جامعة الإمام عبدالرحمن بن فيصل
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Anisotropic Flow Correlations in Heavy-Ion collisions

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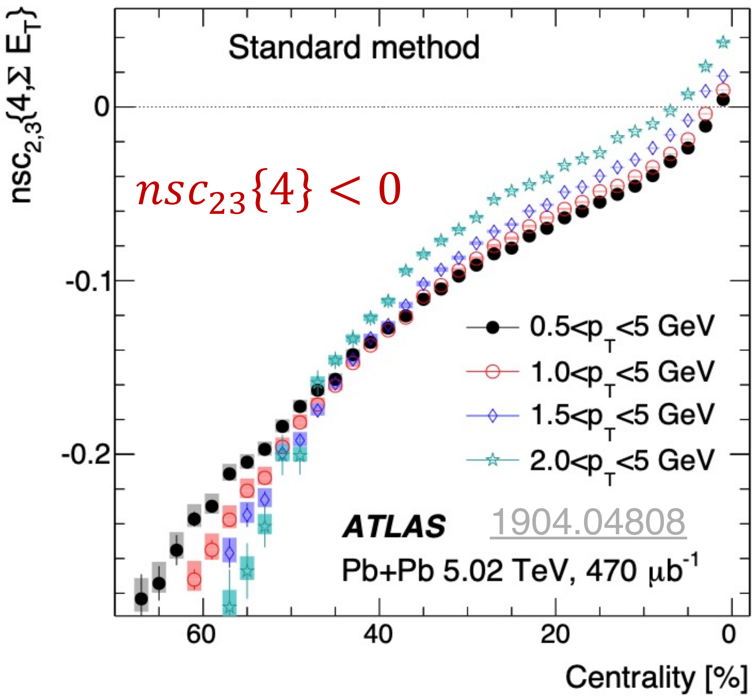
Outline:

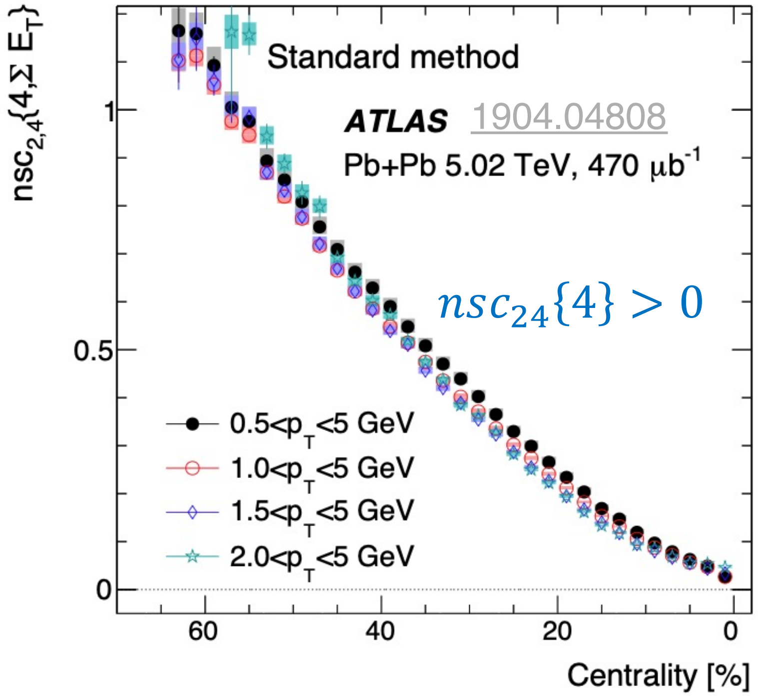
- Collective flow and flow fluctuations
- Elliptic and triangular flow
- Mixed flow cumulants
- Intrinsic cumulants: a new perspective
- Predictions and comparison with data

Based on:
M Alqahtani and Jean-Yves Ollitrault *Phys.Lett.B* 872 (2026) 140066
M Alqahtani and Jean-Yves Ollitrault *Phys.Lett.B* 877 (2026) 140504

Correlations between flow harmonics I

$$\mathrm{nsc}_{n,m}\{4\} \propto \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle$$





- v_2 **and** v_3 are anti-correlated, while v_2 **and** v_4 are positively correlated.

From initial geometry to collective flow

The initial overlap geometry of the collision is quantified by the eccentricities

$$\boldsymbol{\varepsilon_n} = \varepsilon_n e_n^{in\Phi_n}$$

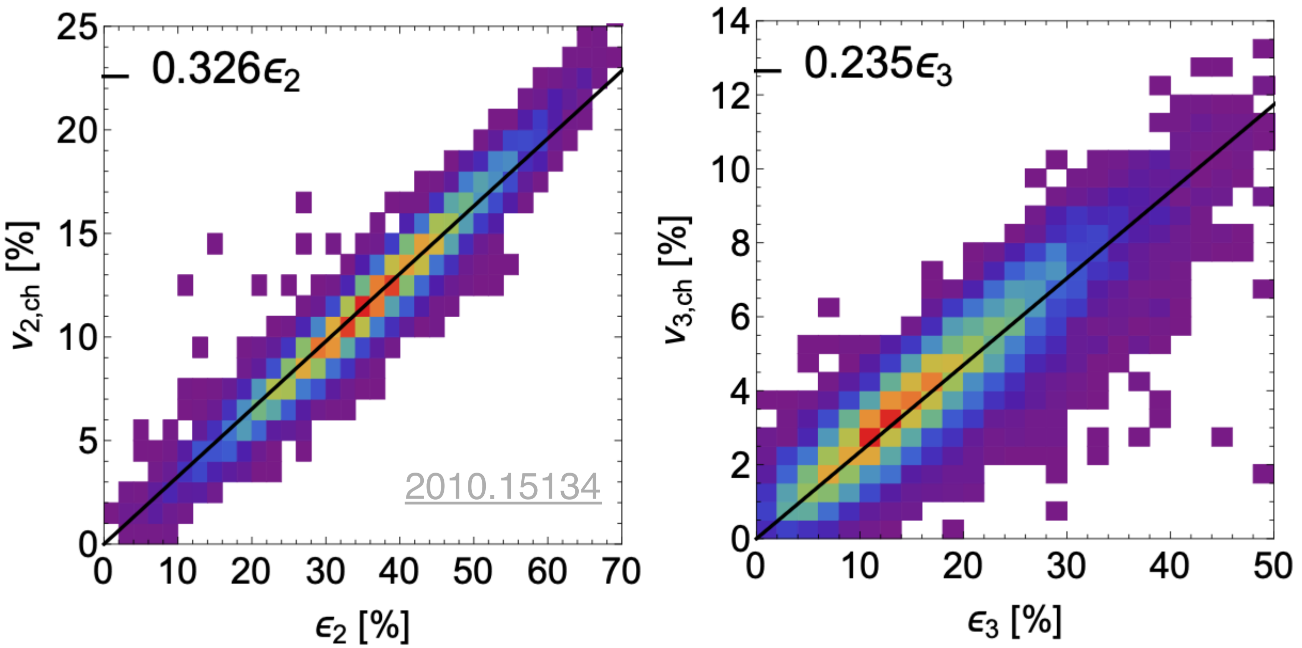
ε_2: ellipticity ε_3: triangularity

The final-state anisotropic flow is characterized by

$$\boldsymbol{V}_n = v_n e_n^{in\Psi_n}$$

v_2: elliptic flow v_3: triangular flow

Trajectum framework



Hydrodynamic evolution converts initial-state eccentricities $\mathcal{E}_n$ into final-state collective flow.

$v_2 \approx \kappa_2 \varepsilon_2$ & $v_3 \approx \kappa_3 \varepsilon_3$

See also H. Niemi et al 1212.1008

Key message:

- Event-by-event fluctuations of the initial geometry are reflected in the flow harmonics v_2 and v_3.
- We will use this approximate relations to understand relations betwenn anisotropic flow coefficients.

Distributions of initial eccentricity

- Hydrodynamic response implies

$$P(v_2) \propto P(\varepsilon_2). \quad \&. \quad P(v_3) \propto P(\varepsilon_3).$$

What one finds (See slide 10 for a clearer illustration)

ε_2 : geometry-dominated.

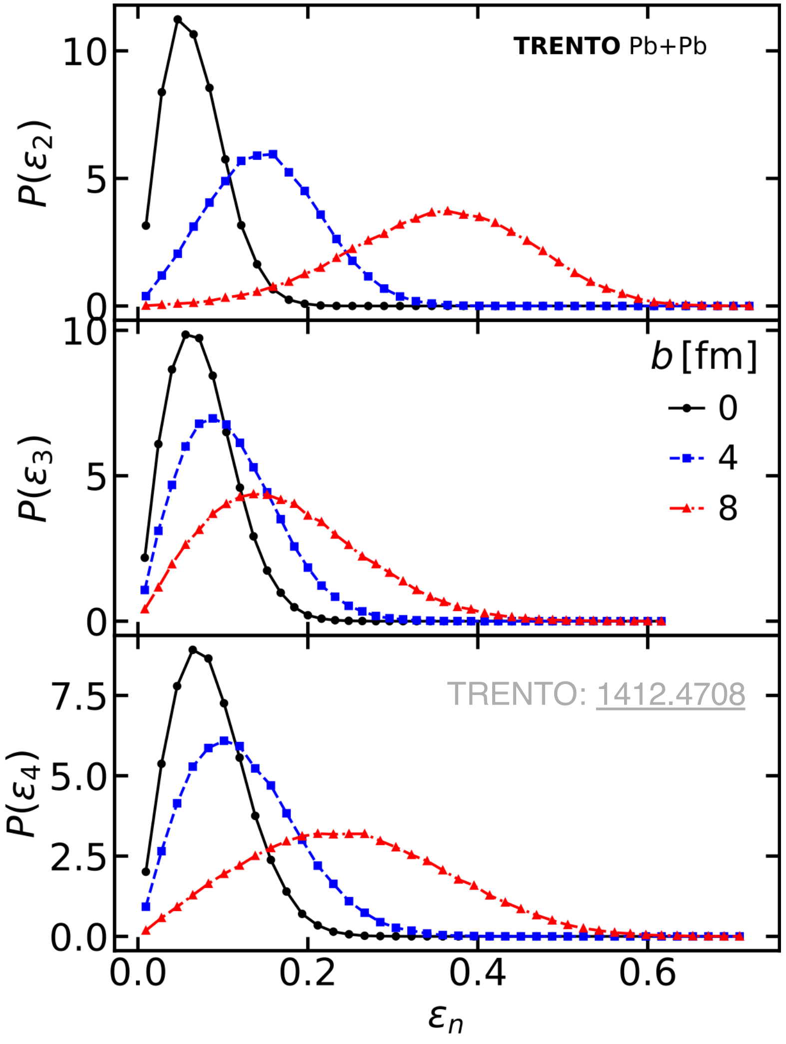
ε_3: fluctuation-dominated.

ε_4: mixed origin (geometry + fluctuations).

- Eccentricity flucuations are approximately Gaussian.
- Deviations from Gaussianity:
  - Bessel-Gaussian distribution
  - Elliptic power distribution

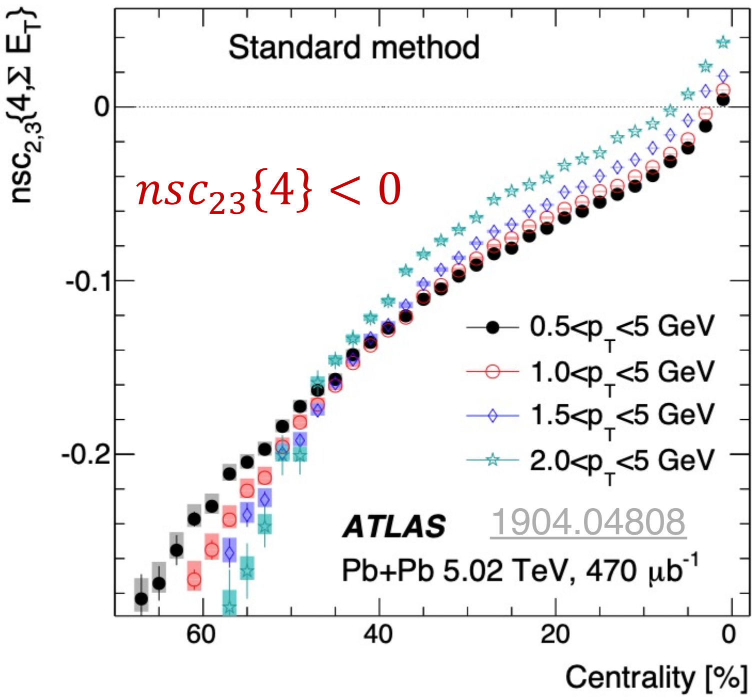
See:
Voloshin et al 0708.0800
Li Yan et al 1405.6595

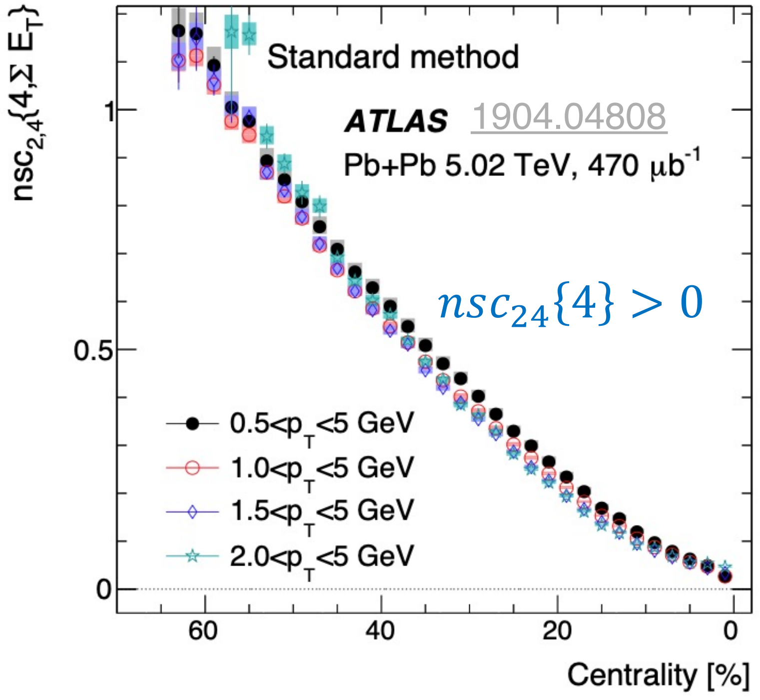
TRENTO predictions for eccentricity distributions at different b



Correlations between flow harmonics II

$$\mathrm{nsc}_{n,m}\{4\} \propto \langle v_n^2 v_m^2\rangle - \langle v_n^2\rangle\langle v_m^2\rangle$$





- v_2 **and** v_3 are anti-correlated, while v_2 **and** v_4 are positively correlated.
- $nsc_{23}\{4\} \ll 1$, can be understood by the smallness of non-Gaussian effects.
- In this talk, we address the question: What drives the v_2-v_3 anti-correlation?

Moment and cumulant generating functions

For a random variable X, the moment-generating function is defined as:

$$M(t) \equiv \langle e^{tX} \rangle = 1 + t\langle X \rangle + \frac{t^2}{2!}\langle X^2 \rangle + \cdots = \sum_{n=0}^{\infty} \frac{t^n}{n!}\langle X^n \rangle .$$

Where t is an expansion parameter. One can isolate any moment by:

$$\langle X^m \rangle = \frac{d^m M(t)}{\mathrm{d}t^m}\Big|_{t=0}$$

The cumulant-generating function is defined by:

$$K(t) \equiv \ln M(t) = \ln\langle e^{tX} \rangle = t\,\kappa_1 + \frac{t^2}{2!}\kappa_2 + \cdots = \sum_{n=1}^{\infty} \frac{t^n}{n!}\kappa_n$$

$\kappa_1 = \langle X \rangle$; mean μ
$\kappa_2 = \langle X^2 \rangle - \langle X \rangle^2$; variance σ^2
$\kappa_3 = \langle X^3 \rangle - 3\langle X \rangle\langle X^2 \rangle + 2\langle X \rangle^3$
$\kappa_4 = \langle X^4 \rangle - 4\langle X \rangle\langle X^3 \rangle - 3\langle X^2 \rangle^2 + 12\langle X \rangle^2\langle X^2 \rangle - 6\langle X \rangle^4$
… and so on

$\mu = \kappa_1$
$\sigma^2 = \kappa_2$
skw $= \kappa_3/\sigma^3$
Excess kurt $= \frac{\kappa_4}{\sigma^4}$

Remark: $\kappa_n = 0 \;\; (n \geq 3)$
for a Gaussian Distribution

See Rajeev Bhalerao, arXiv:2009.09586

Measuring correlations between v_2 and v_3

- Correlations between v_2 and v_3 can be quantified using moments or cumulants.
- Mixed-harmonic cumulants (MHC) isolate **genuine correlations** between v_2 and v_3.
- MHC comes in different orders: $MHC\left(v_2^{2m}, v_3^{2q}\right)$

For example:

$MHC(v_2^2, v_3^2) = \langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle$; Lowest-order cumulant (covariance)

$$MHC(v_2^4, v_3^2) = \langle v_2^4 v_3^2 \rangle - 4\langle v_2^2 v_3^2 \rangle \langle v_2^2 \rangle - \langle v_2^4 \rangle \langle v_3^2 \rangle + 4\langle v_2^2 \rangle^2 \langle v_3^2 \rangle$$

...

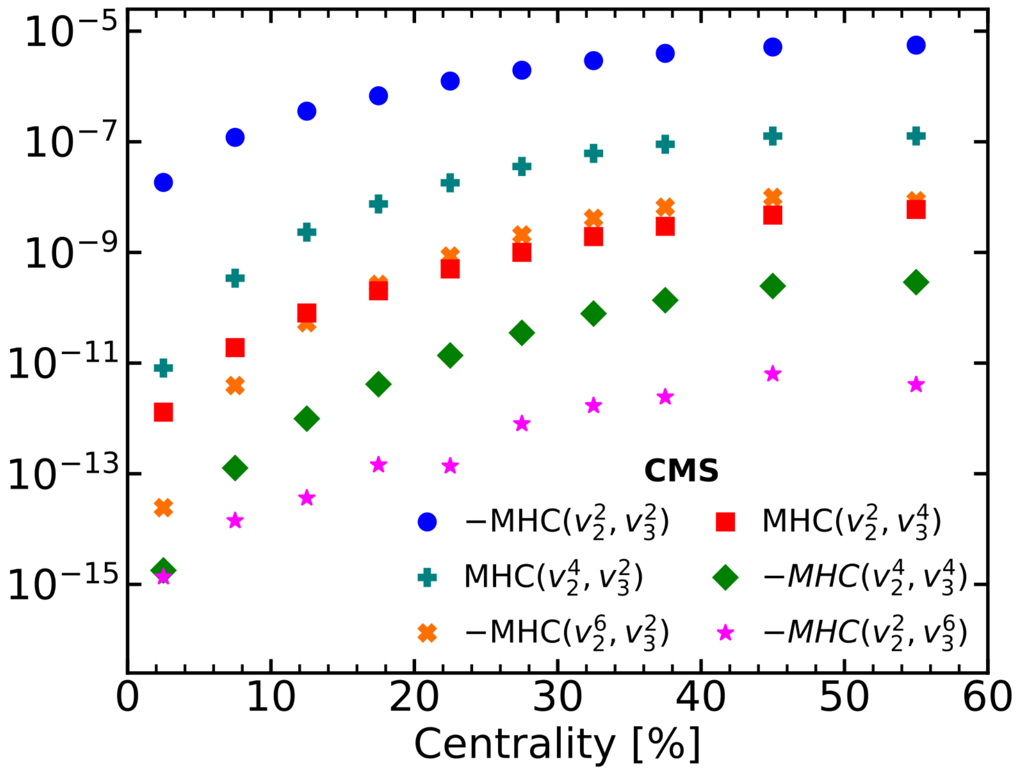
- To compare different orders and systems, cumulants are often normalized.

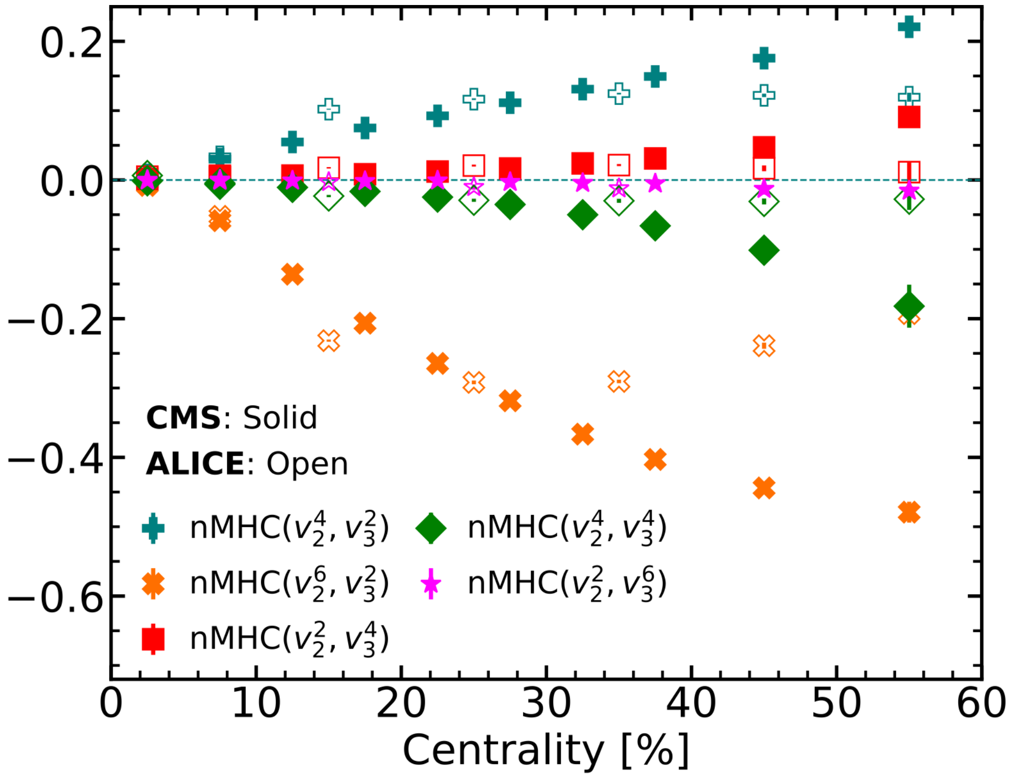
Normalized correlation strength

$$nMHC\left(v_2^{2m}, v_3^{2q}\right) \equiv \frac{MHC\left(v_2^{2m}, v_3^{2q}\right)}{\langle v_2^{2m} \rangle \langle v_3^{2q} \rangle}$$

0 → no correlation
+1 → correlated
–1 → anti-correlated

Measured v_2-v_3 mixed-harmonic cumulants





- Higher-order MHCs are suppressed by several orders of magnitude.

- Non-trivial higher-order v_2-v_3 correlation patterns.
- The sign changes with increasing m or q.

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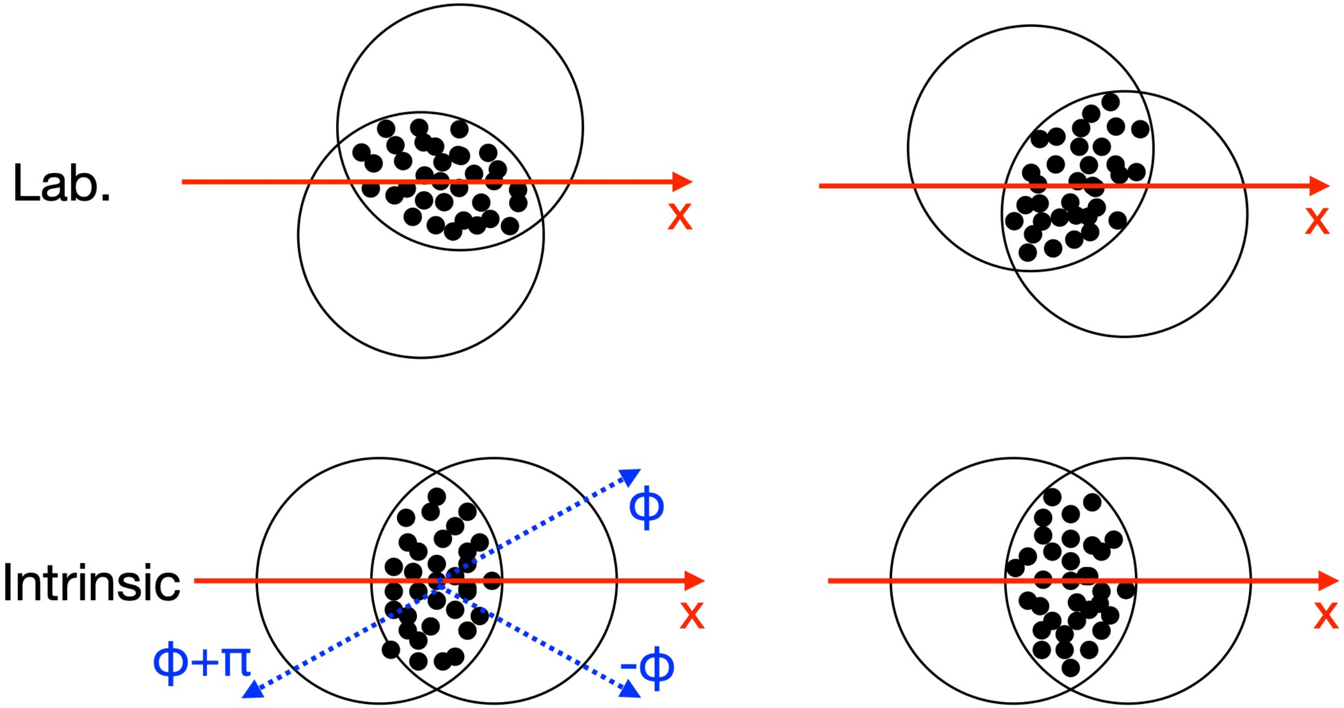
ALICE: Phys. Lett. B 818 (2021) 136354.
CMS: Phys. Lett. B 876 (2026) 140359.

Lab vs. intrinsic frames

Two collision events with the same impact parameter b



Intrinsic frame revelas the underlying anisotropy washed out in the lab frame

LAB Frame:

- Random reaction-plane Ψ_{RP}
- Azimuthal averaging

$$\langle v_{2x}\rangle_{\text{lab}} = 0.$$

- **Intrinsic Frame:**
- All events are aligned, $b \parallel x$
- Preserves reaction-plane information
- Nonzero mean elliptic flow in the reaction plane

$$\langle v_{2x}\rangle_{\text{int}} \equiv \bar{V}_2 \neq 0,$$

- At b=0, rotational symmetry is exact:

$$\langle v_{2x}\rangle = 0,$$

intrinsic and lab cumulants coincide.

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Initial ellipticity and triangularity

TRENTO initial conditions at fixed impact parameter

- Probability distribution of the eccentricity vector $\vec{\varepsilon}_n = (\varepsilon_{nx}, \varepsilon_{ny})$

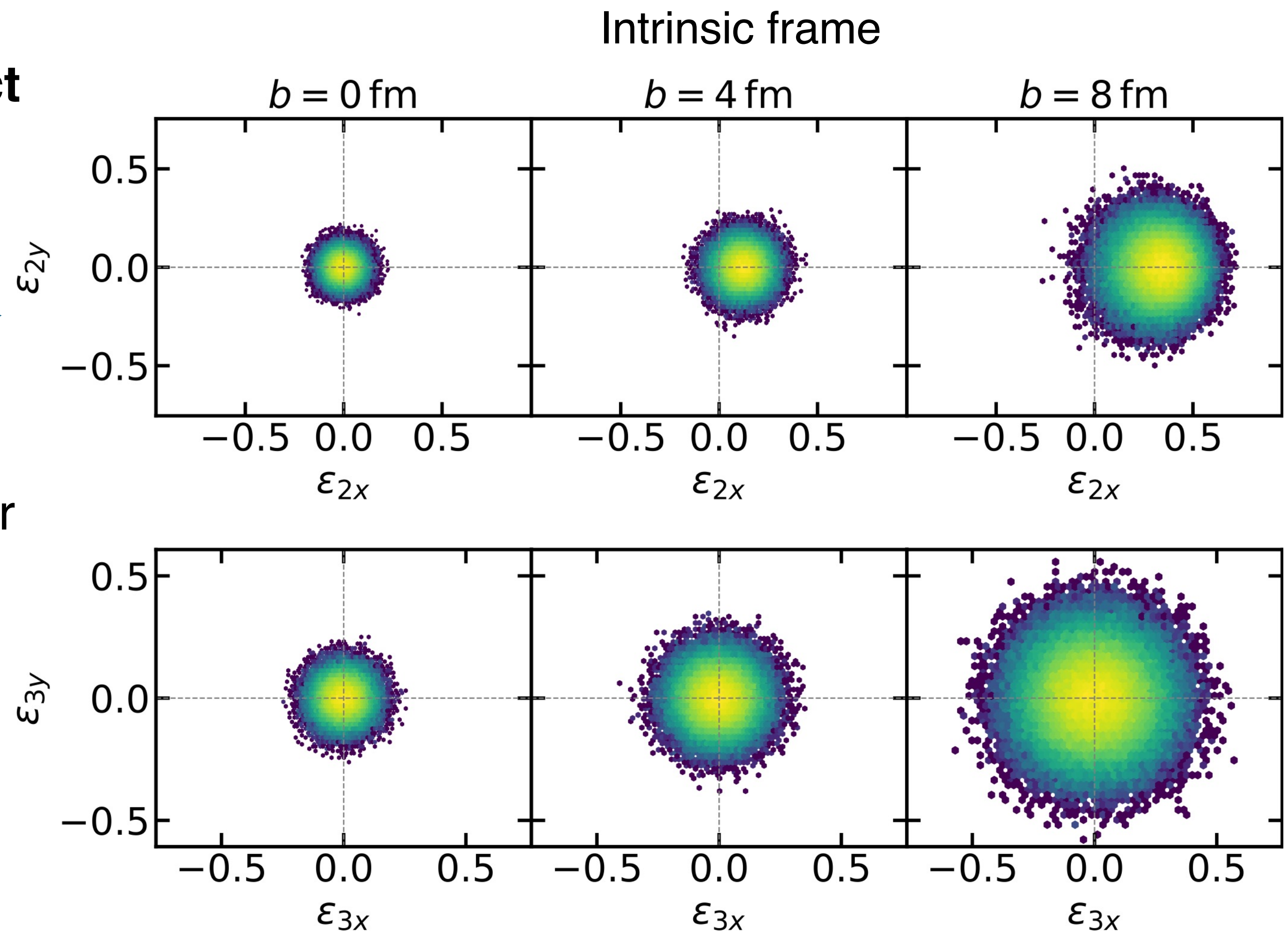
  $\varepsilon_{nx} = \varepsilon_n \cos(n\Phi_n)$ & $\varepsilon_{ny} = \varepsilon_n \sin(n\Phi_n)$

- Only ellipticity develops a nonzero mean for $b > 0$.

  $\langle \epsilon_{2x} \rangle \neq 0$, $\langle \epsilon_{2y} \rangle = 0$

  $\langle \epsilon_{3x} \rangle = \langle \epsilon_{3y} \rangle = 0$

- v_2 is geometry-driven, whereas v_3 is fluctuation-driven.



Predictions from intrinsic cumulants

Intrinsic cumulants are defined as *

$$c_{mpqr} = \langle V_2^m (V_2^*)^p V_3^q (V_3^*)^r \rangle_c.$$

where $\langle \ldots \rangle_c$ denotes the connected cumulant

Lab-frame mixed cumulants can be expanded in terms of intrinsic cumulants.

$$MHC(v_2^2, v_3^2) = 2\bar{V}_2 c_{0111} + \cdots$$
$$MHC(v_2^4, v_3^2) = -4\bar{V}_2^3 c_{0111} + \cdots$$
$$MHC(v_2^6, v_3^2) = 24\bar{V}_2^5 c_{0111} + \cdots$$

- Higher-order terms are neglected (see slide 11).
- $\bar{V}_2 \approx v_2\{4\}$ See: arXiv:0708.0800

c_{0111} quantifies the linear correlation between elliptic flow in the reaction plane and the triangularity.

$$c_{0111} = \langle v_{2x}\, v_3^2 \rangle - \langle v_{2x} \rangle \langle v_3^2 \rangle$$ mixed skewness

Taking ratios yields

$$\frac{MHC(v_2^6, v_3^2)}{v_2\{4\}^2\, MHC(v_2^4, v_3^2)} = -6 \qquad \frac{MHC(v_2^4, v_3^2)}{v_2\{4\}^2\, MHC(v_2^2, v_3^2)} \approx -2$$

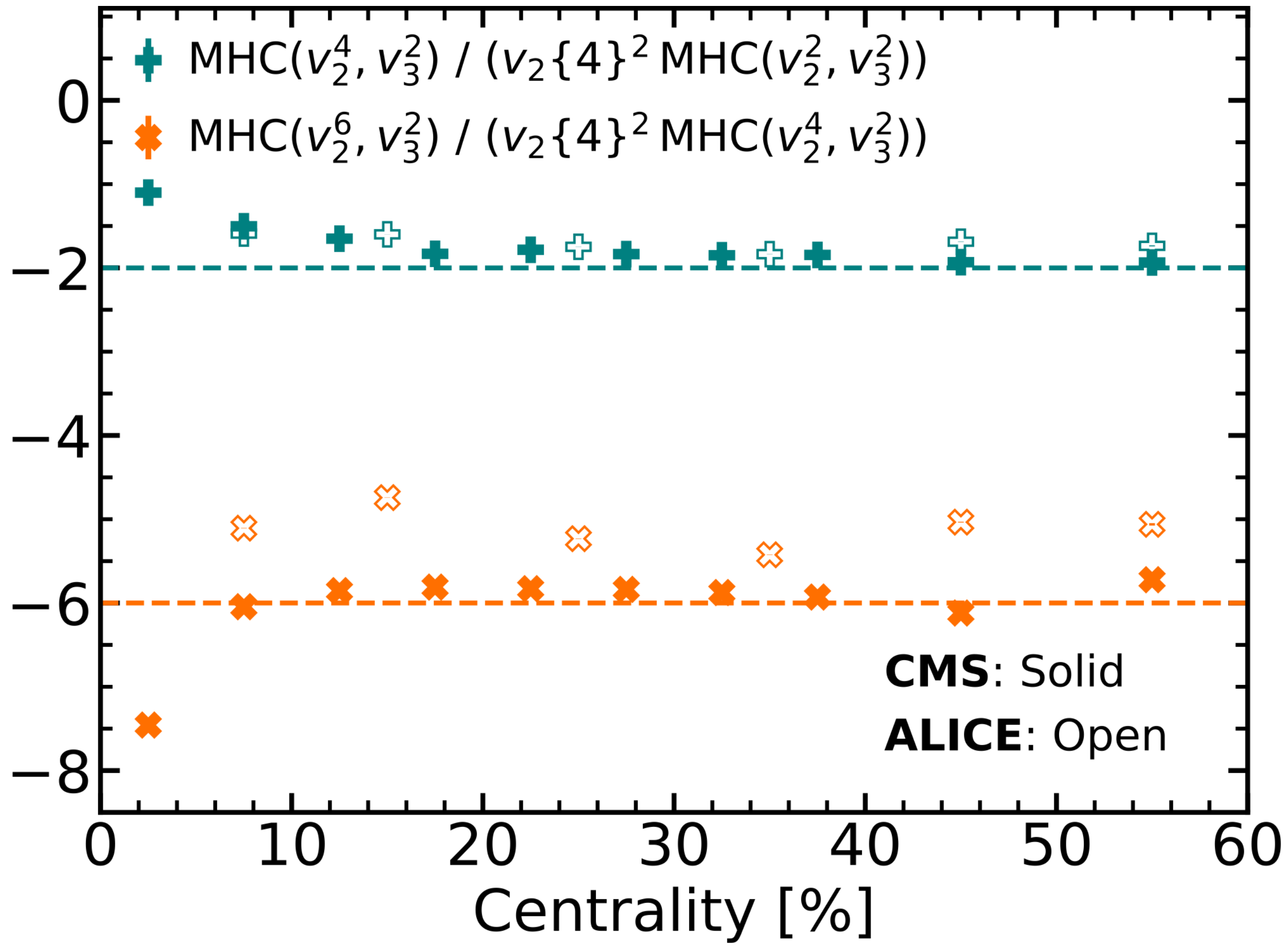
...

Parameter-free predictions from c_{0111} dominance

*They can be generated systematically using the cumulant generating function

Predicted ratios vs. data

e.g.



- The predicted ratios are in good agreement with CMS data.
- ALICE data show deviations, possibly due to wider centrality bins.
- Data are broadly consistent with the intrinsic-cumulant hierarchy.

ALICE: Phys. Lett. B 818 (2021) 136354.
CMS: Phys. Lett. B 876 (2026) 140359.
F. G. Gardim et al 1608.02982

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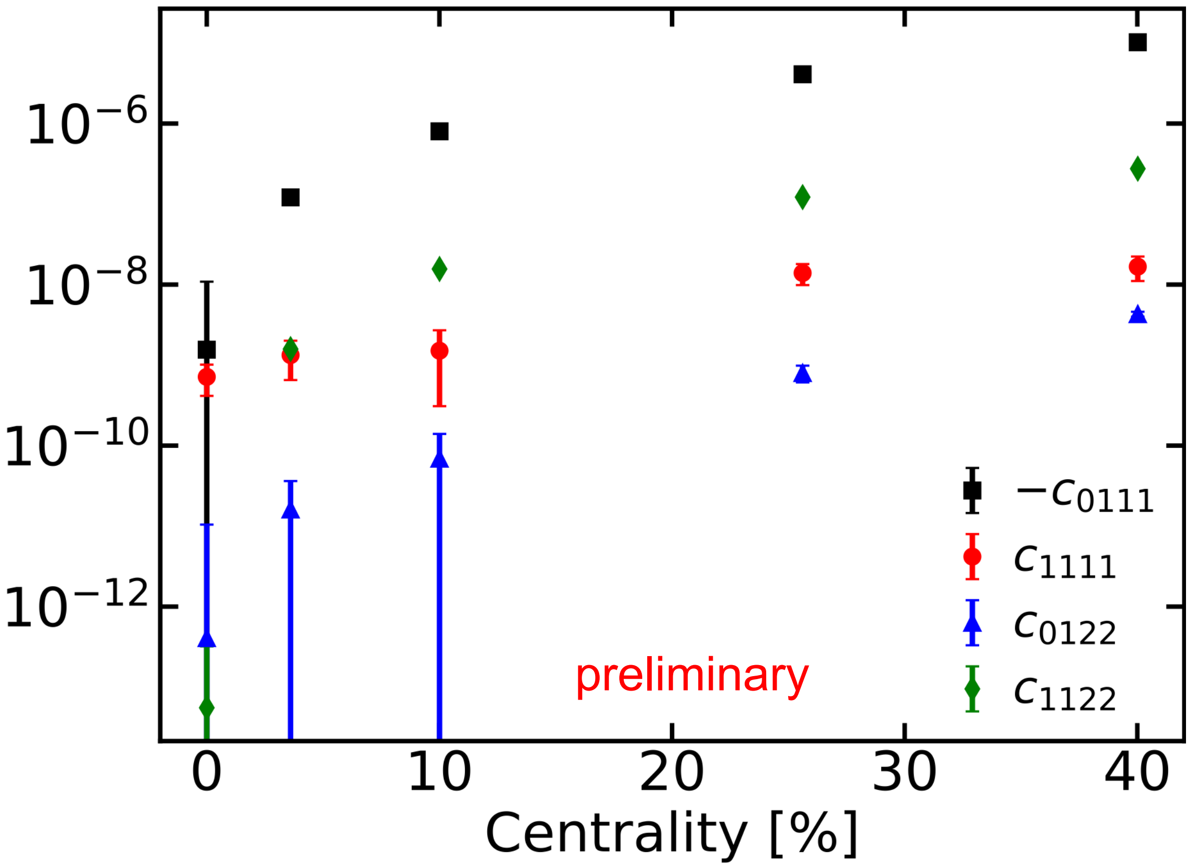
Hierarchy of intrinsc cumulants

How important are higher-order intrinsic cumulants?

- Fix b to eliminate centrality fluctuations.
- Run hydrodynamics at fixed b.
- Compute intrinsic cumulants c^{int}_{mpqr}.

Main Result:

- c_{0111} is the Dominant cumulant for $b > 0$.
- Higher-order cumulants are strongly suppressed by several orders of magnitude.



Work in progress with Andreas Kirchner and Jean-Yves Ollitrault.

Summary and outlook

- Correlations between v_2 and v_3 are driven by non-Gaussian properties of the joint distribution $P(v_2, v_3)$.
- The intrinsic frame reveals anisotropic information hidden by azimuthal averaging in the lab frame.
- Intrinsic cumulants exhibit a strong hierarchy:

$$c_{0111} \gg \text{higher-order intrinsic cumulants}$$

- This hierarchy leads to simple relations between mixed-harmonic cumulants, e.g.

$$\frac{MHC(v_2^4, v_3^2)}{v_2\{4\}^2\, MHC(v_2^2, v_3^2)} \approx -2$$

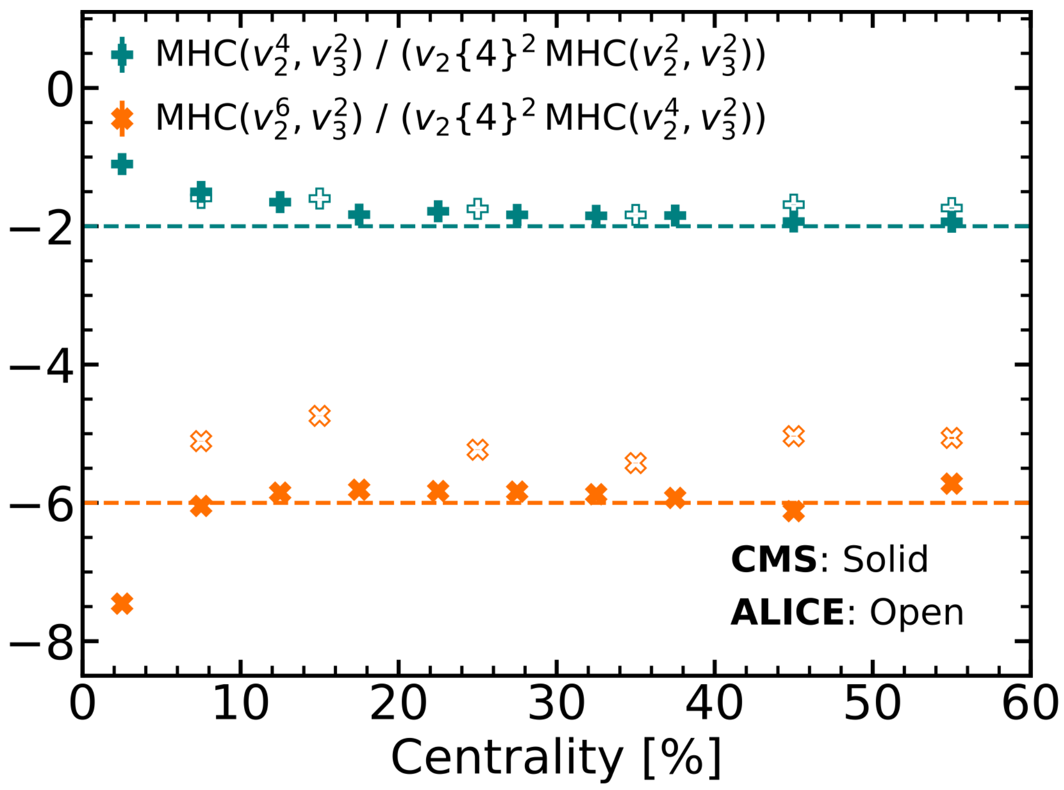
- Further explore intrinsic cumulants using hydrodynamic simulations at fixed b.

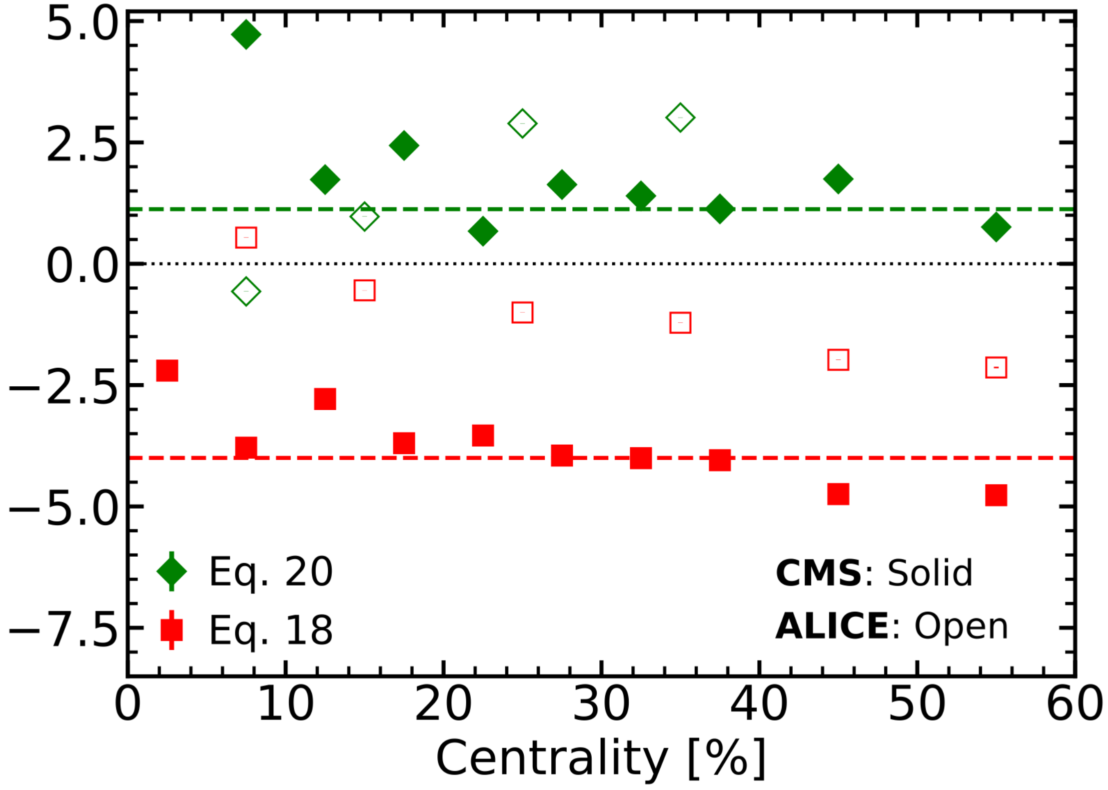
Work in progress with Andreas Kirchner and Jean-Yves Ollitrault.

- Extend the analysis to v_2-v_4 correlations (not trivial!) Left for future work

Backup Slides

Predicted ratios vs. data





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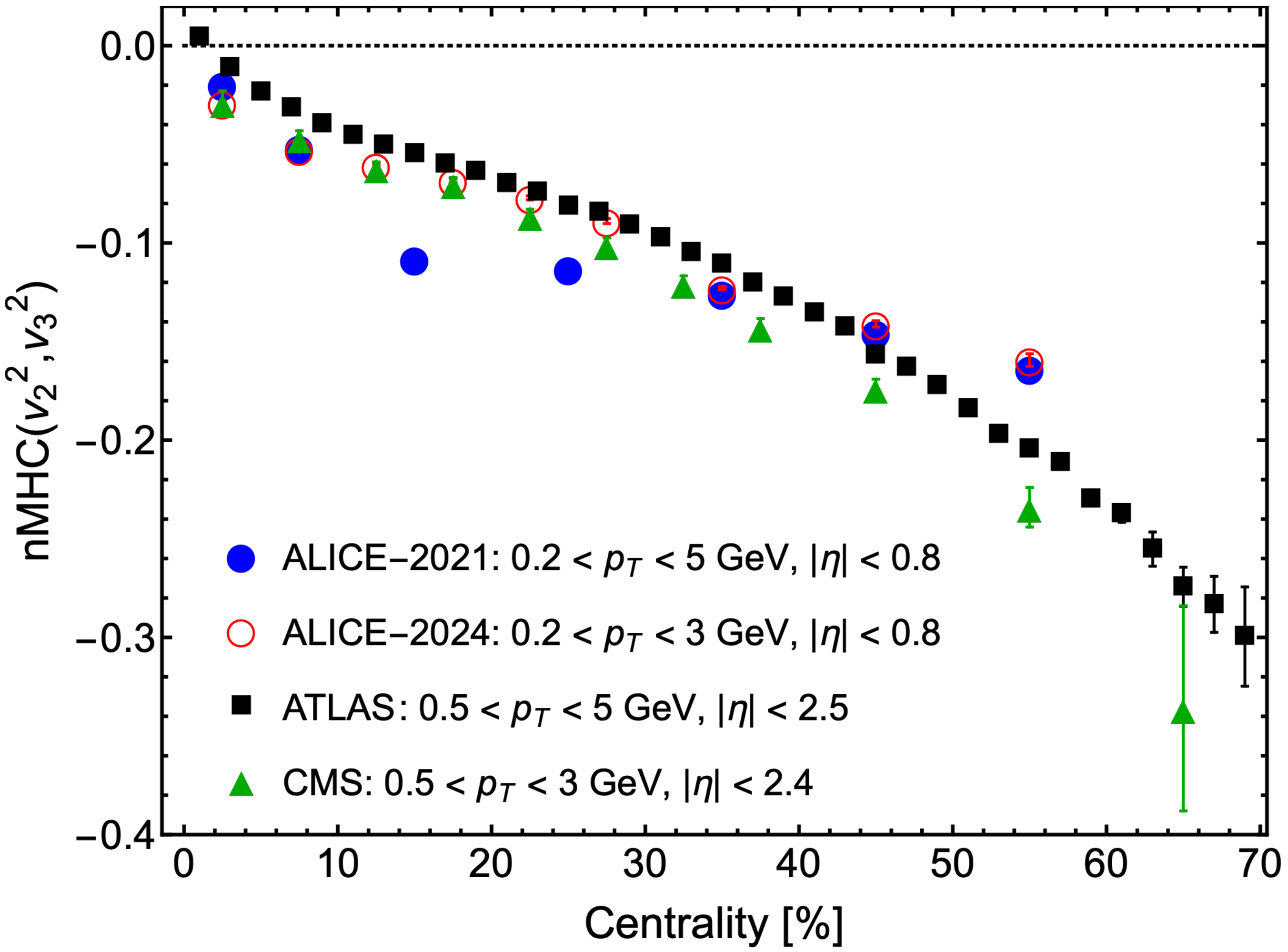
Comparison of experimental measurements



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Estimating the hierarchy of intrinsc cumulants

- Two relevant sources of fluctuations: size and shape.
- **Size:** a cumulant of order k varies with N like N^{1-k} (Here $N \sim N_{part}$)
- **Shape:** for $\bar{V}_2 \ll 1$, intrinsic cumulants scale with powers of $\bar{V}_2$ according to

$$c_{mpqr} \sim \mathcal{O}\left(\mathcal{F}^{2(m+p+q+r-1)} \bar{V}_2^{\left|m-p+\frac{3}{2}(q-r)\right|}\right)$$

$\mathcal{F}$: typical magnitude of fluctuations

Consistency check: $c_{1100} \sim c_{0011} \sim \mathcal{O}(\mathcal{F}^2)$ (variance)

Predicted Hierarchy:

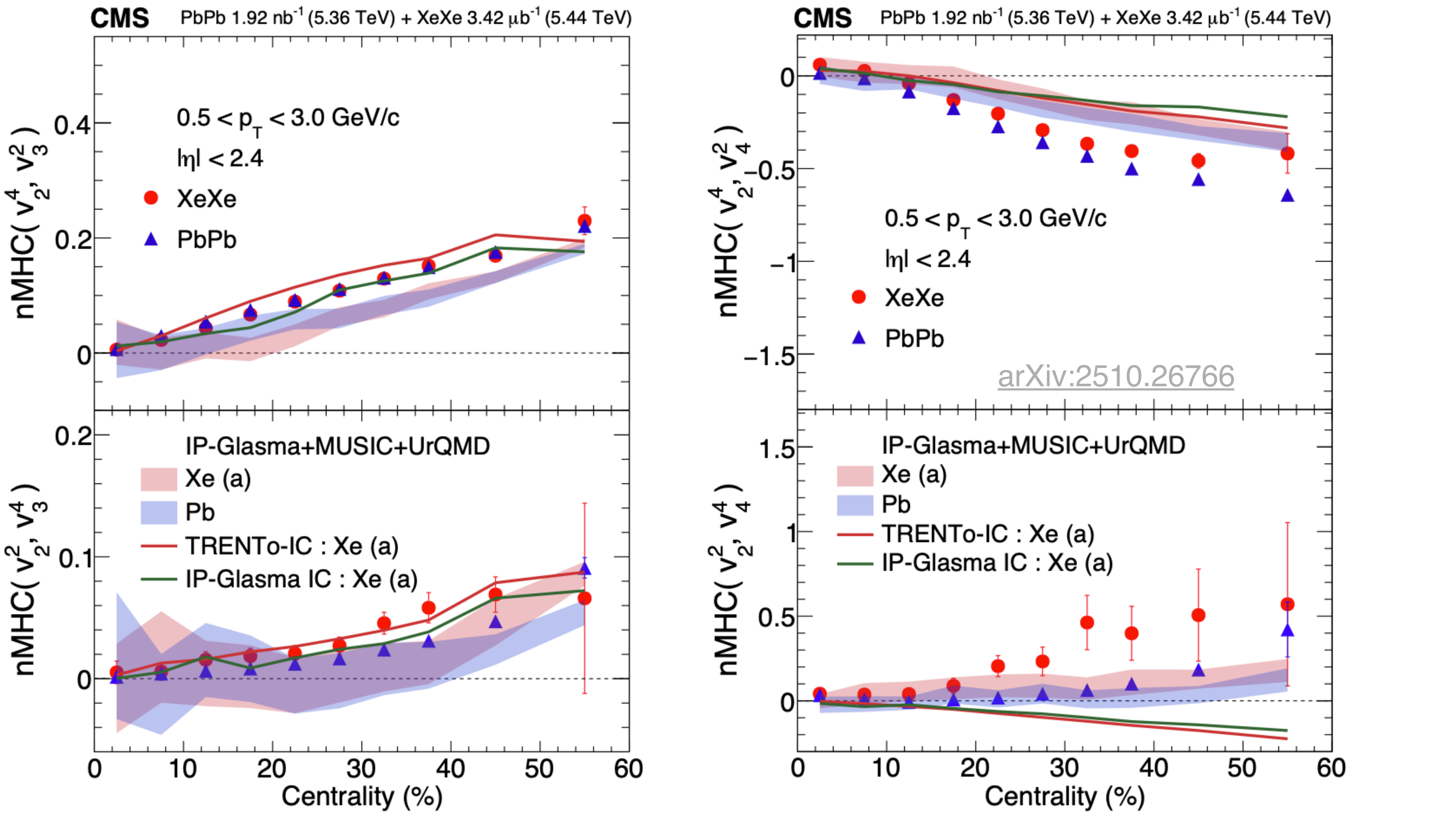
$$c_{0111} \sim \mathcal{O}(\mathcal{F}^4 \bar{V}_2) \sim \mathcal{O}(\mathcal{F}^5)$$

$$c_{0122} \sim \mathcal{O}(\mathcal{F}^8 \bar{V}_2)$$

...

$\Rightarrow c_{0111}$ is the dominant cumulant

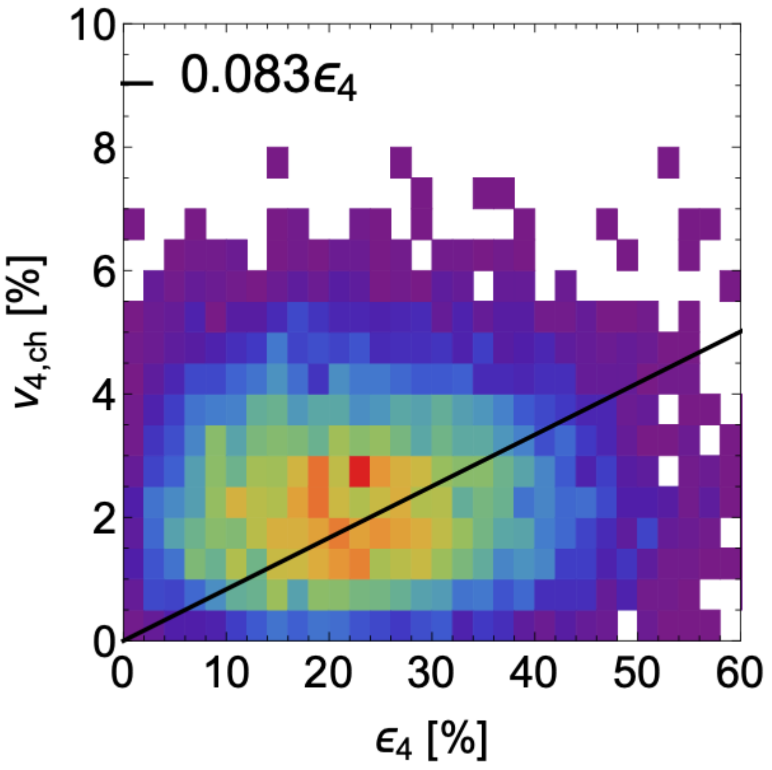
Mixed cumulants: Hydrodynamics vs Data

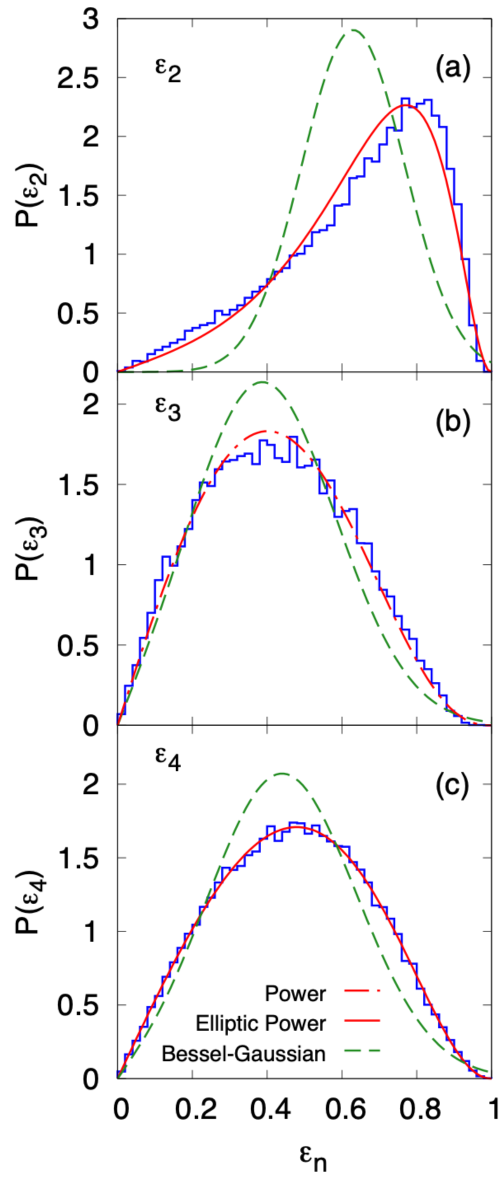


Overall agreement between hydrodynamics and CMS data for both XeXe and PbPb.

The v_4-ε_4 relation is not purely linear due to nonlinear mode coupling.

$$V_4 = V_{4L} + \chi_4 V_2^2$$

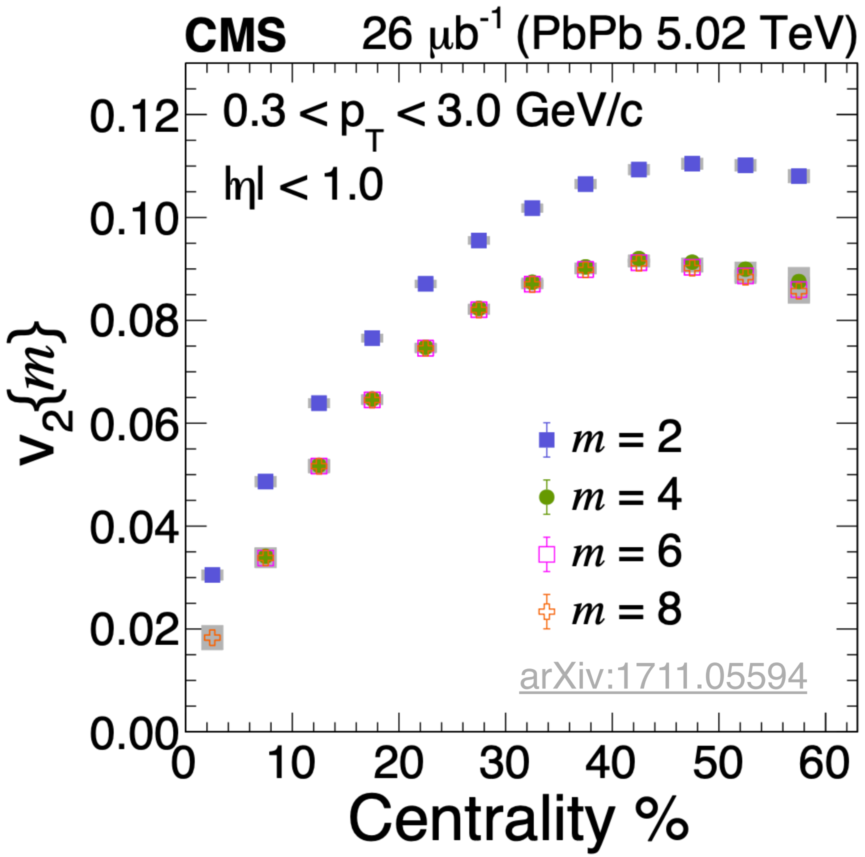


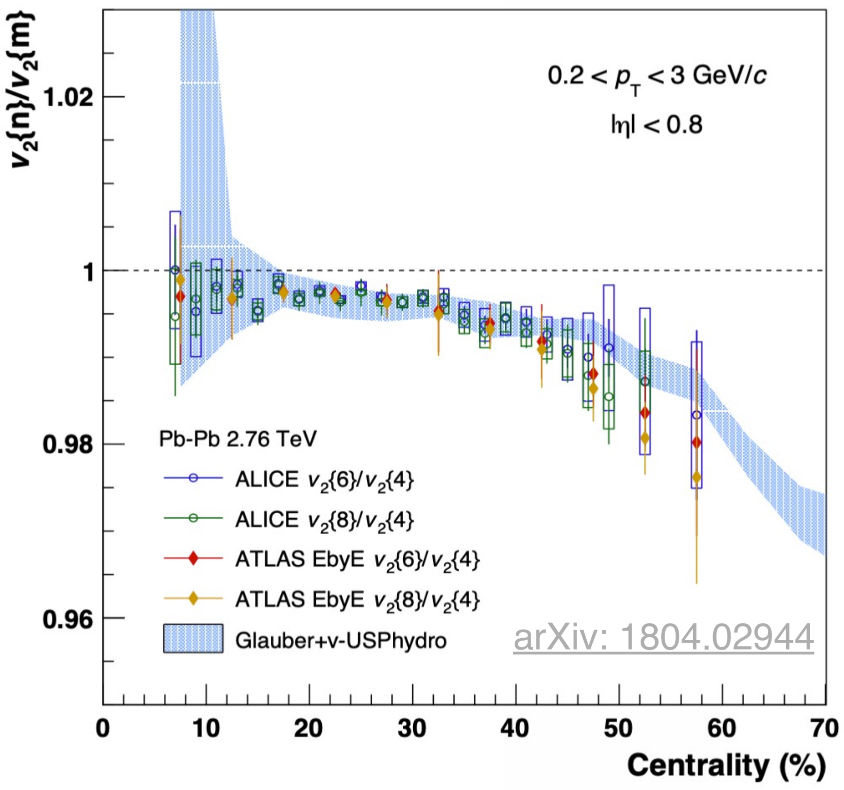


See e.g.
Teaney and Yan: 1206.1905
Jacquelyn Noronha-Hostler et al. 1511.03896

See Li Yan et al 1405.6595

Non-Gaussian fluctuations





- Gaussian fluctuations imply

$$v_2\{4\} \simeq v_2\{6\} \simeq v_2\{8\}$$

- Small deviations provide a sensitive probe

- A small splitting is observed ($< 1\%$).
- The splitting increases towards peripheral collisions
- Offers a sensitive test for hydrodynamic models