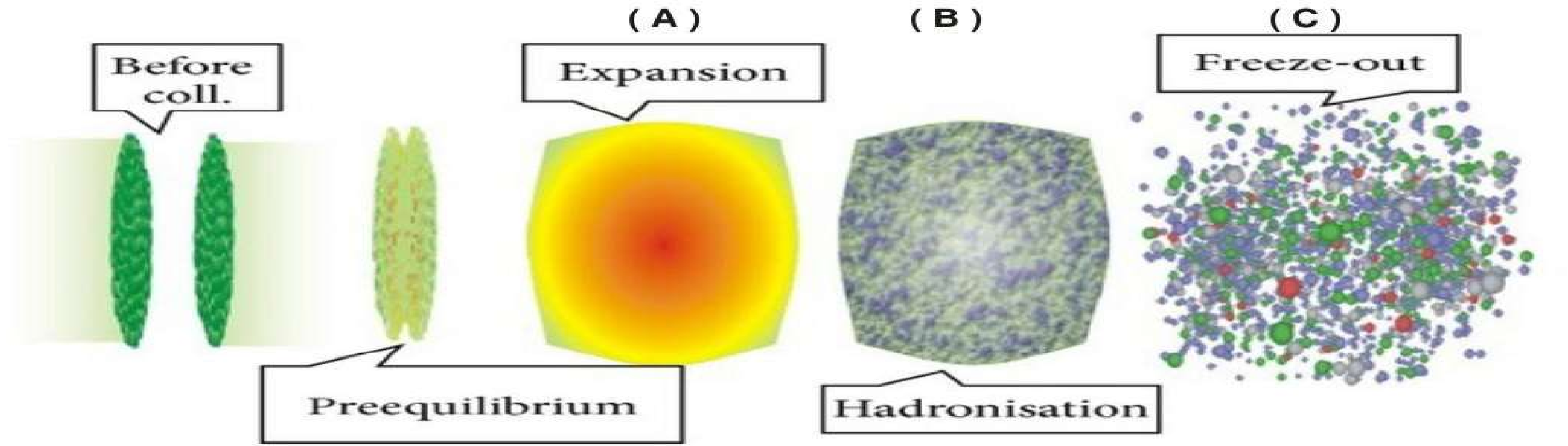
Contents:

1. Thermal spectra from fluctuating strings – Kinetic eq. aproach
2. Confining density functionals from string-breaking or string-flip
3. Generalized Beth-Uhlenbeck description of quark-hadron matter

High
Energy
Density
Institute

18. June 2026

Particle Production in Strong, Time-dependent Fields



Generic kinetic equation with scalar (mass) and color meanfields, Schwinger source terms and collision integrals for hadronization and rescattering

$$\left[\partial_t + \frac{1}{E_X}\vec{p}\cdot\vec{\nabla} - \frac{m_X(\vec{x},t)}{E_X}\vec{\nabla}m_X(\vec{x},t)\cdot\vec{\nabla}_p + \vec{F}(\vec{x},t)\cdot\vec{\nabla}_p\right] f_X(\vec{p},\vec{x};t)$$
$$= S_X^{\rm Schwinger}\{f_q, f_{\bar{q}}, f_\pi, \dots\} + C_X^{\rm gain}\{f_q, f_{\bar{q}}, f_\pi, \dots\} - C_X^{\rm loss}\{f_q, f_{\bar{q}}, f_\pi, \dots\}$$

(A) quark-antiquark pair creation in time-dependent color electric background field
(B) quantum kinetics of pre-hadron inelastic rescattering in the dense quark plasma
(C) chemical freeze-out by Mott-Anderson localization of bound states

4 November 1999
PHYSICS LETTERS B
ELSEVIER
Physics Letters B 466 (1999) 301–304

Lesson 1:

Fluctuations of the string tension and transverse mass distribution

A. Bialas [a,b]

"Schwinger" **"Thermal"**

$$\frac{dn_\kappa}{d^2p_\perp} \sim e^{-\pi m_\perp^2/\kappa^2}, \; m_\perp = \sqrt{p_\perp^2 + m^2}. \quad \longrightarrow \quad \frac{dn}{d^2p_\perp} \sim \exp\left(-m_\perp\sqrt{\frac{2\pi}{\langle\kappa^2\rangle}}\right), \quad T = \sqrt{\frac{\langle\kappa^2\rangle}{2\pi}}.$$

(universal temperature)

$$P(\kappa)\,d\kappa = \sqrt{\frac{2}{\pi\langle\kappa^2\rangle}}\exp\left(-\frac{\kappa^2}{2\langle\kappa^2\rangle}\right)d\kappa, \quad \langle\kappa^2\rangle = \int_0^\infty P(\kappa)\,\kappa^2\,d\kappa.$$

(averaged string tension)

$$\frac{dn}{d^2p_\perp} \sim \int_0^\infty d\kappa\, P(\kappa)\, e^{-\pi m_\perp^2/\kappa^2} = \frac{\sqrt{2}}{\sqrt{\pi\langle\kappa^2\rangle}}\int_0^\infty d\kappa\, e^{-\kappa^2/2\langle\kappa^2\rangle} e^{-\pi m_\perp^2/\kappa^2} \sim \exp\left(-m_\perp\sqrt{\frac{2\pi}{\langle\kappa^2\rangle}}\right)$$

$$\int_0^\infty dt\, e^{-st}\frac{u}{2\sqrt{\pi t^3}} e^{-u^2/4t} = e^{-u\sqrt{s}}.$$

See also:
D. Blaschke et al., arXiv:1704.04147; Particles 2 (2019) 166

Vol. 35 (2004) ACTA PHYSICA POLONICA B No 2

SCHWINGER TUNNELING AND THERMAL CHARACTER OF HADRON SPECTRA

Wojciech Florkowski

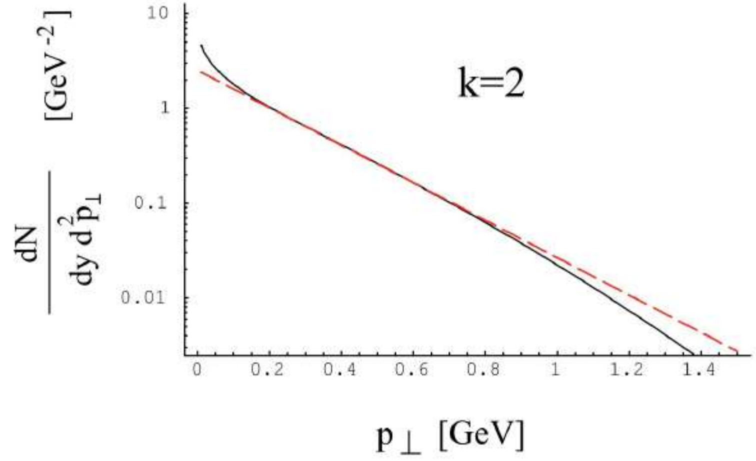
$$\left(p^\mu \partial_\mu \pm g\boldsymbol{\epsilon}_i \cdot \boldsymbol{F}^{\mu\nu} p_\nu \partial^p_\mu\right) G_i^\pm(x,p) = \frac{dN_i^\pm}{d\Gamma}$$

Parton kinetic equation with Schwinger source term

$$\frac{dN}{d\Gamma} = p^0 \frac{dN}{d^4x\, d^3p} = \frac{F}{4\pi^3}\left|\ln\left(1 \mp \exp\left(-\frac{\pi p_\perp^2}{F}\right)\right)\right| \delta\left(w - w_0\right) v\,, \quad w_0 = -\frac{p_\perp^2}{2F}\,,$$

$$\frac{dN}{dy\, d^2p_\perp} = \int d^4x \frac{dN}{d\Gamma} = \pi R^2 \int\limits_0^\infty d\tau'\, \tau' \int\limits_{-\infty}^{+\infty} d\eta\, \mathcal{R}(\tau', p_\perp) \delta\left(w \mp w_0\right) v = \pi R^2 \int\limits_0^\infty d\tau'\, \tau'\, \mathcal{R}(\tau', p_\perp)\,,$$

$$\frac{dN}{dy\, d^2p_\perp} = \frac{R^2}{4\pi^2} \sum_{\text{all partons}} \int\limits_0^\infty d\tau'\, \tau'\, F(\tau') \left|\ln\left(1 \mp \exp\left(-\frac{\pi p_\perp^2}{F(\tau')}\right)\right)\right|.$$



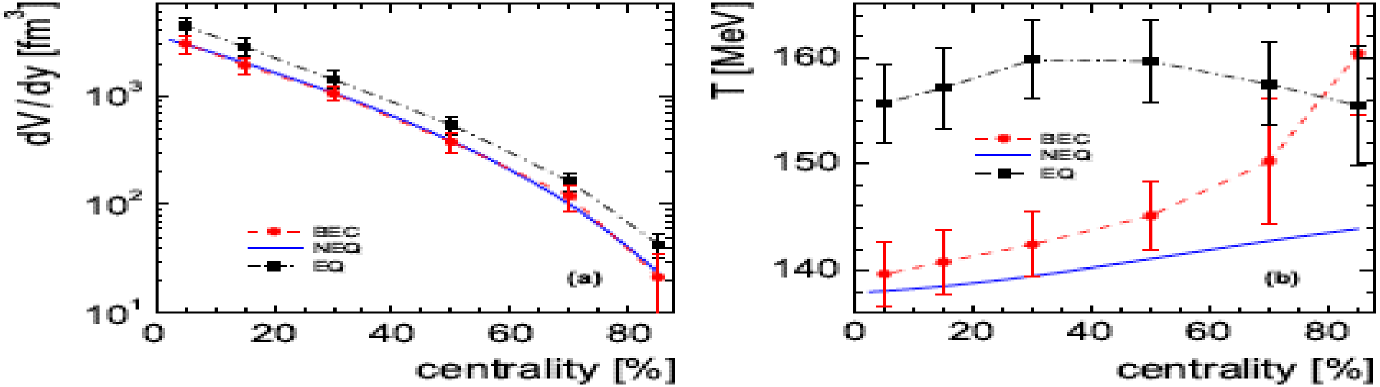
PHYSICAL REVIEW D **88**, 034028 (2013)

Equilibration of anisotropic quark-gluon plasma produced by decays of color flux tubes

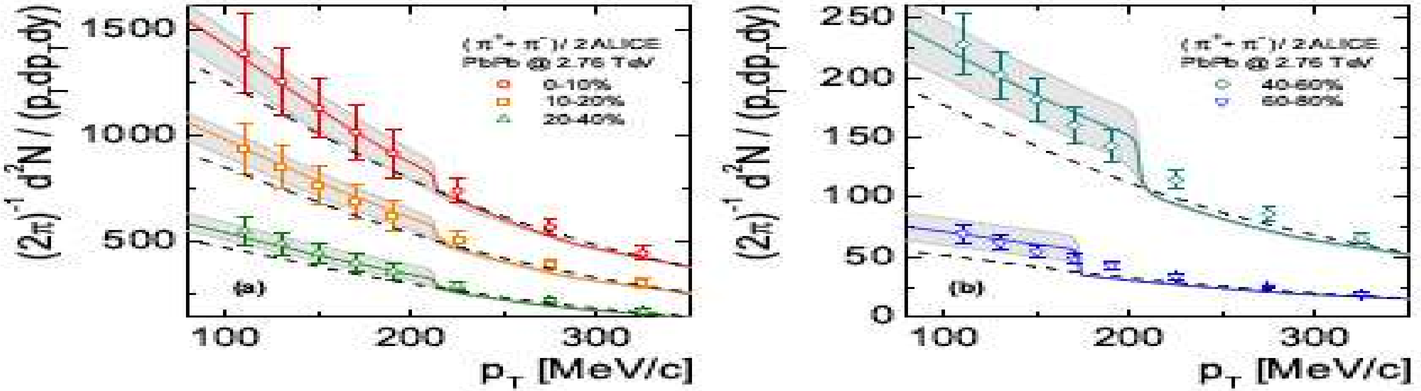
Radoslaw Ryblewski[1,*] and Wojciech Florkowski[1,2,†]

Low-momentum pion enhancement at LHC - Onset of Bose-Einstein Condensation of pions ?

$$n = \int d^3p \, \frac{1}{(2\pi)^3} \frac{g}{\exp\left(\frac{\sqrt{p^2+m^2}-\mu}{T}\right)-1} \left[1 + \frac{(2\pi)^3}{V} \delta(p_x)\,\delta(p_y)\,\delta(p_z)\right]$$



V. Begun, W. Florkowski, PRD (2015); Arxiv:1503.04040



See also:
O. Vitiuk et al.
PRC 113 (2026)
Arxiv:2409.09019
(in Zubarev approach)

Low-momentum pion enhancement from quantum kinetics of oversaturated phase space

$$\frac{\partial f_\pi}{\partial t}(t,\vec{p}_1) = \int\int \frac{|M_{\pi\pi\to\pi\pi}|^2}{64\pi^3\varepsilon_1}\frac{p_3p_4}{\varepsilon_3\varepsilon_4}\mathrm{d}p_3\mathrm{d}p_4\mathrm{D}F[f_\pi]$$
$$+ (1+f_\pi(t,p_1))\int\int \frac{|M_{gg\to\pi\pi}|^2}{64\pi^3\varepsilon_1}\frac{p_3p_4}{\varepsilon_3\varepsilon_4}\mathrm{d}p_3\mathrm{d}p_4\mathrm{D}\left(1+f_\pi(t,p_2)\right)f_g(t,p_3)f_g(t,p_4)$$
$$- f_\pi(t,p_1)\int\int \frac{|M_{\pi\pi\to gg}|^2}{64\pi^3\varepsilon_1}\frac{p_3p_4}{\varepsilon_3\varepsilon_4}\mathrm{d}p_3\mathrm{d}p_4\mathrm{D}f_\pi(t,p_2)\left(1+f_g(t,p_3)\right)\left(1+f_g(t,p_4)\right)$$

$$\frac{\partial f_g}{\partial t}(t,\vec{p}_1) = \int\int \frac{|M_{gg\to gg}|^2}{64\pi^3\varepsilon_1}\frac{p_3p_4}{\varepsilon_3\varepsilon_4}\mathrm{d}p_3\mathrm{d}p_4\mathrm{D}F[f_g]$$
$$+ (1+f_g(t,p_1))\int\int \frac{|M_{\pi\pi\to gg}|^2}{64\pi^3\varepsilon_1}\frac{p_3p_4}{\varepsilon_3\varepsilon_4}\mathrm{d}p_3\mathrm{d}p_4\mathrm{D}\left(1+f_g(t,p_2)\right)f_\pi(t,p_3)f_\pi(t,p_4)$$
$$- f_g(t,p_1)\int\int \frac{|M_{gg\to\pi\pi}|^2}{64\pi^3\varepsilon_1}\frac{p_3p_4}{\varepsilon_3\varepsilon_4}\mathrm{d}p_3\mathrm{d}p_4\mathrm{D}f_g(t,p_2)\left(1+f_\pi(t,p_3)\right)\left(1+f_\pi(t,p_4)\right)$$

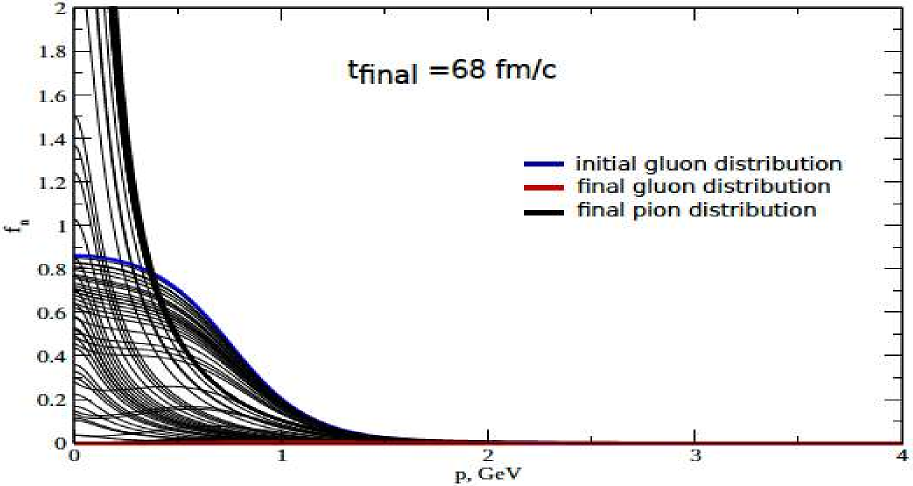
Lesson 2:

Kinetics of g→π conversion maps (over)saturated g phase space to low-momentum π enhancement / Bose-Einstein Condensation (BEC)

Initial conditions:

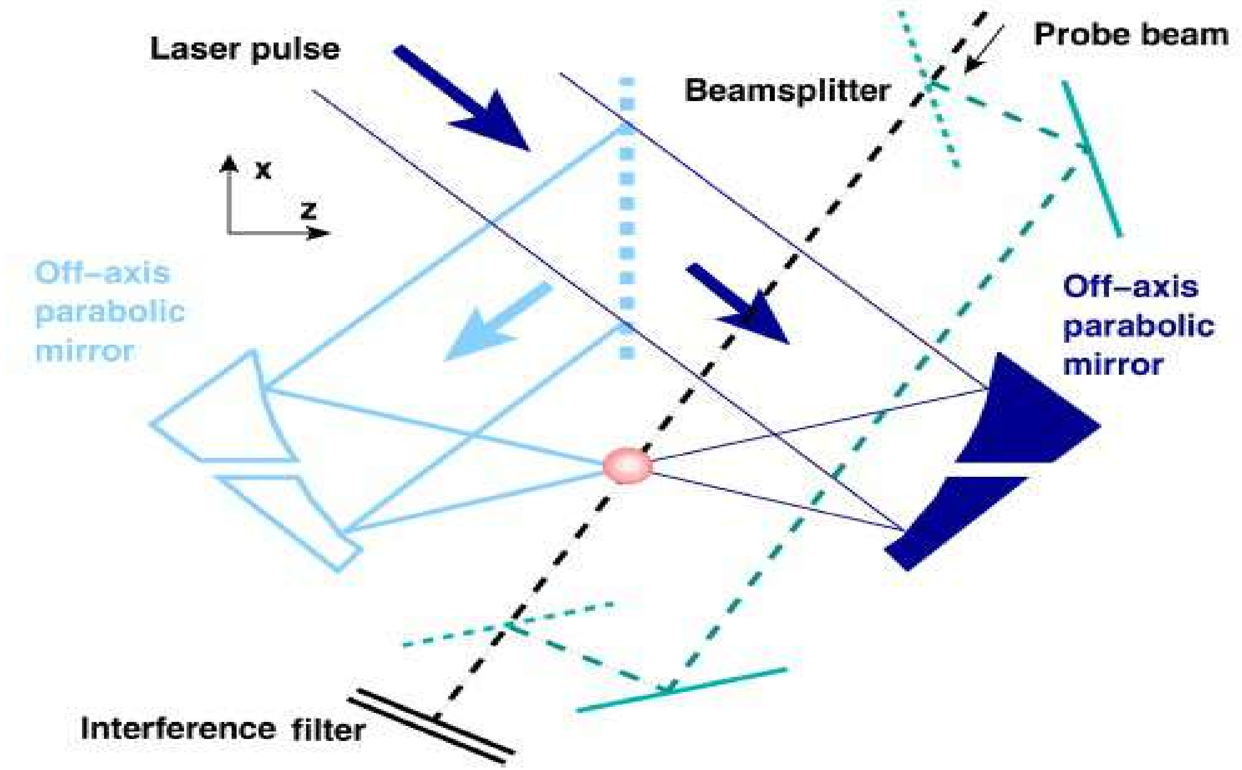
$$f_g(t,p)\Big|_{t=0} = f_0\,\theta(1-p/Q_s)$$
$$f_\pi(t,p)\Big|_{t=0} = 0$$

Time evolution as solution of the kinetic equations →



E. Nazarova, L. Juchnowski, D. Blaschke, T. Fischer, Particles 2 (2019) 140 - 149

"Pump & Probe" of the QED Vacuum: Bifrequent Laser Beams



Why is it interesting?

- pump (HI optical laser) & Probe (XFEL) experiment exploring modification of QED vacuum structure
- refraction & birefringence
- "assisted" dynamical Schwinger effect

A. Otto, D. Seipt, D. Blaschke, B. Kämpfer, S.A. Smolyansky, PLB 740, 335 (2015)
D. Blaschke, L. Juchnowski, HIBEF kickoff meeting, DESY (2013)

Dynamical Schwinger process in a bifrequent electric field

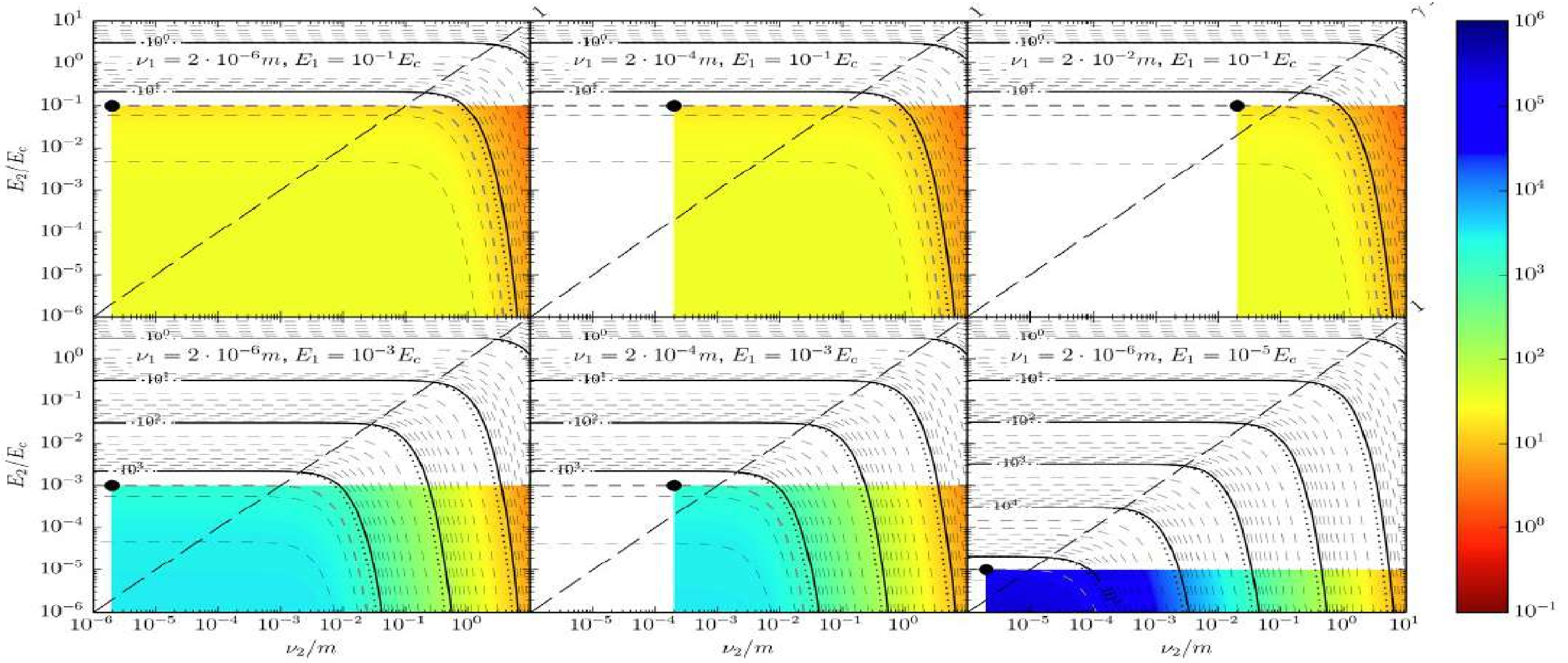


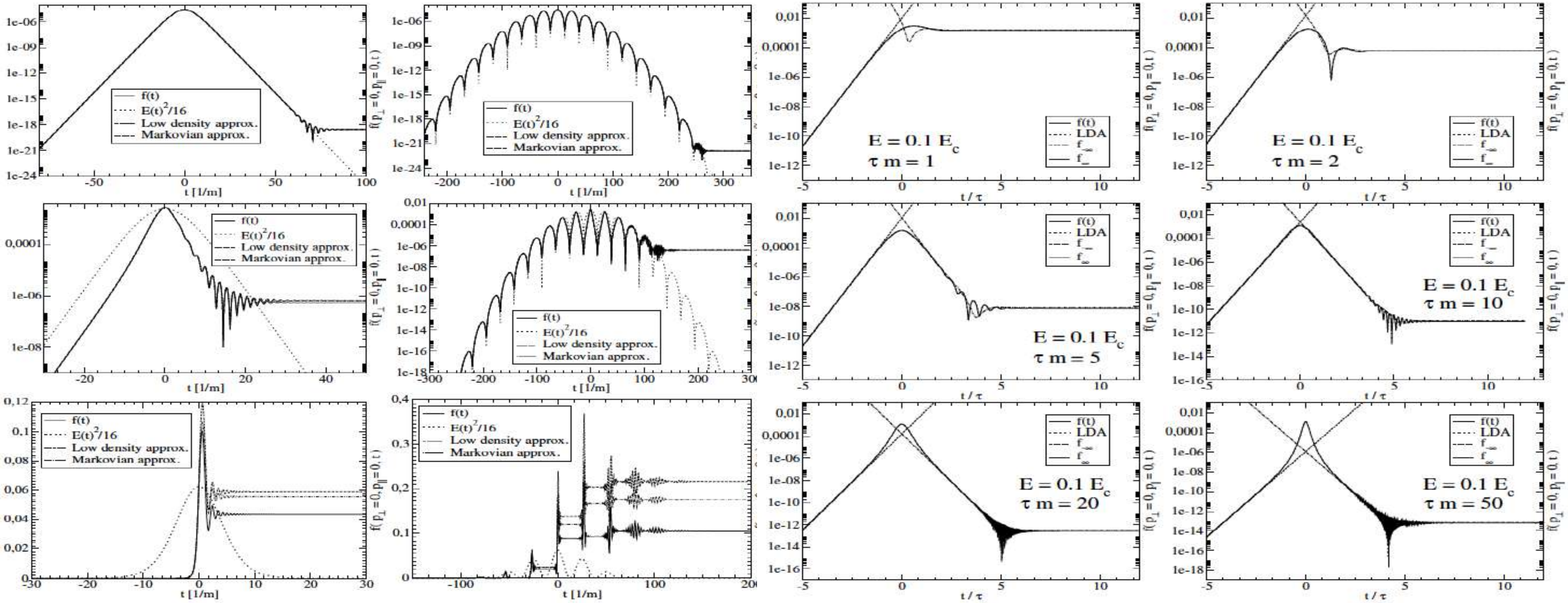
FIG. 3 (color online). Contour plots of the exponential $4\frac{m}{\nu_1}G(p_\perp \ll m, \gamma_1, \gamma_2, N)$ for six given fields ν_1, E_1 in the adiabatic region (positions depicted by the bullets, which are the loci of field doubling) over the field-frequency (E_2/E_c vs ν_2/m) plane, i.e. actually $4\frac{m}{\nu_1}G(p_\perp \ll m, \nu_1, E_1, \nu_2, E_2)$. Despite the displayed smooth distribution, our results are strictly valid only for $E_2 < E_1$ and $\nu_2 = (4n+1)\nu_1$, $n = 0, 1, 2 \ldots$ [solid curves: using (6) for G, dotted curves: the approximation (9)]. The heavy grey dashed contour curves are constructed to go through the bullets. An amplification beyond the field doubling occurs in the colored [grey] rectangular regions right to these curves.

A. Otto, D. Seipt, D. Blaschke, S.A. Smolyansky, B. Kämpfer, Phys. Rev. D 91, 105018 (2015); arxiv:1503.08675 [hep-ph]

Kinetic equation approach to strong, time-dependent external fields

Different field strengths: E_0/E_c = 0.02, 0.2, 1.0

Different pulse duration: τm = 1, 2, 5, 10, 20, 50



Lesson 3: **To enhance particle production in external fields, make pulses strong & short!**

D. Blaschke, L. Juchnowski, A. Otto, Particles 2 (2019) 166 - 179

Confining Density Functionals (CDFs) - String-Flip Model

The linear confinement potential is given by

$$V(\vec{r}) = \sigma|\vec{r}|$$

the effective screened potential is given by

$$V^{\rm eff}(r,n) = V(r)c(r,n)$$

with the non-normalized nearest-neighbor distribution function

$$c(r,n) = \exp\left(-\frac{4\pi\, n}{3N_c}r^3\right) .$$

The density functional is an average over string lengths

$$U(n) = \frac{n^2}{N_c}\int V^{\rm eff}(r,n)d^3r = \sigma\,\Gamma\left(\frac{4}{3}\right)\left(\frac{3N_c}{4\pi}\right)^{1/3} n^{2/3} .$$

Keep in mind the dependence $n^{2/3}$

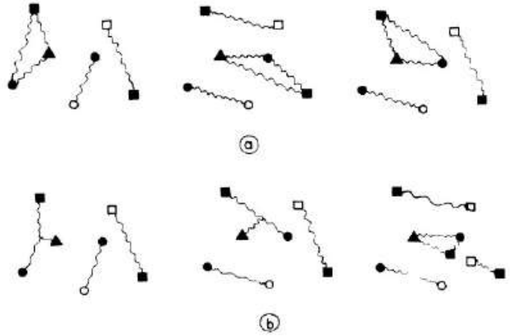


FIG. 1. (a) String configurations attributed to identical quark positions: two-body strings. (b) Three-body string and $q\bar{q}$ pair creation.

PHYSICAL REVIEW D VOLUME 34, NUMBER 11 1 DECEMBER 1986

Pauli quenching effects in a simple string model of quark/nuclear matter

G. Röpke and D. Blaschke
Department of Physics, Wilhelm-Pieck-Universtiy, 2500 Rostock, German Democratic Republic

H. Schulz
Central Institute for Nuclear Research, Rossendorf, 8051 Dresden, German Democratic Republic and The Niels Bohr Institute, 2100 Copenhagen, Denmark
(Received 16 December 1985)

Confining Density Functionals – String-breaking model

Regularized linear confinement in 1 dim., parameter μ

$$V(\vec{r}) = \lim_{\mu\to 0} \frac{\sigma}{\mu}\left(1 - e^{-\mu|\vec{r}|}\right)$$

Fourier transform: $V(\vec{k}) = \lim_{\mu\to 0} \frac{\sigma}{\mu}\left[(2\pi)^3\delta(\vec{k})^{(3)} - \frac{8\pi\mu}{(\mu^2+k^2)^2}\right]$

The density functional is

$$\begin{aligned} U(k_F, q_F) &= \frac{1}{2}\int \frac{d^3\vec{k}}{(2\pi)^3}\int \frac{d^3\vec{q}}{(2\pi)^3} V(\vec{k}-\vec{q})n(\vec{k})n(\vec{q}) \\ &= \frac{1}{2}\int_{|\vec{k}|\le k_F} \frac{d^3k}{(2\pi)^3}\int_{|\vec{q}|\le q_F} \frac{d^3q}{(2\pi)^3} V(\vec{k}-\vec{q}) = \frac{\sigma}{2\mu}\left\{\frac{k_F^3}{2\pi^2} - I(k_F, q_F)\right\} \end{aligned}$$

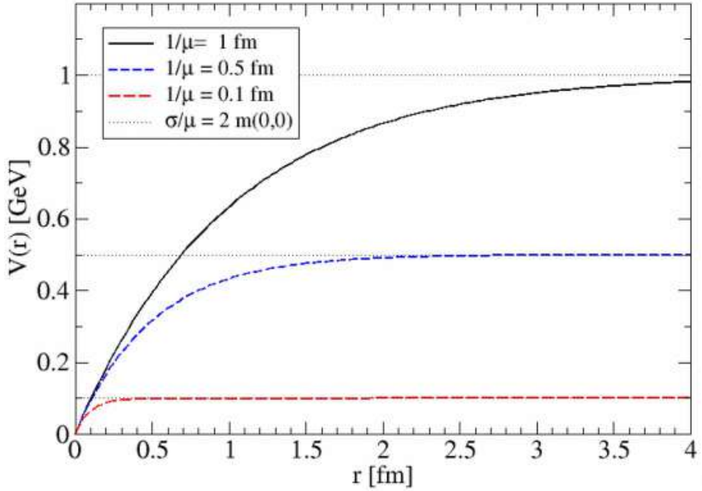
$$I(k_F, k_F) = \frac{\mu}{4\pi^3}\left\{\frac{2}{3}k_F^2 + \left(\frac{k_F^2}{2} + \frac{\mu^2}{3}\right)\ln\left[\frac{\mu^2}{4k_F^2+\mu^2}\right] + \left(\frac{4}{3}\frac{k_F^3}{\mu} - 6\mu k_F\right)\arctan\left[\frac{2k_F\mu}{\mu^2}\right]\right\}$$

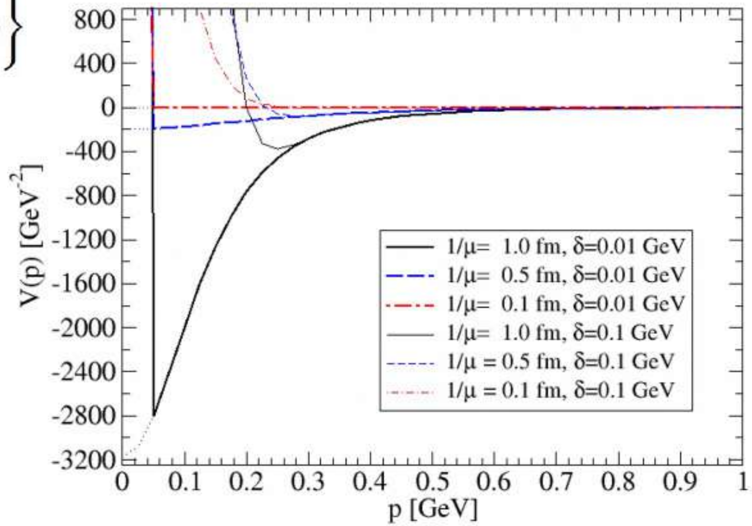
Screening parameter $\mu = \frac{1}{r_D} = \left(\frac{3N_c}{2\pi}\right)^{-\frac{1}{3}}\left(\frac{2N_f N_c}{6\pi^2}k_F^3\right)^{\frac{1}{3}} = \xi k_F$

$$U(k_F, k_F) = \frac{\sigma\chi}{2}k_F^2 = D\,n^{2/3}$$

Again, the $n^{2/3}$ rule !
D ~ σ

Oliver Heymer, D.B., in preparation (2026)





Confining Density Functionals (CDFs) - String-Flip Model

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - m)q - \mathcal{U}(\overline{q}q, \overline{q}\gamma_0 q)$$

- Scalar and vector densities

$$\langle \overline{q}q \rangle = n_s, \quad \langle \overline{q}\gamma_0 q \rangle = n_v$$

- Scalar and vector self-energies

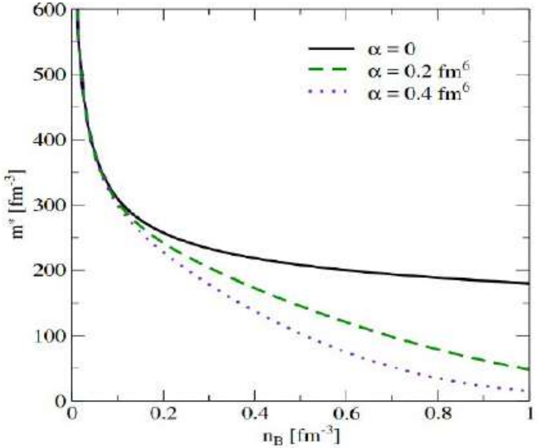
$$\Sigma_s = \frac{\partial \mathcal{U}(n_s, n_v)}{\partial n_s}, \quad \Sigma_v = \frac{\partial \mathcal{U}(n_s, n_v)}{\partial n_v}$$

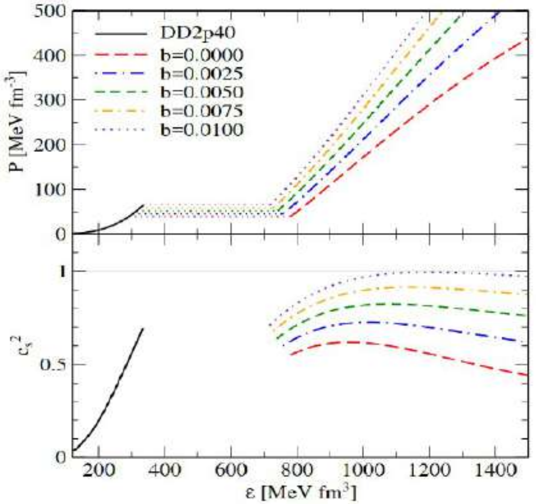
- String-flip model

$$\mathcal{U} = \underbrace{D(n_v) n_s^{2/3}}_{\text{confinement}} + \underbrace{a n_v^2 + \frac{b n_v^4}{1 + c n_v^2}}_{\text{modified vector repulsion}}$$

- Effective mass

$$m^* = m + \Sigma_s = m + \frac{2}{3} D(n_v) n_s^{-1/3}$$





M. Kaltenborn, N.-U. Bastian, and D. Blaschke. PRD 96, 056024 (2017)

Confining Density Functionals (CDFs) - String-Flip Model

- $2M_\odot$ stars formation? (accretion is too slow)
- Supernovae with progenitor mass $\sim 50\ M_\odot$
- Quark-hadron transition stabilizes collapse

T. Fischer et al., Nature Astronomy 2, 980–986 (2018)

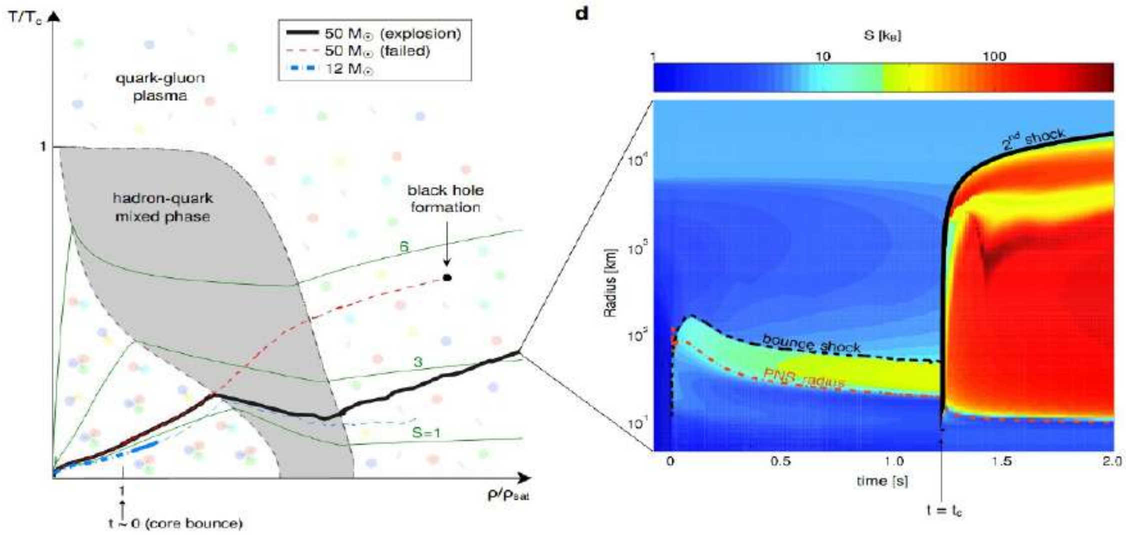


Table 1 | Summary of the supernova simulation results with hadron–quark phase transition

M_{ZAMS} ($M_\odot$)	t_{onset} (s)	$t_{collapse}$ (s)	$\rho\|_{collapse}$ (ρ_{sat})	$T\|_{collapse}$ (MeV)	$M_{PNS,collapse}$[a] ($M_\odot$)	t_{final} (s)	$\rho\|_{final}$ (ρ_{sat})	$T\|_{final}$ (MeV)	$M_{PNS,final}$[a] ($M_\odot$)	E^*_{expl} (10^{51} erg)
12[12]	3.251	3.489	2.49	28	1.727	3.598	5.5	17	1.732	0.1
18[12]	1.465	1.518	2.53	27	1.958	1.575	5.9	18	1.964	1.6
25[12]	0.905	0.976	2.40	31	2.163	0.983	9.6	19	2.171[b]	-
50[13]	1.110	1.215	2.37	32	2.105	1.224	5.8	31	2.092	2.3

Deconfinement is a supernova engine for massive blue giants

Confining Density Functional (CDF) & chirally sym. Model

What is new?

O. Ivanytskyi & D.B., Phys. Rev. D 105 (2022) 114042

Interaction $\mathcal{U} = D_0 \left[(1+\alpha)\langle\bar{q}q\rangle_0^2 - (\bar{q}q)^2 - (\bar{q}i\vec{\tau}\gamma_5 q)^2\right]^{\varkappa}$

- **Parameters**

  D_0 - dimensionfull coupling, controls interaction strength

  α - dimensionless constant, controls vacuum quark mass

  $\langle\bar{q}q\rangle_0$ - χ-condensate in vacuum (introduced for the sake of convenience)

$\varkappa = 1/3$ $\Downarrow$ motivated by String Flip model

$\varkappa = 1$ $\Downarrow$ Nambu–Jona-Lasinio model

$$\mathcal{U}_{SFM} \propto \langle q^+ q\rangle^{2/3}$$

$$\Sigma_{SFM} = \frac{\partial \mathcal{U}_{SFM}}{\partial\langle q^+ q\rangle} \propto \langle q^+ q\rangle^{-1/3} \propto separation$$

- **Dimensionality**

$$[\mathcal{U}] = energy^4, \quad [\bar{q}q] = energy^3 \quad \Rightarrow \quad [D_0]_{\varkappa=1/3} = energy^2 = [string\ tension]$$

self energy = string tension × separation $\Rightarrow$ **confinement**

Confining Density Functionals (CDFs) – Chiral Sym. Model

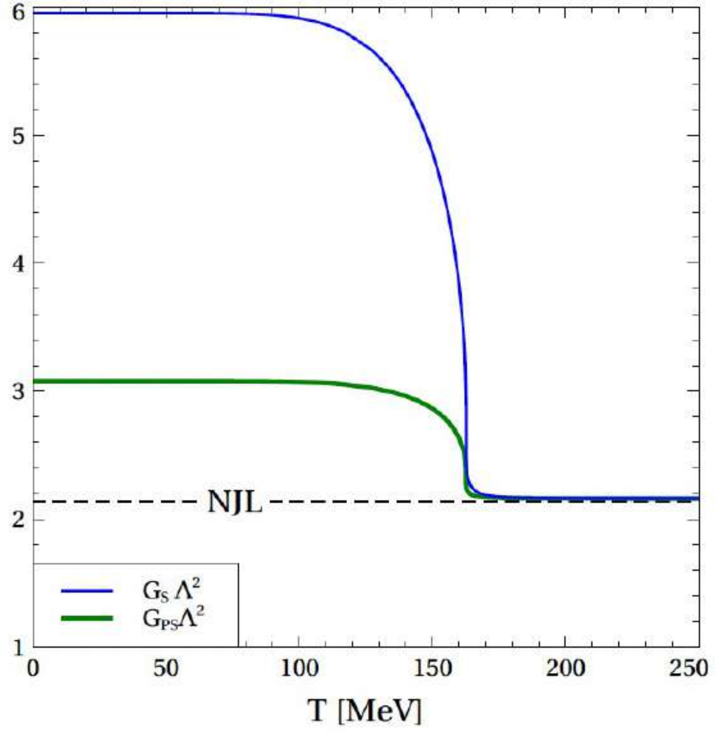
$$\mathcal{U} = \underbrace{U}_{0^{\text{th}}\ \text{order}} + \underbrace{\left(\overline{q}q - \langle\overline{q}q\rangle\right)\Sigma_{MF}}_{1^{\text{st}}\ \text{order}} - \underbrace{G_S\left(\overline{q}q - \langle\overline{q}q\rangle\right)^2 - G_{PS}\left(\overline{q}i\vec{\tau}\gamma_5 q\right)^2}_{2^{\text{nd}}\ \text{order}} + \ldots$$

- **Mean-field self-energy**

$$\Sigma_{MF} = \frac{\partial U}{\partial\langle\overline{q}q\rangle}$$

- **Effective medium dependent couplings**

$$G_S = -\frac{1}{2}\frac{\partial^2 U}{\partial\langle\overline{q}q\rangle^2}, \quad G_{PS} = -\frac{1}{6}\frac{\partial^2 U}{\partial\langle\overline{q}i\vec{\tau}\gamma_5 q\rangle^2}$$



Confining Density Functional (CDF) & CSC Quark Matter

Lagrangian $\mathcal{L} = \overline{q}(i\partial\!\!\!/ - \hat{m})q - \mathcal{U} + \mathcal{L}_V + \mathcal{L}_I + \mathcal{L}_D$

- **Scalar & pseudoscalar interaction channels**

$$\mathcal{U} = G_0 \left[(1+\alpha)\langle\overline{q}q\rangle_0^2 - (\overline{q}q)^2 - (\overline{q}i\vec{\tau}\gamma_5 q)^2\right]^{\frac{1}{3}}$$

(motivated by String Flip Model, χ-dynamics, quark "confinement")

- **Vector-isoscalar interaction channel**

$$\mathcal{L}_V = -G_V(\overline{q}\gamma_\mu q)^2$$

(motivated by gluon exchange, stiff EoS needed to reach $2M_\odot$)

- **Vector-isovector interaction channel**

$$\mathcal{L}_I = -G_I(\overline{q}\gamma_\mu\vec{\tau}q)^2$$

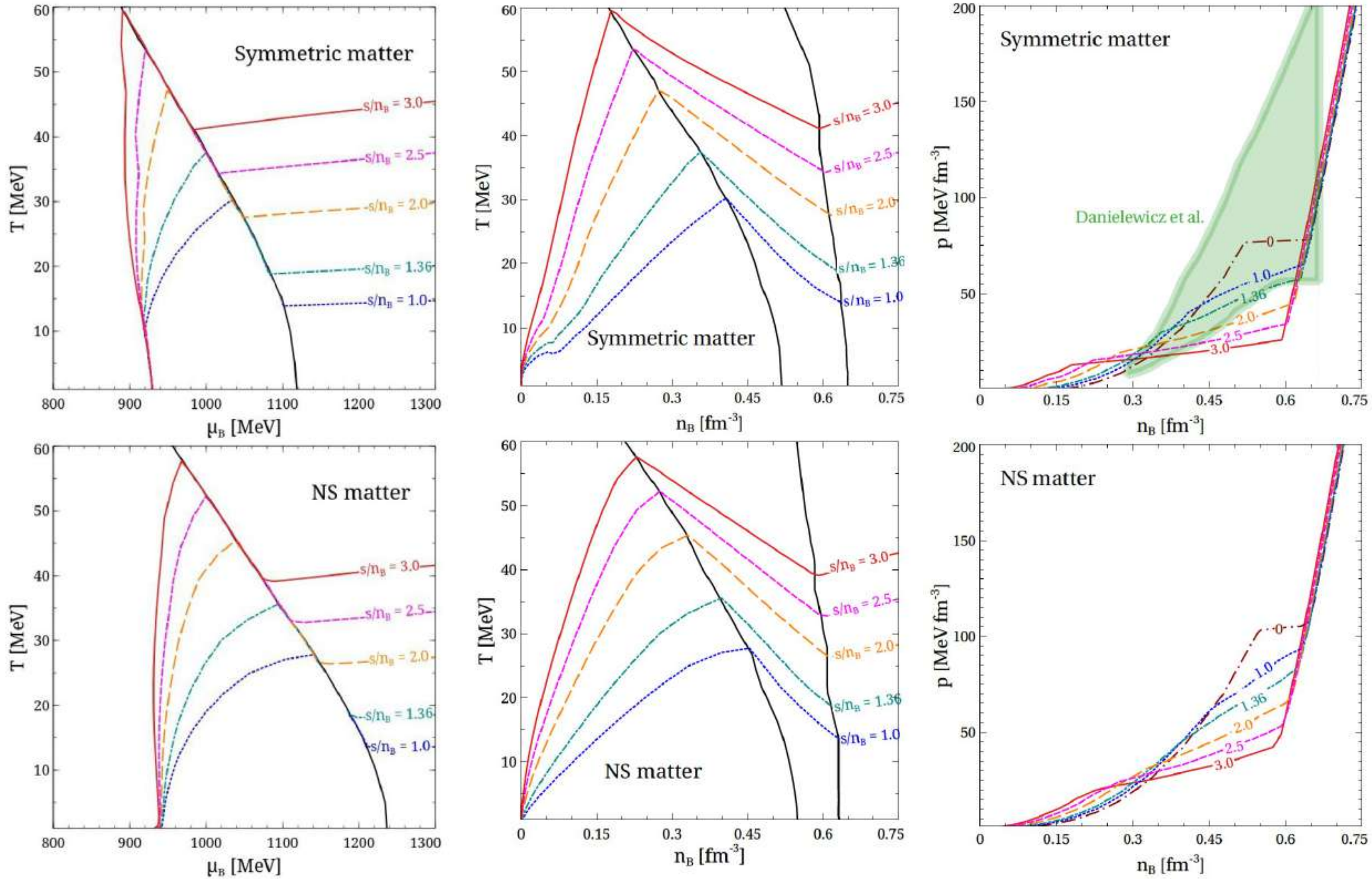
(motivated by gluon exchange, isospin sensitive interaction)

- **Diquark interaction channel**

$$\mathcal{L}_D = G_D \sum_{A=2,5,7} (\overline{q}i\gamma_5\tau_2\lambda_A q^c)(\overline{q}^c i\gamma_5\tau_2\lambda_A q)$$

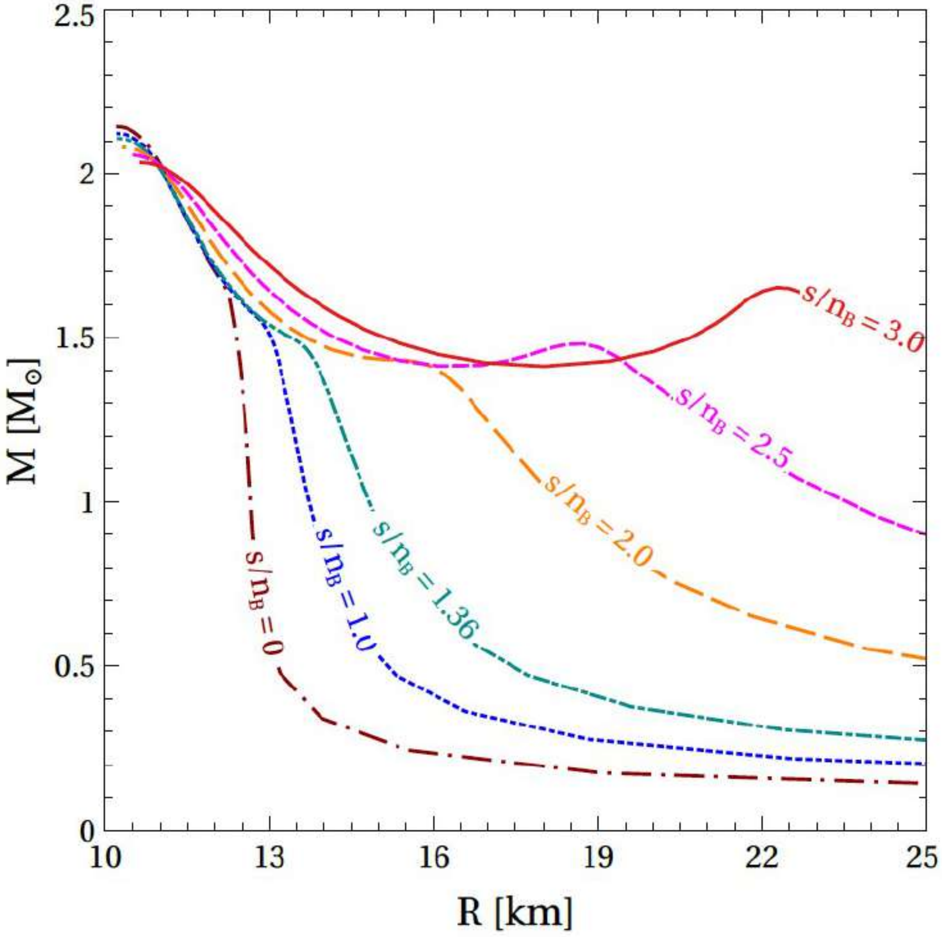
(motivated by Cooper theorem, color superconductivity)

Confining Density Functional (CDF) & CSC Quark Matter

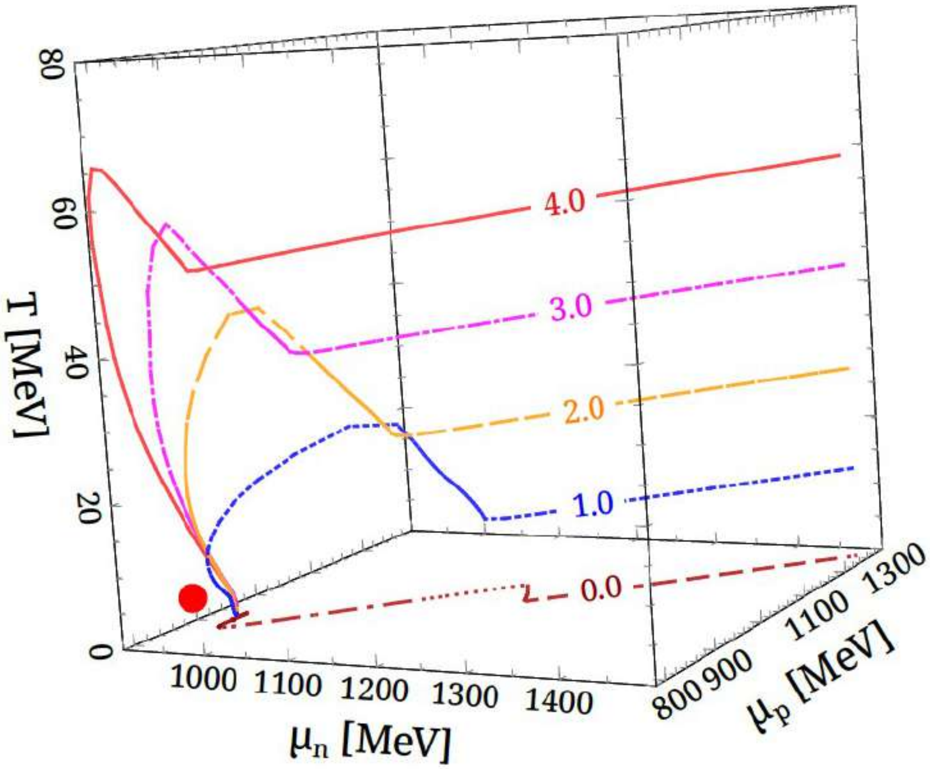


CDF Without CSC

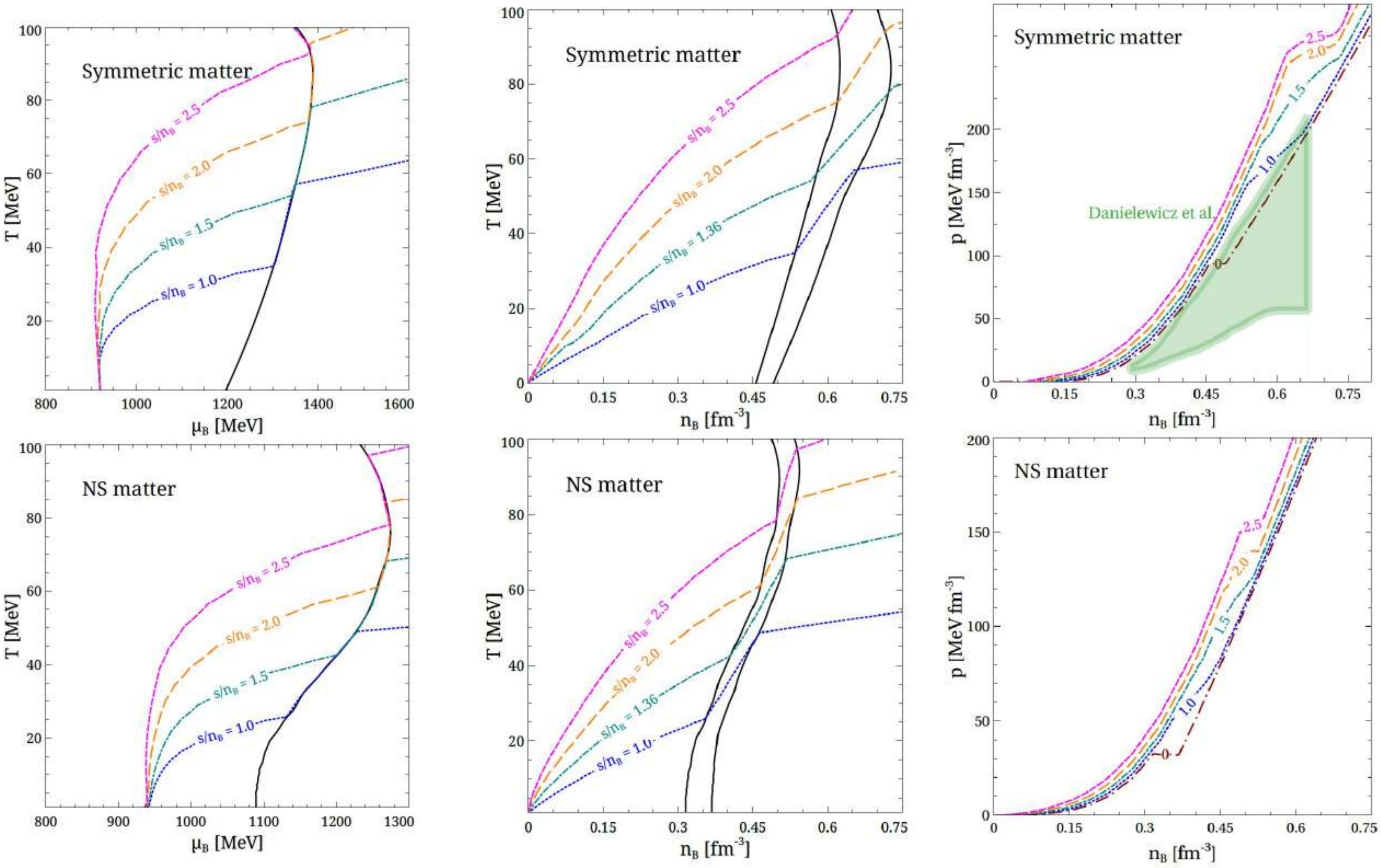
Confining Density Functional (CDF) & CSC Quark Matter



CDF Without CSC



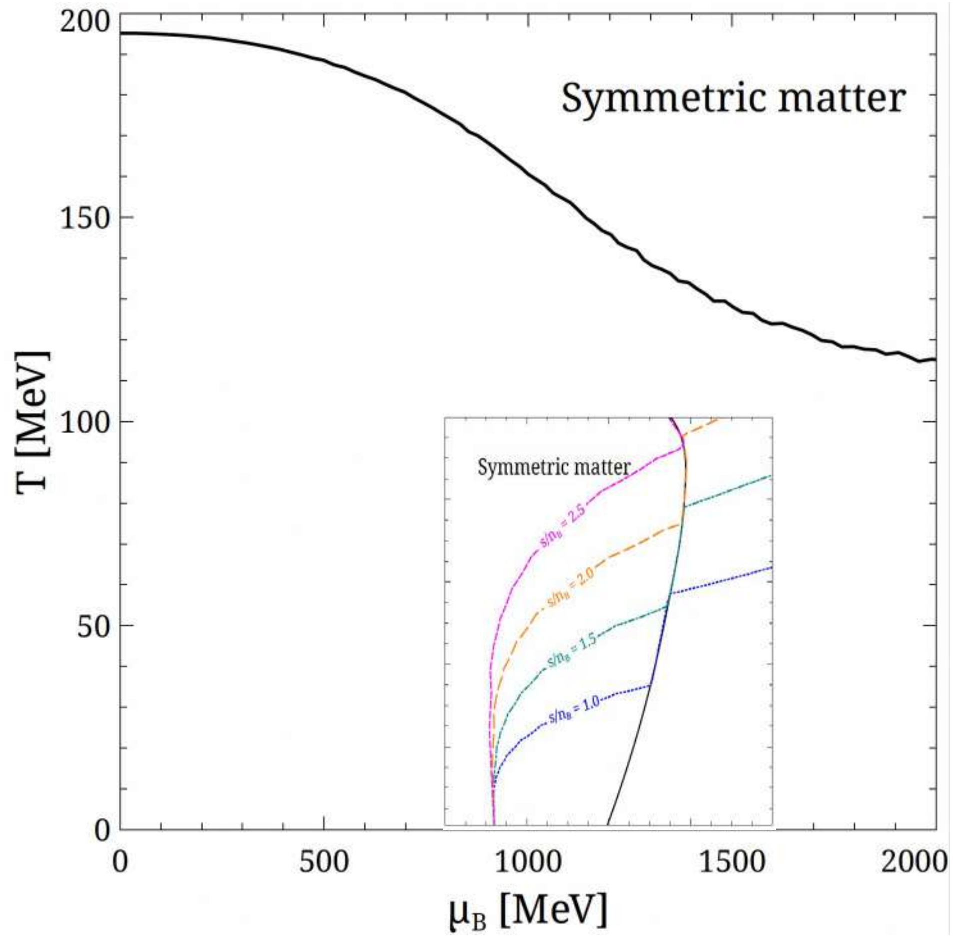
Confining Density Functional (CDF) & CSC Quark Matter



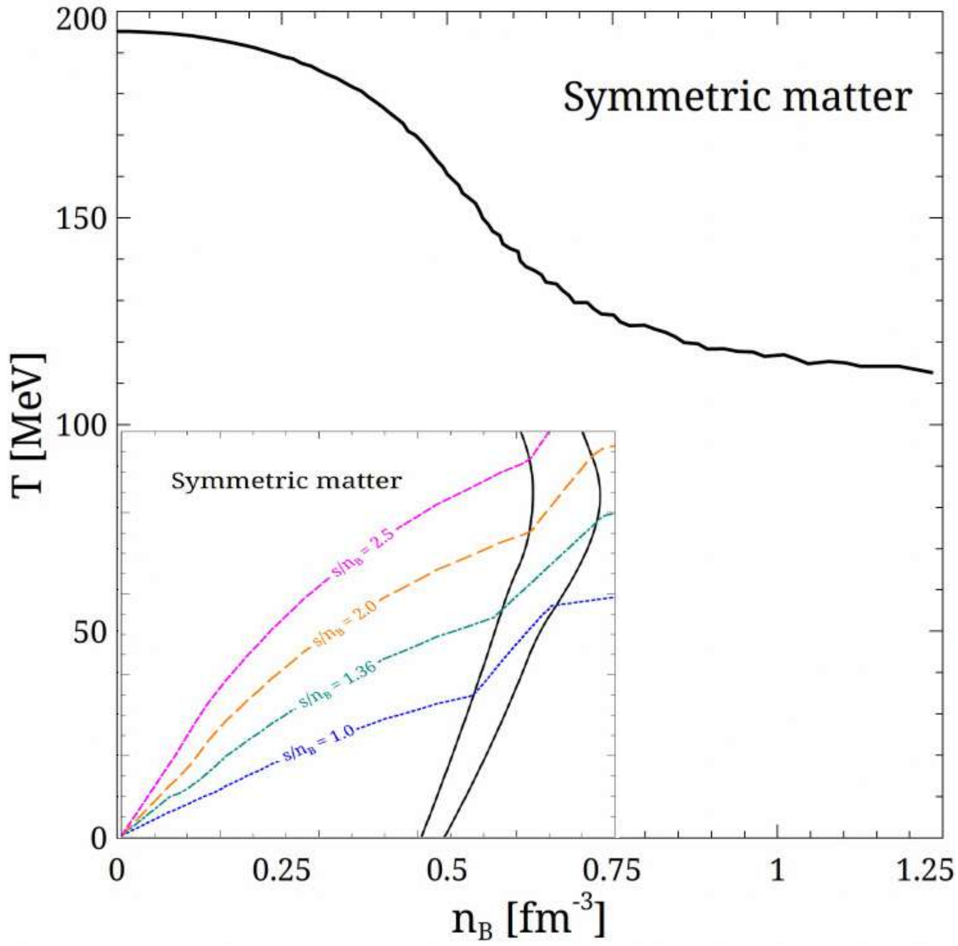
CDF With CSC

Confining Density Functional (CDF) & CSC Quark Matter

CDF with CSC

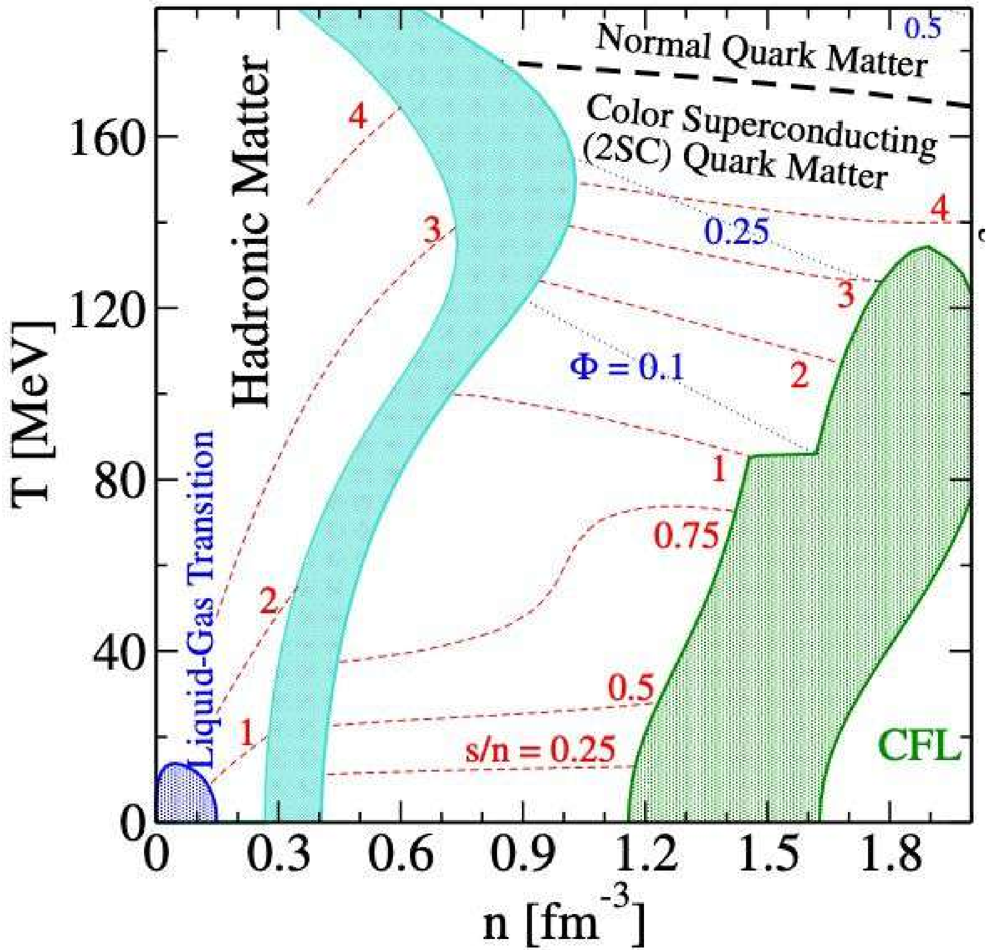


CDF with CSC

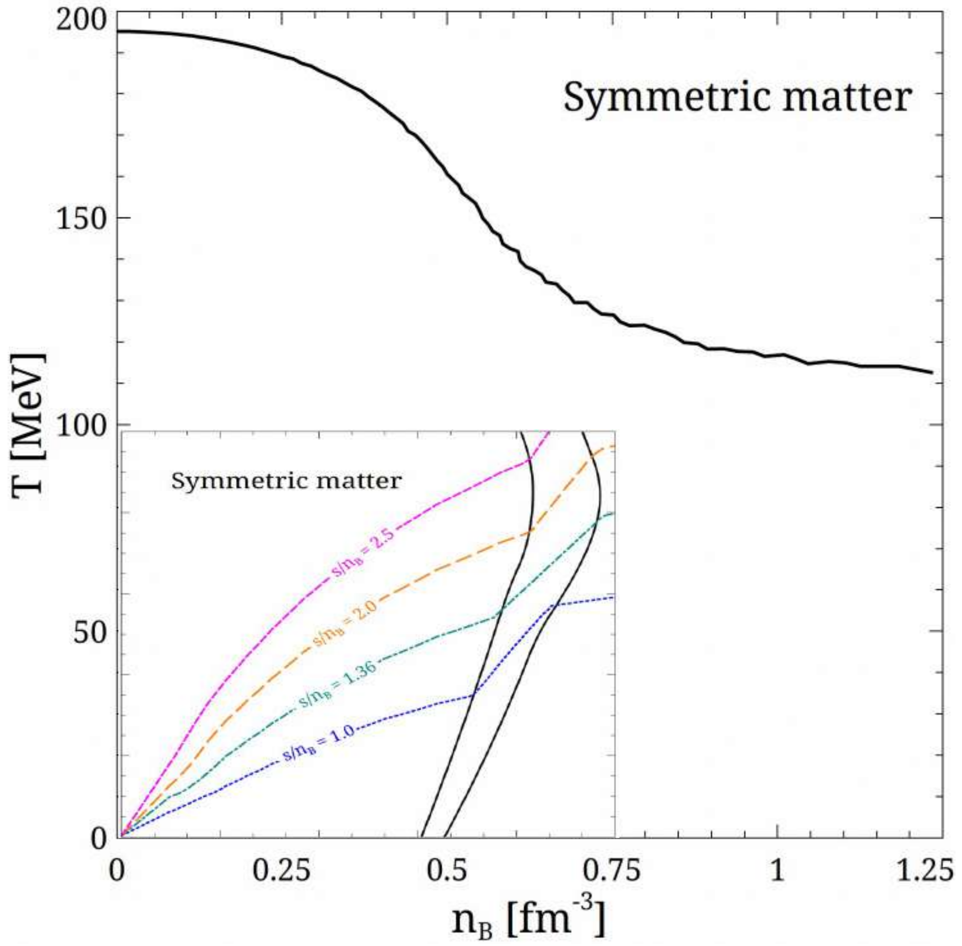


Confining Density Functional (CDF) & CSC Quark Matter

PNJL with CSC

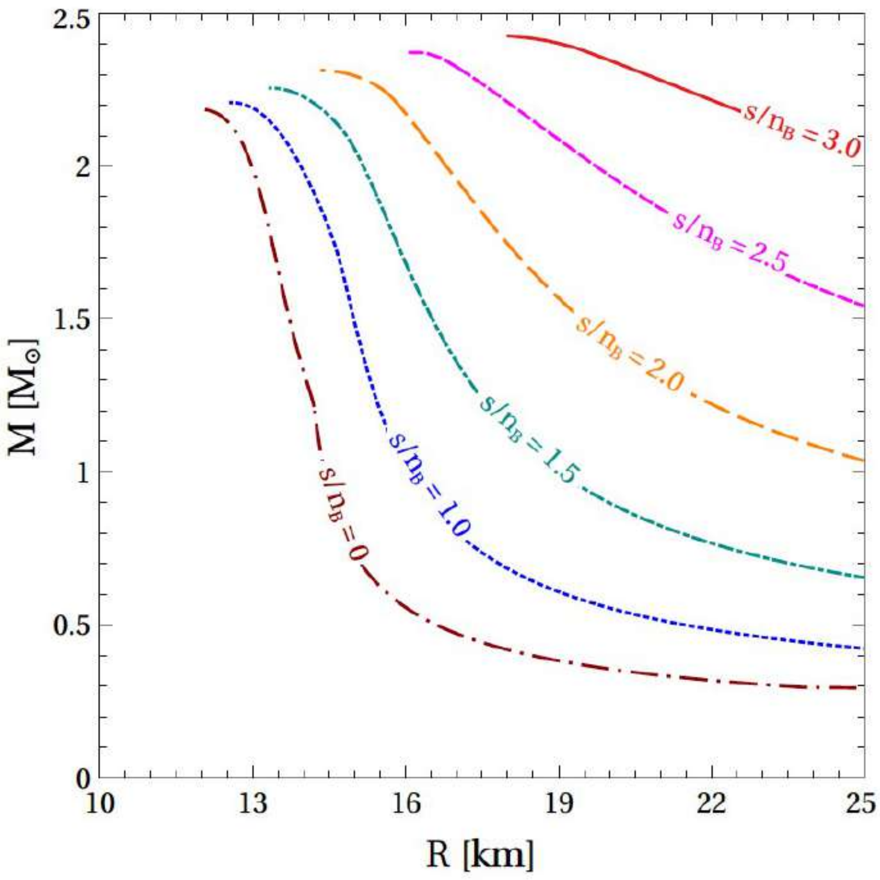


CDF with CSC

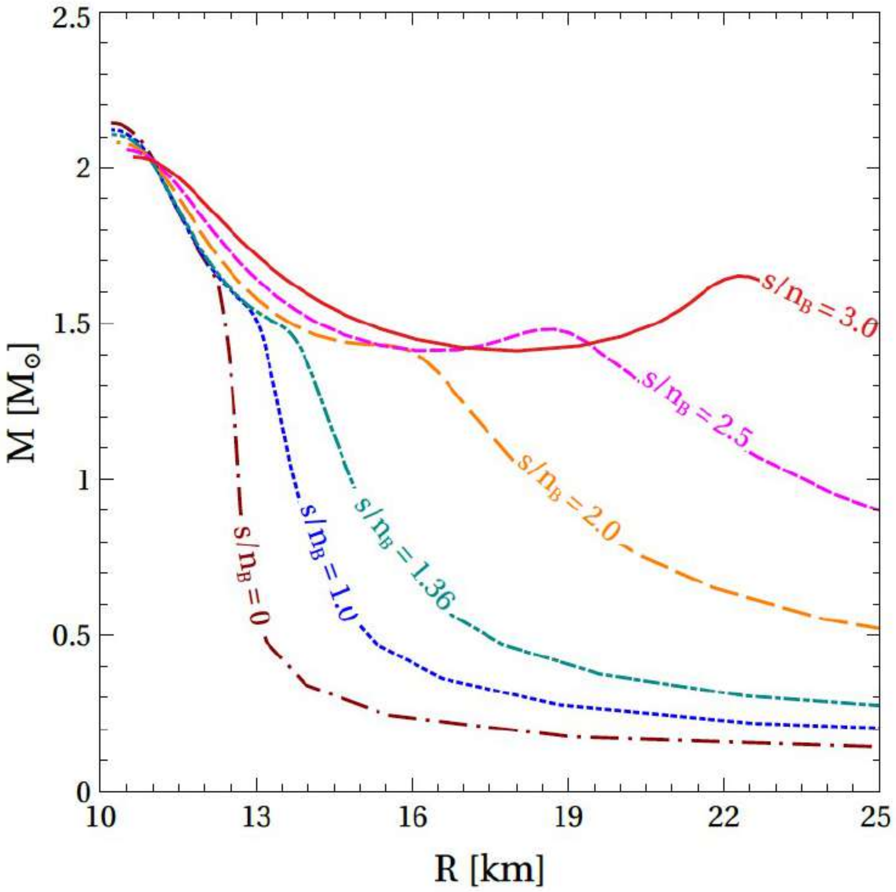


Confining Density Functional (CDF) & CSC Quark Matter

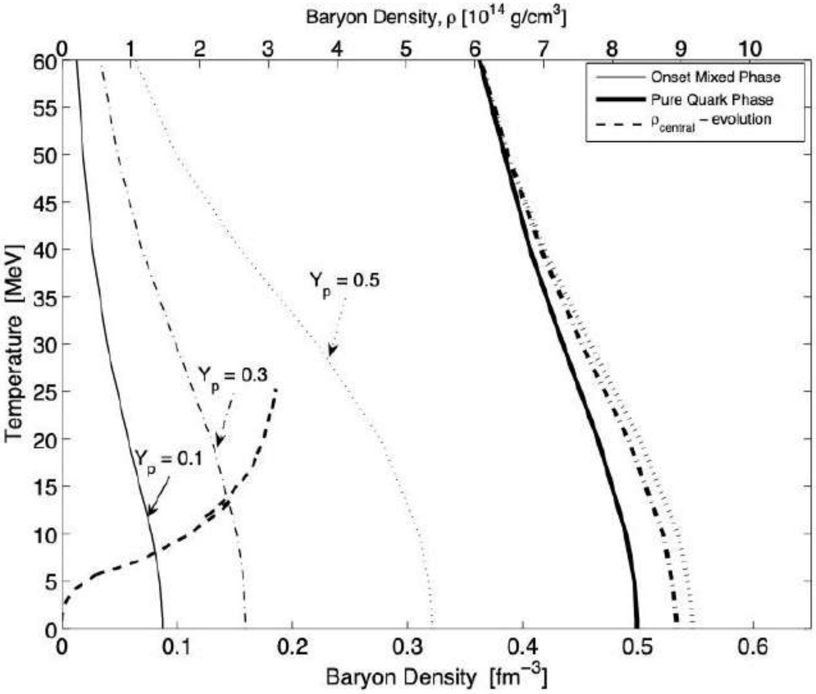
CDF with CSC



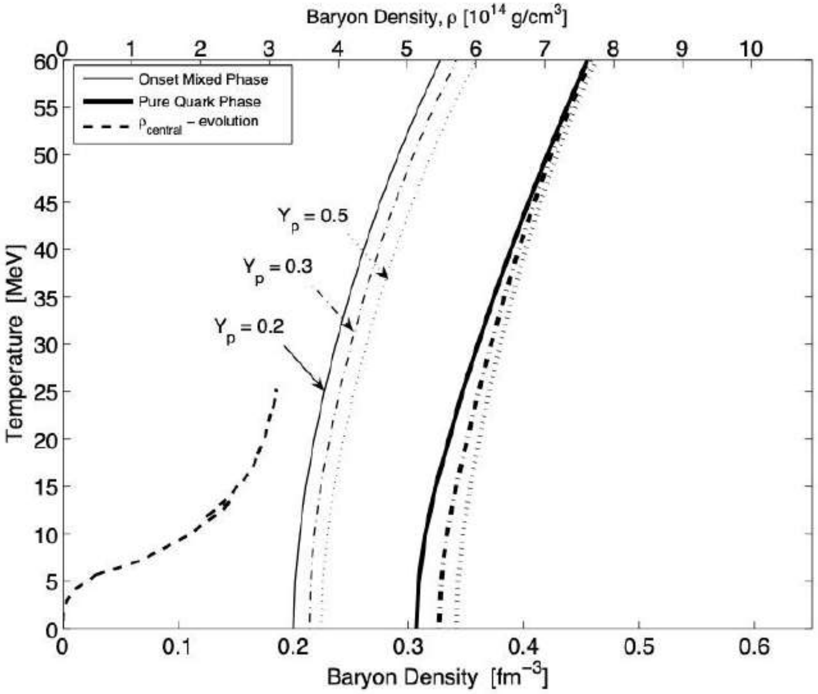
CDF without CSC



Confining Density Functional (CDF) & CSC Quark Matter



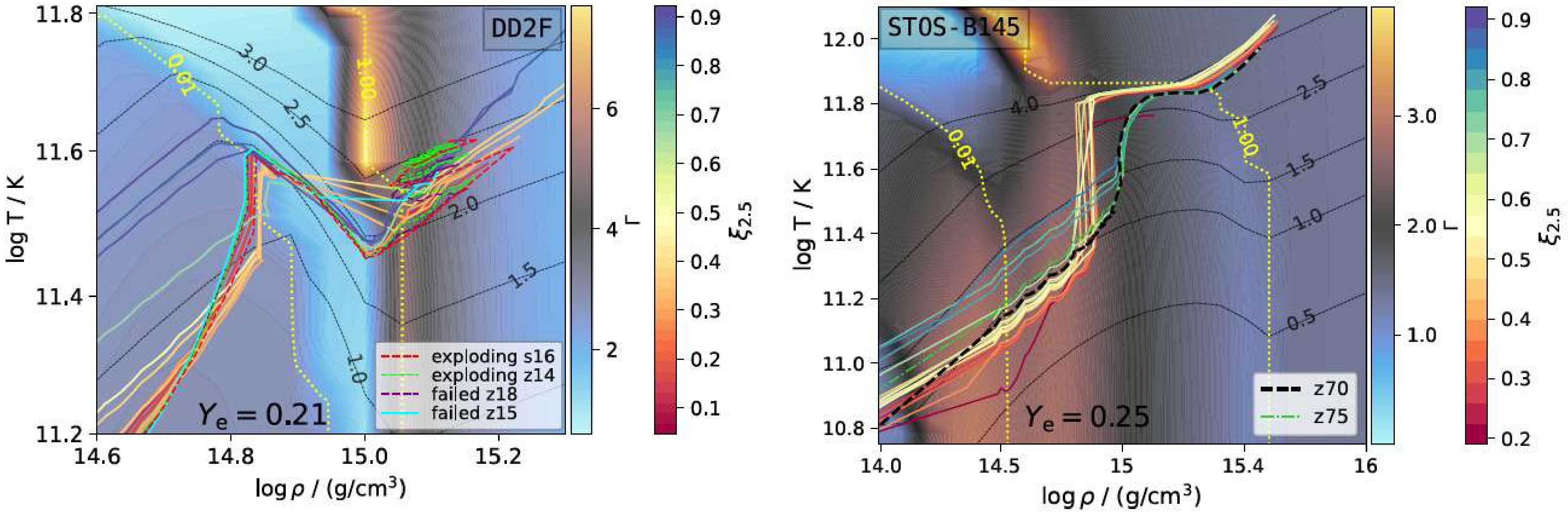
(a) Quark bag model ($B^{1/4} = 165$ MeV), for $Y_p = 0.1$ (solid lines), $Y_p = 0.3$ (dash-dotted lines) and $Y_p = 0.5$ (dotted lines)



(b) PNJL model ($\eta_D = 1.02$, $\eta_V = 0.25$), for $Y_p = 0.2$ (solid lines), $Y_p = 0.3$ (dash-dotted lines) and $Y_p = 0.5$ (dotted lines)

Thermal twin stars – Indicators of CCSN explodability ?

Successful explosion of massive progenitor stars* for hybrid EoS with entropic first-order transition (thermal twin stars)



Important for explodability: Postbounce mass accretion rate (metallicity) vs. Time to reach the onset of deconfinement**)

*) Pia Jakobus et al., MNRAS 516, 2554 (2022); arxiv:2204.10397 [astro-ph.HE]
**) Noshad Khosravi Largani et al., Astrophys. J. 964, 143 (2024); arxiv:2304.12316 [astro-ph.HE]

Cluster virial expansion for quark-hadron matter

The cluster decomposition of the thermodynamic potential is given as

$$\Omega_{\rm total}(T,\mu,\phi,\bar\phi) = \Omega_{PNJL}(T,\mu,\phi,\bar\phi) + \Omega_{\rm pert}(T,\mu,\phi,\bar\phi) + \Omega_{MHRG}(T,\mu,\phi,\bar\phi),$$

where the first two terms describe the quark and gluon degrees of freedom via the mean–field thermodynamic potential for quark matter in a gluon background field $\mathcal{U}$

$$\Omega_{PNJL}(T,\mu,\phi,\bar\phi) = \Omega_Q(T,\mu,\phi,\bar\phi) + \mathcal{U}(T,\phi,\bar\phi)$$

with a perturbative correction $\Omega_{\rm pert}(T,\mu,\phi,\bar\phi)$.
The Mott-Hadron-Resonance-Gas (MHRG) part for the multi-quark clusters is

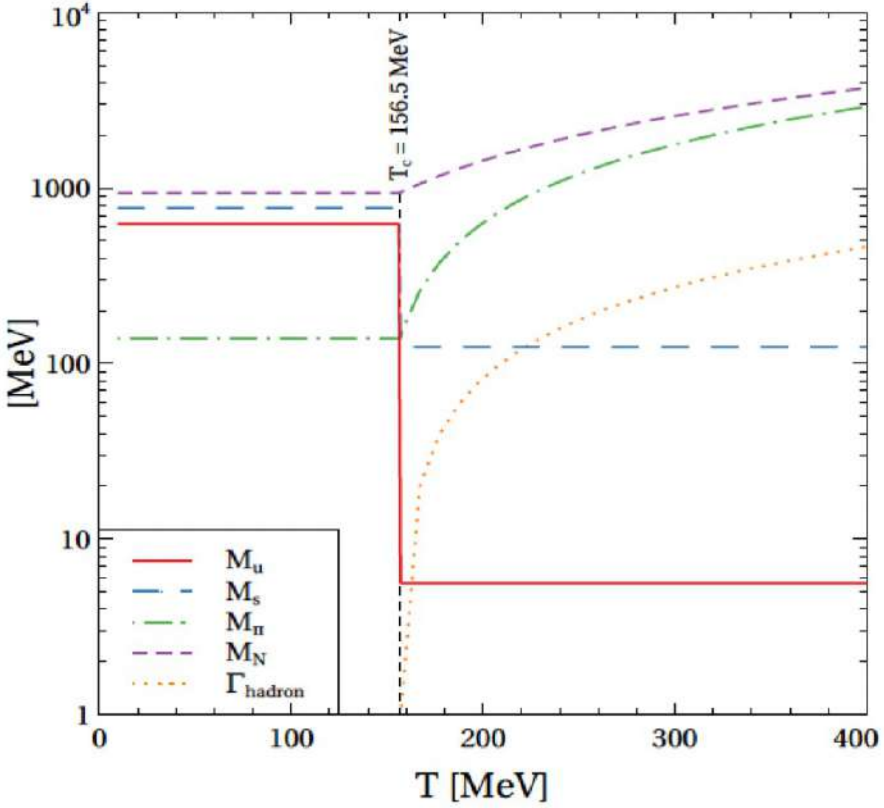
$$\Omega_{MHRG}(T,\mu,\phi,\bar\phi) = \sum_{i=M,B,\dots} \Omega_i(T,\mu,\phi,\bar\phi),$$

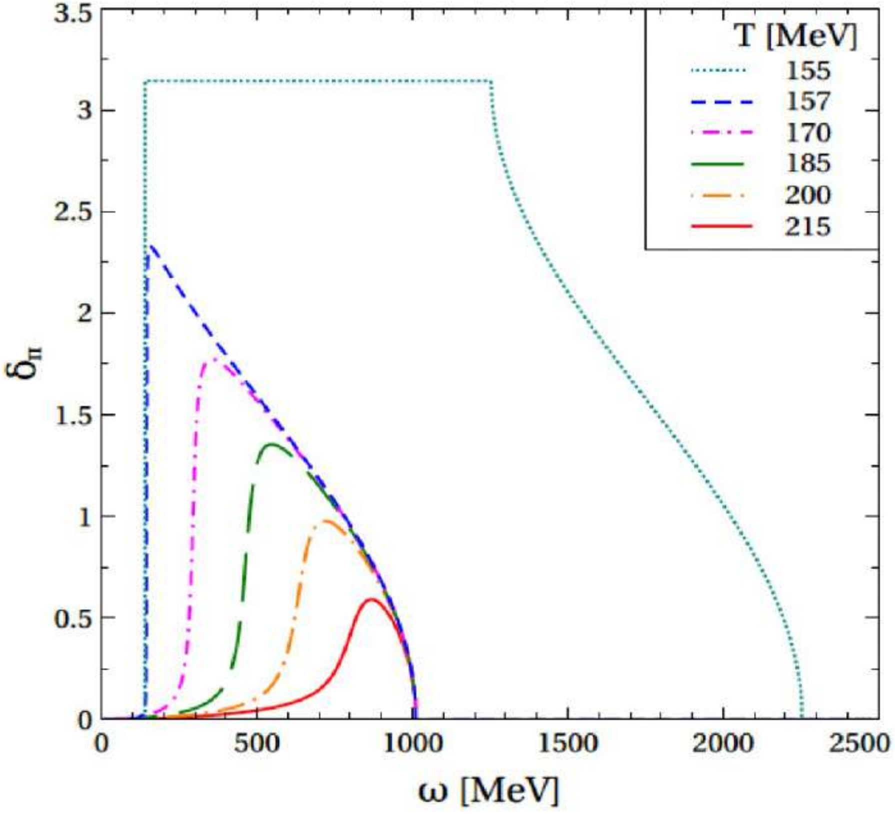
where the multi–quark states are described by the GBU formula:

$$\begin{aligned} n &= -\frac{\partial\Omega}{\partial\mu} = \sum_a a\, n_a(T,\mu) \\ &= \sum_a a\, d_a \int \frac{d\omega}{\pi} \int \frac{d^3q}{(2\pi)^3} \left\{ f_\phi^{(a),+} - \left[f_\phi^{(a),-} \right]^* \right\} 2\sin^2\delta_a(\omega,q) \frac{\partial\delta_a(\omega,q)}{\partial\omega}, \end{aligned}$$

where d_i is the degeneracy factor, a is the number of valence quarks in the cluster an $f_\phi^{(a),+}$, $\left[f_\phi^{(a),-}\right]^*$ are the Polyakov-loop modified distribution functions.
Analogous for the entropy density $s = -\partial\Omega/\partial T$.

Inputs: mass spectrum & phase shifts (models)





D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14

Inputs: mass spectrum (Particle Data Tables)

Mesons

PDG mesons	d_i	$M_{\rm PDG}$ [MeV]	M_i [MeV]	$M^<_{{\rm th},i}$ [MeV]	$M^>_{{\rm th},i}$ [MeV]
π^+/π^0	3	140	140	1254	11.2
K^+/K^0	4	494	494	1397	129.6
η	1	548	878	1349	90.1
ρ^+/ρ^0	9	775	783	1254	11.2
ω	9	783	783	1254	11.2
K^{*+}/K^{*0}	12	895	$806^{*)}$	2651	140.8
η'	1	960	878	1349	90.1
a_0	3	980	$1095^{*)}$	2508	22.4
f_0	1	980	$1095^{*)}$	2508	22.4
ϕ	3	1020	1069	1540	248
..					
$\pi_2(1880)$	15	1895	$1095^{*)}$	2508	22.4
$f_2(1950)$	5	1944	$1095^{*)}$	2508	22.4
$a_4(2040)$	27	1996	$1095^{*)}$	2508	22.4
$f_2(2010)$	5	2011	$1095^{*)}$	2508	22.4
$f_4(2050)$	9	2018	$1095^{*)}$	2508	22.4
$K_4^*(2045)$	36	2045	$1238^{*)}$	2651	140.8
$\phi(2170)$	3	2175	$1381^{*)}$	2794	259.2
$f_2(2300)$	5	2297	$1095^{*)}$	2508	22.4
$f_2(2340)$	5	2339	$1095^{*)}$	2508	22.4

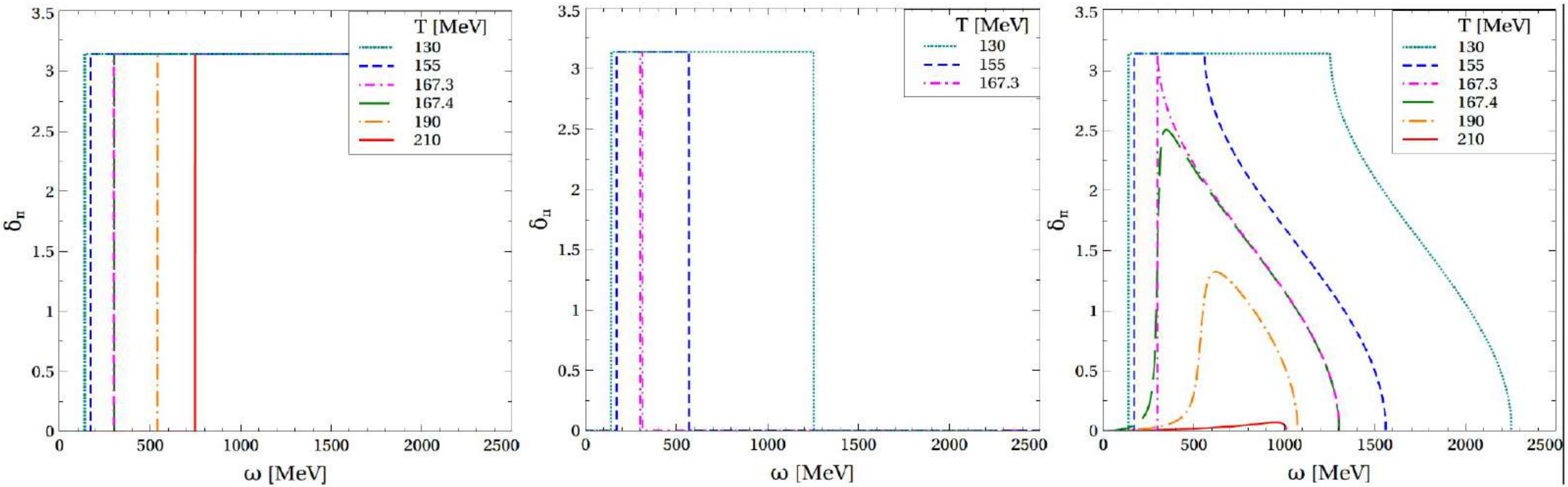
Baryons

PDG baryons	d_i	$M_{\rm PDG}$ [MeV]	M_i [MeV]	$M^<_{{\rm th},i}$ [MeV]	$M^>_{{\rm th},i}$ [MeV]
n/p	4	939	939	1881	16.8
Λ	2	1116	1082	2024	135.2
Σ	6	1193	1082	2024	135.2
Δ	16	1232	$1251^{**)}$	3135	28
Ξ^0	2	1315	1225	2167	253.6
Ξ^-	2	1322	1225	2167	253.6
$\Sigma(1385)$	6	1385	$1394^{**)}$	3278	146.4
$\Lambda(1405)$	2	1405	$1394^{**)}$	3278	146.4
$N(1440)$	4	1440	$1251^{**)}$	3135	28
..					
$N(2195)$	36	2220	$1251^{**)}$	3135	28
$\Sigma(2250)$	6	2250	$1394^{**)}$	3278	146.4
$\Omega^-(2250)$	2	2252	$1680^{**)}$	3564	383.2
$N(2250)$	20	2275	$1251^{**)}$	3135	28
$\Lambda(2350)$	10	2350	$1394^{**)}$	3278	146.4
$\Delta(2420)$	48	2420	$1251^{**)}$	3135	28
$N(2600)$	24	2600	$1251^{**)}$	3135	28

… and colored clusters (model) !

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14

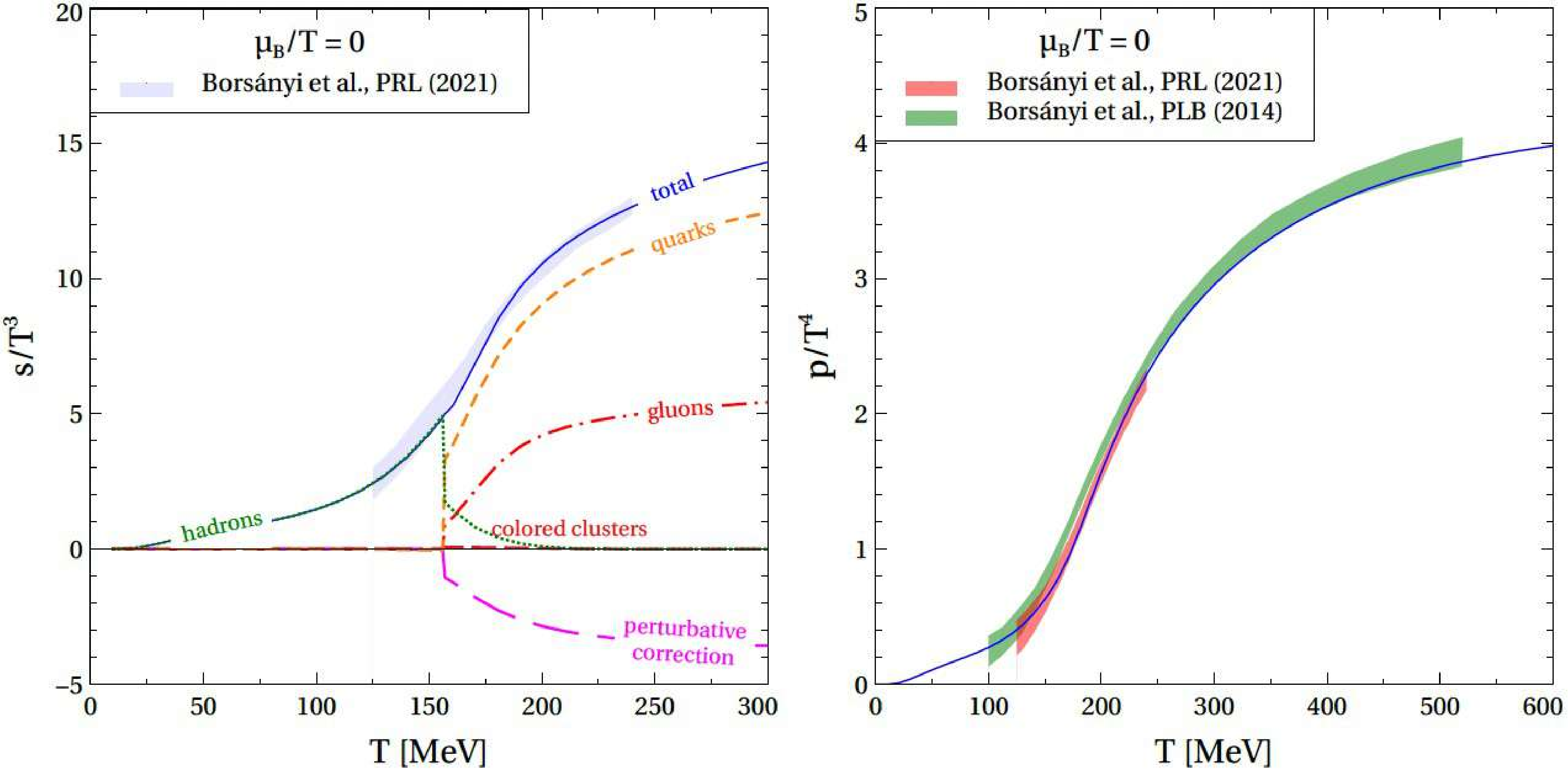
Inputs for the phase shifts (models)



Step-up (SU) model → Hadron Resonance Gas

Step-up-step-down model Step-up-continuum model → Mott Hadron Resonance Gas (MHRG)

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14



$\mu_B/T = 0$
Borsányi et al., PRL (2021)
total
quarks
gluons
hadrons
colored clusters
perturbative
correction
s/T^3
T [MeV]
0
50
100
150
200
250
300
−5
0
5
10
15
20
$\mu_B/T = 0$
Borsányi et al., PRL (2021)
Borsányi et al., PLB (2014)
p/T^4
T [MeV]
0
100
200
300
400
500
600
0
1
2
3
4
5

„Sudden Switch“ from HRG to QGP

Chemical Freeze-out: „inverse“ Mott effect – hadron localization = collapse of the wave function

$$H_{\rm exp}(T_{{\rm cf},i}) = \tau_i^{-1}(T_{{\rm cf},i})$$

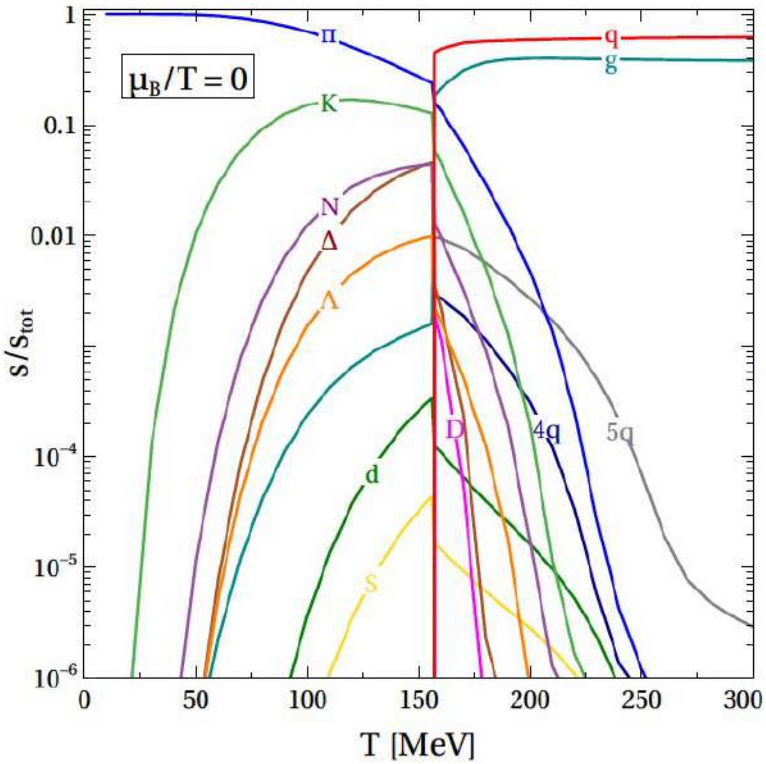
$$H_{\rm exp} = \frac{1}{\tau_{\rm exp}} = \frac{s^{1/3}}{a},$$

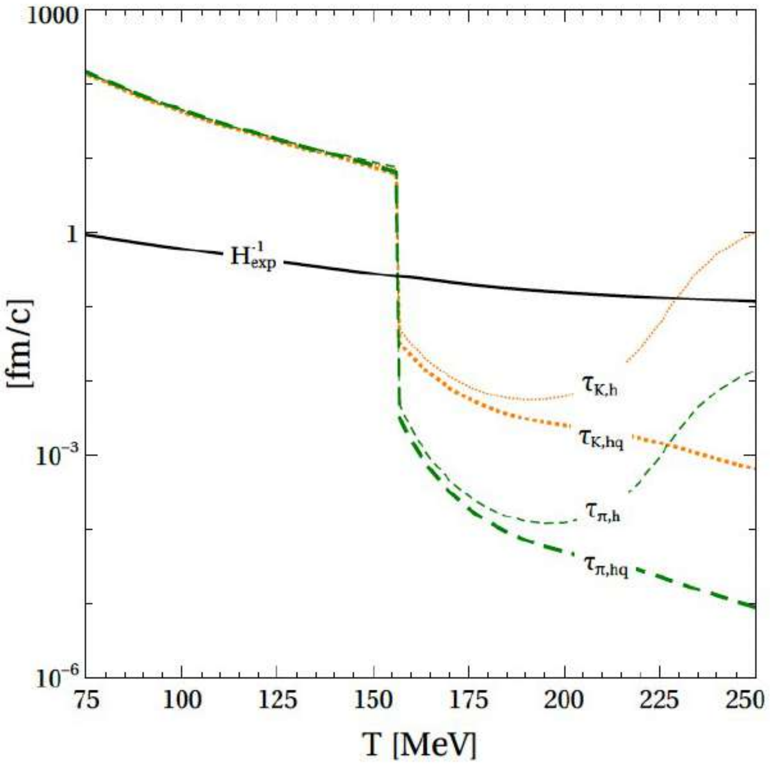
$$\tau_i^{-1} = \sum_j \sigma_{ij} v_{\rm rel} n_j,$$

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle,$$

$$\langle r^2 \rangle_\pi^{1/2} \sim |T - T_{{\rm Mott},\pi}|^{-1/2}$$

$$\langle r^2 \rangle_\pi \sim E_{B,\pi}(T)^{-1/2}$$

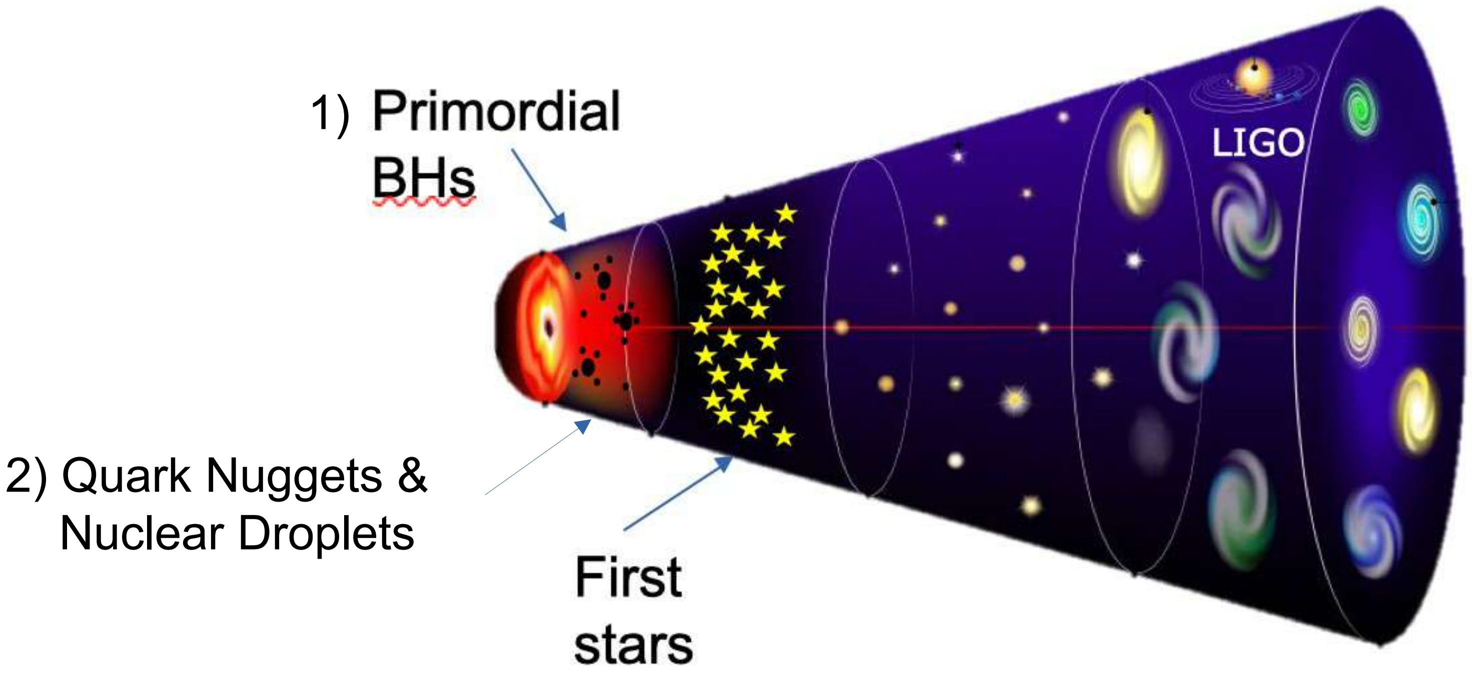




arXiv:2507.10497; PoS (QCHS2024) 247

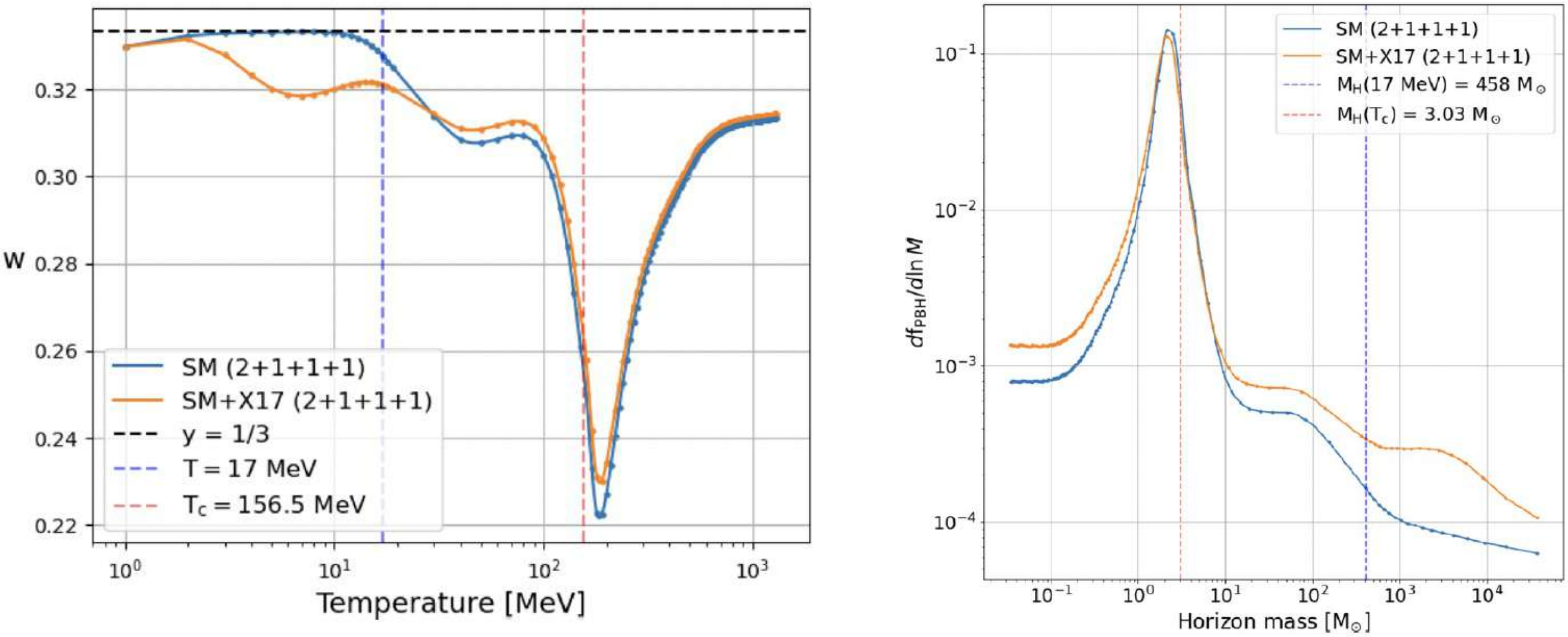
PBH Formation and CFO of heavy elements?

JWST results – primordial black holes !



QCD hadronization transition plays key role for PBH formation !

PBH Formation and CFO of heavy elements?



QCD hadronization transition plays key role plays for PBH formation !

PBH Formation and CFO of heavy elements?

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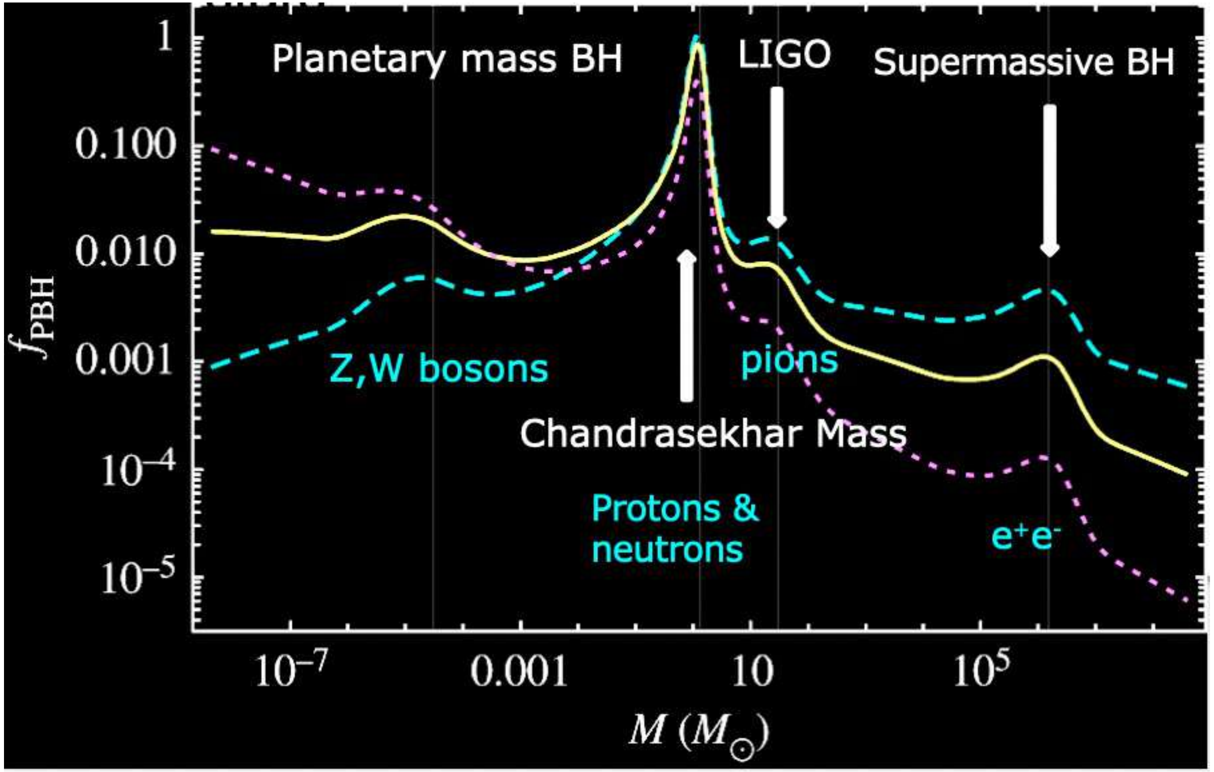

Different peaks correspond to different particles created at the early universe phase transitions and the corresponding reduction in the sound velocity.

BH mass corresponds to the horizon size at each time.

Only requirement is enough fluctuation power in a volume fraction of 10^{-9} of the early Universe.

Carr, Clesse, García-Bellido 2019

QCD hadronization transition plays key role plays for PBH formation !

International Conference on Quark Confinement and the Hadron Spectrum (QCHS 2026)
29.6. - 4.7.2026, Wroclaw, Poland

David Blaschke (Uni Wroclaw & HZDR/CASUS),
Nora Brambilla (TU Munich),
Pior Surowka (Wroclaw Univ. Science & Technology)

https://indico.cern.ch/event/1531304