

An optimization approach to understand biological search

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Darwinian view of life

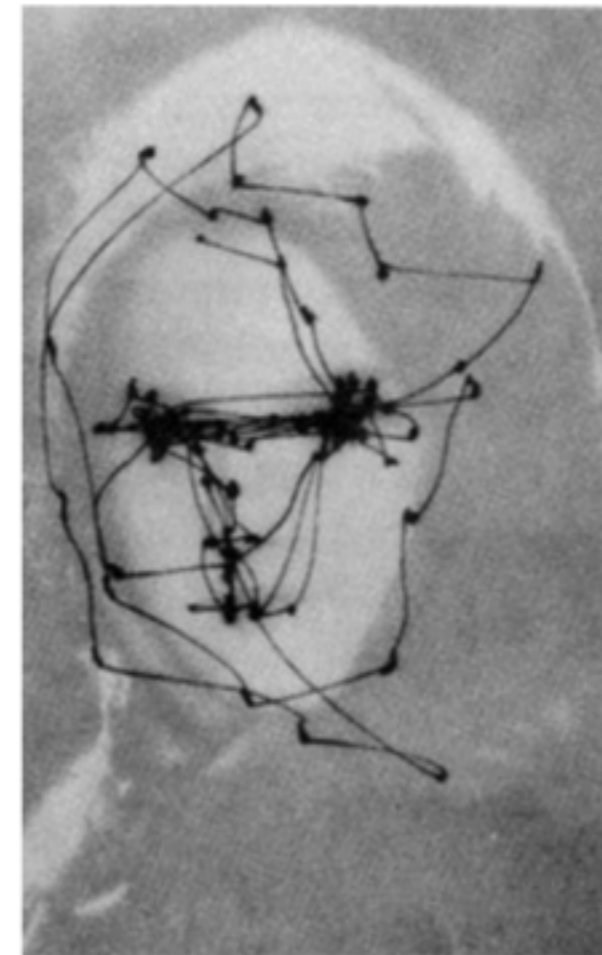
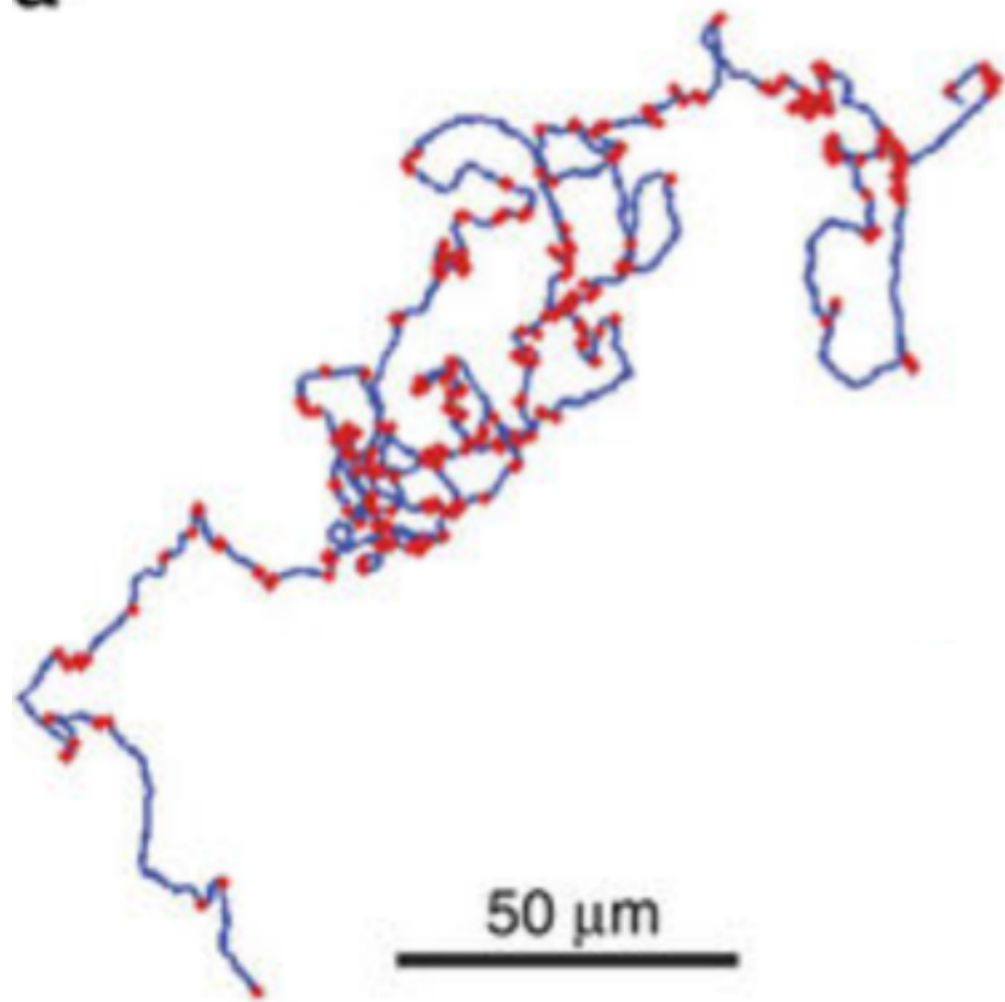


An evolutionally conserved search strategy: Levy flight

Bacteria

Eye position

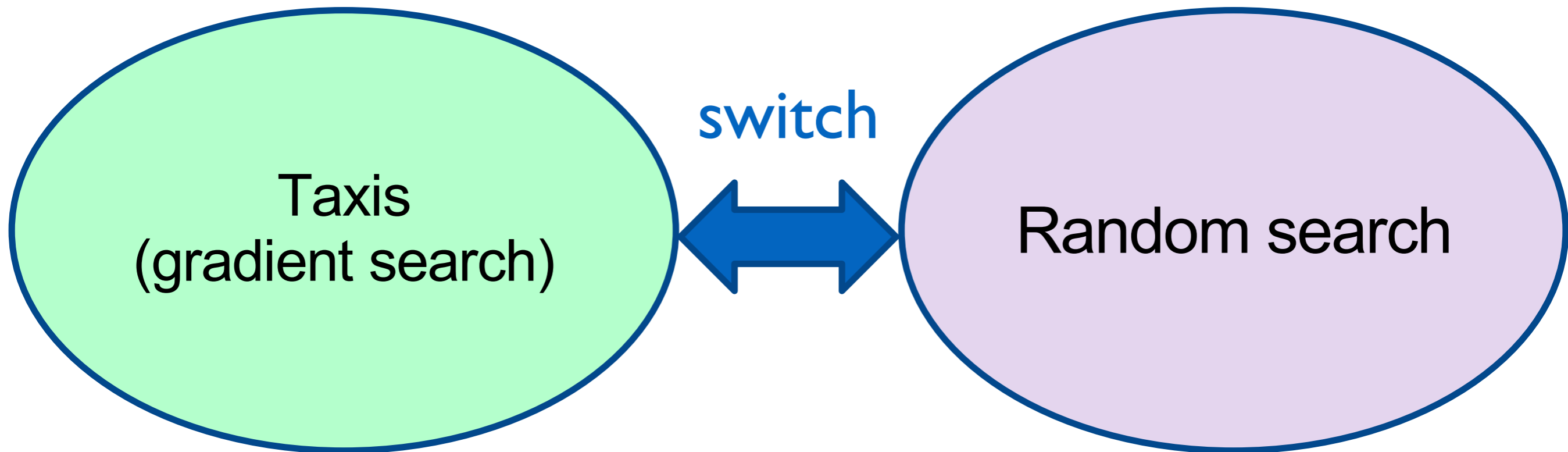
a



Ariel et al. 2015

<http://www.androidblues.com/visualperception.html>

Conventional view



Levy flight emerges from the 2nd-order stochastic optimization

smoothness

gradient

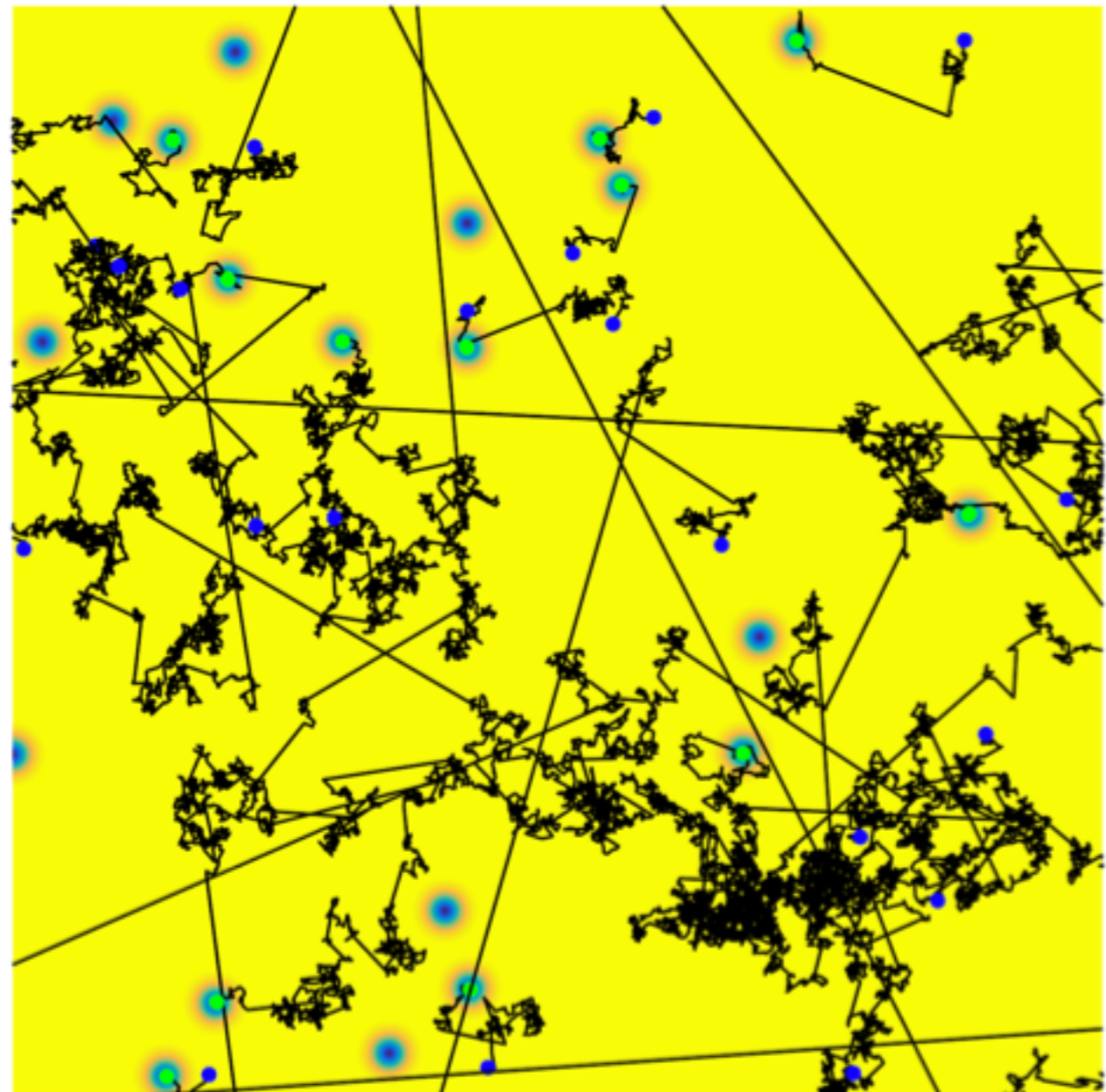
$$\Delta \propto (H + \beta I + \xi_H)^{-1} (\vec{g} + \xi_G)$$

Step-size

$$\Delta = \frac{\xi_G}{\xi_H}$$

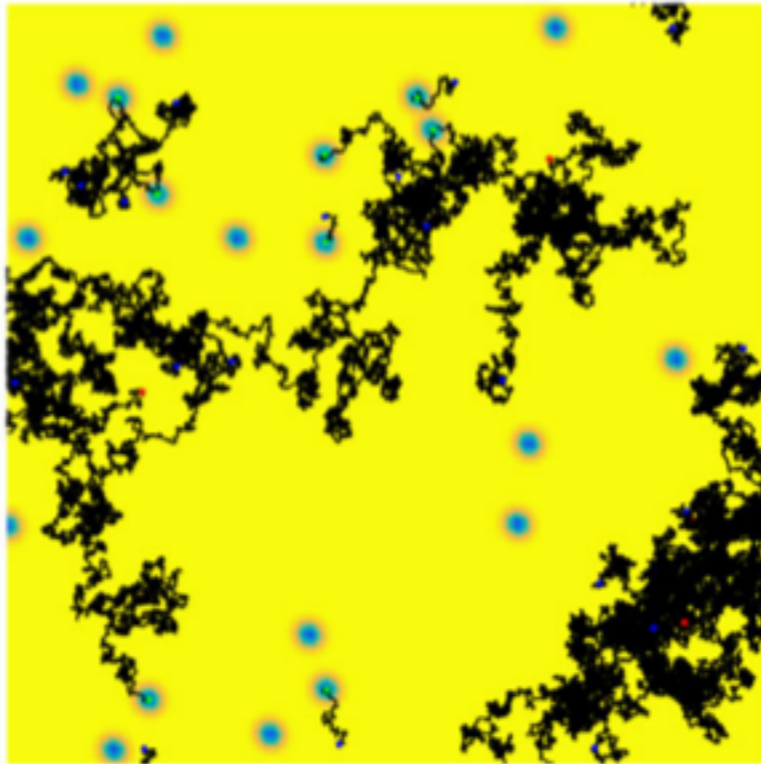
Step-size distribution

$$p(\Delta) = \frac{1}{\pi} \frac{1}{1 + \Delta^2}$$

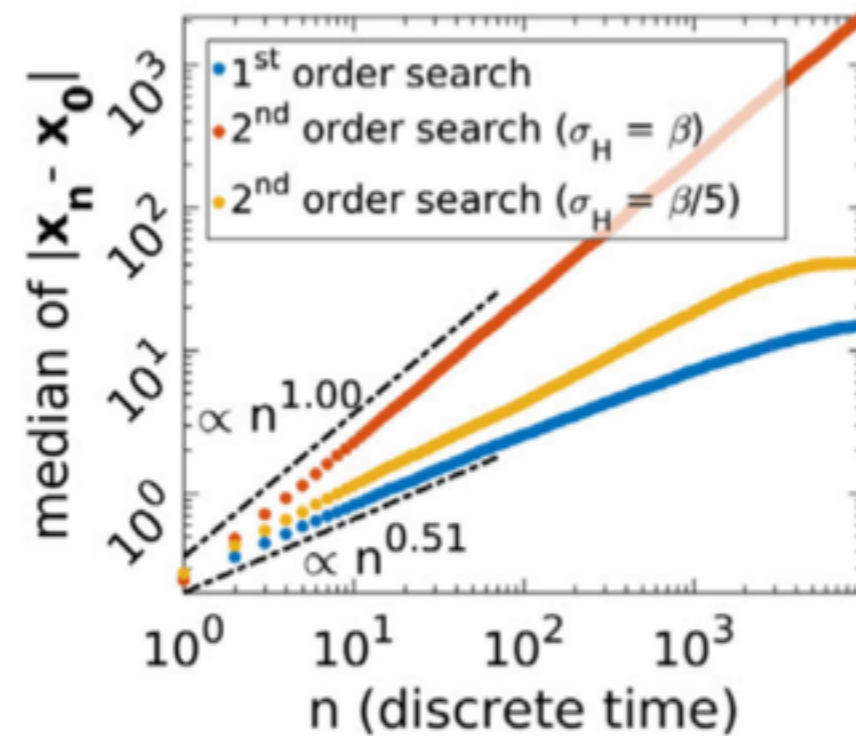
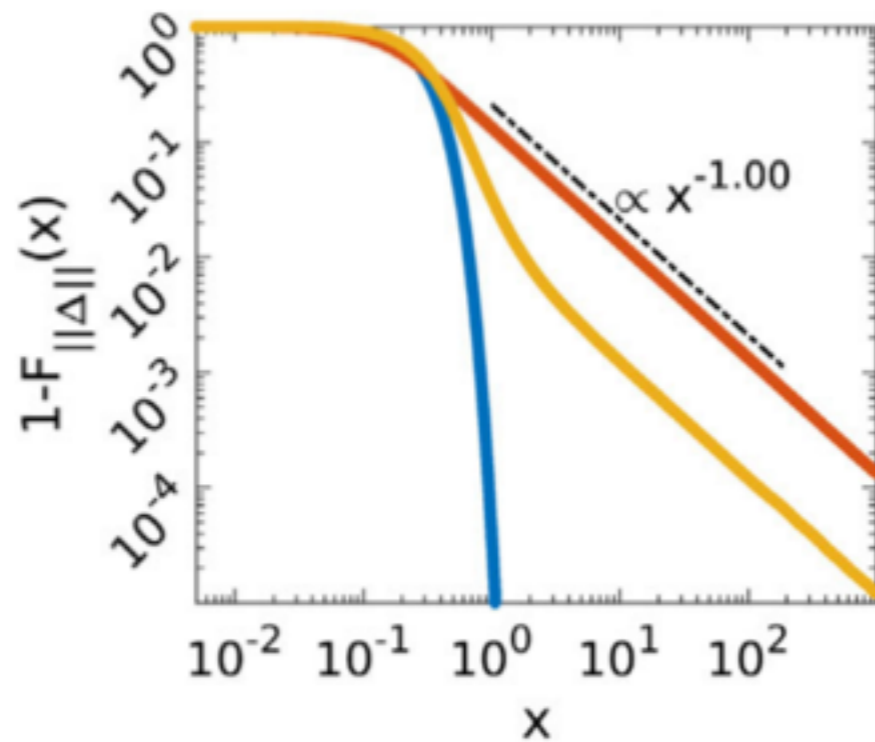
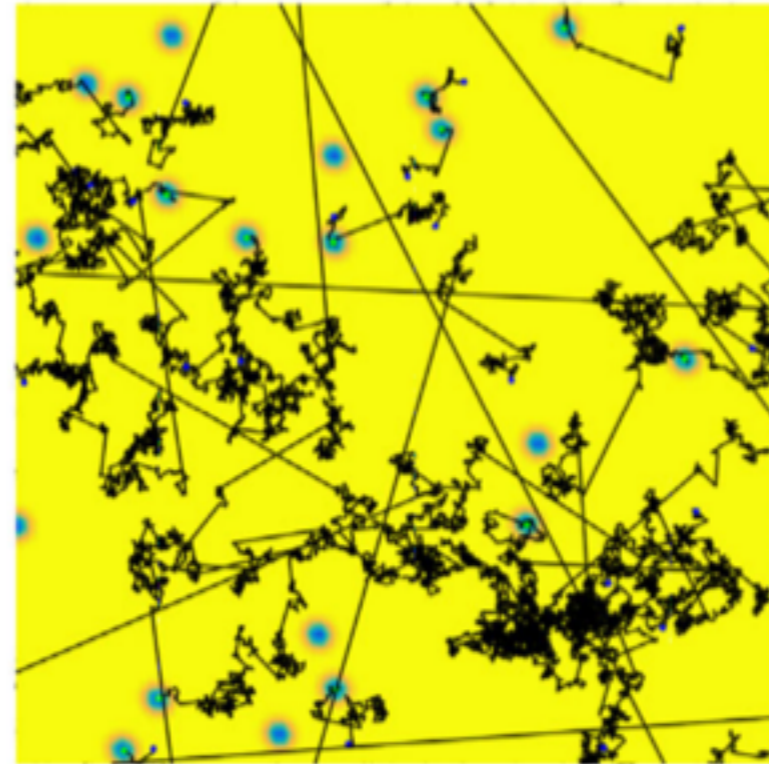


The 2nd-order stochastic optimization: The middle between Brownian and Levy searches

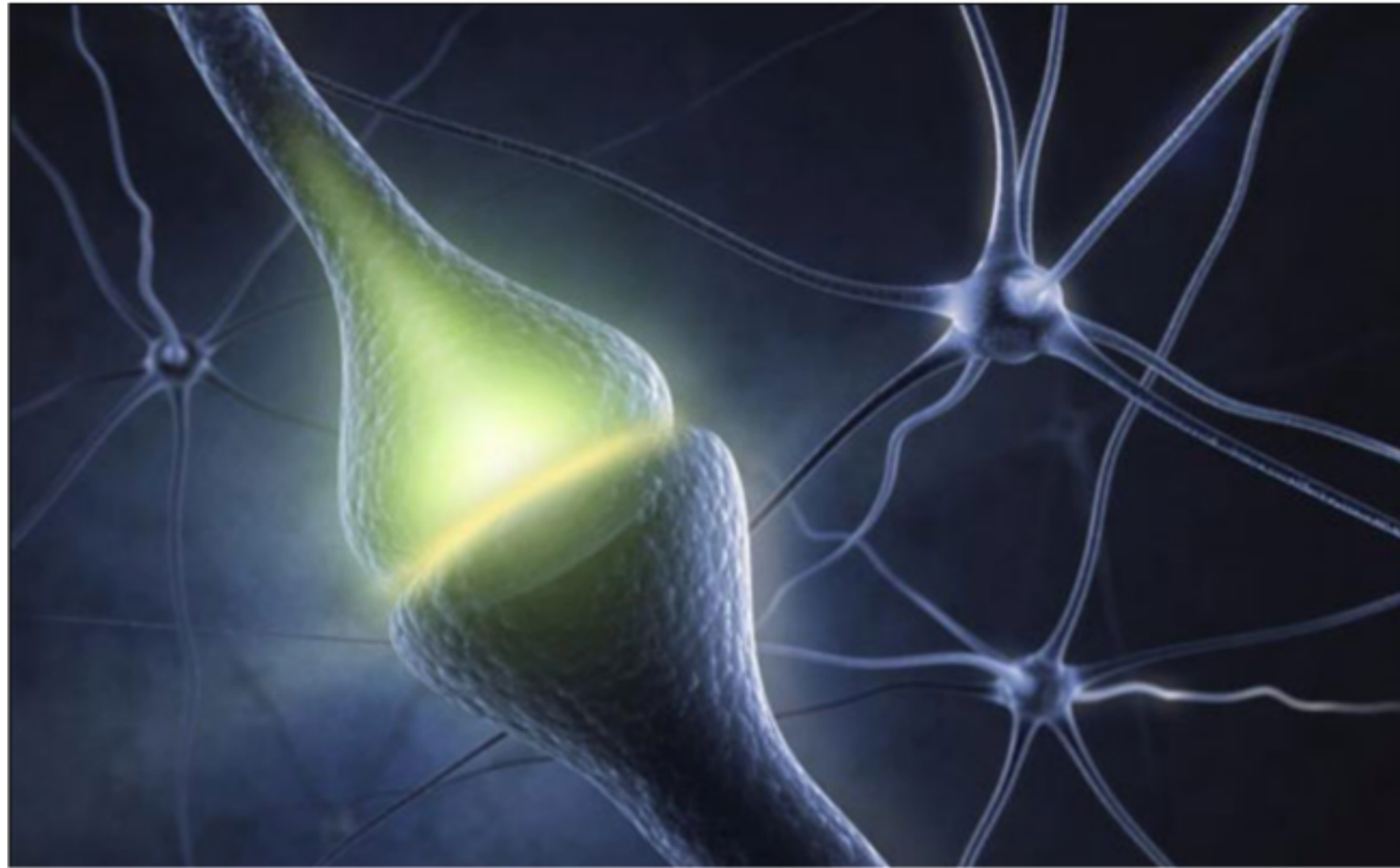
1st order search ($\sigma_G = \beta/5$)



2nd order search ($\sigma_H = \sigma_G = \beta/5$)



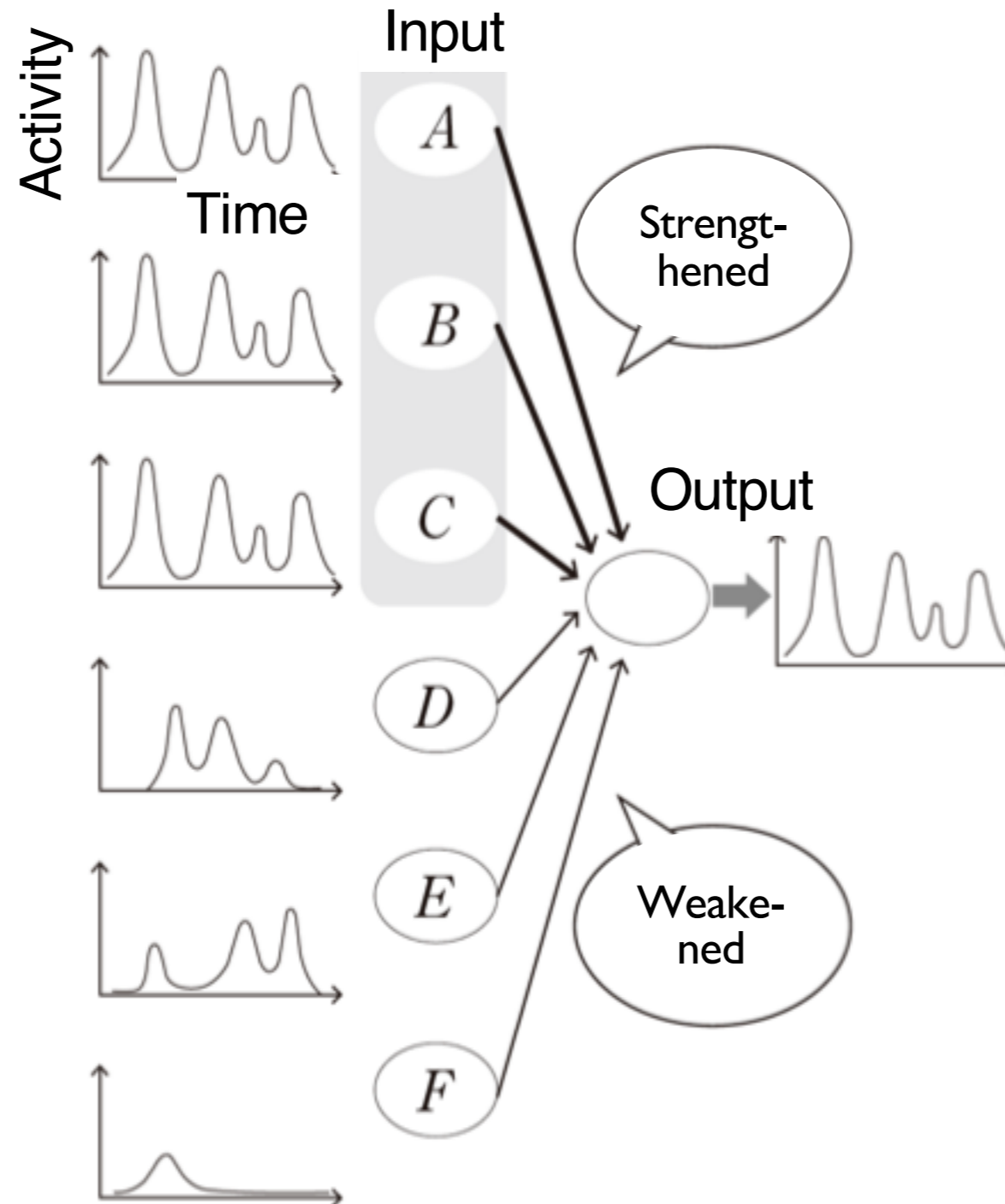
Synaptic plasticity



Optimal synaptic plasticity:

A synaptic plasticity rule for maximizing information transmission

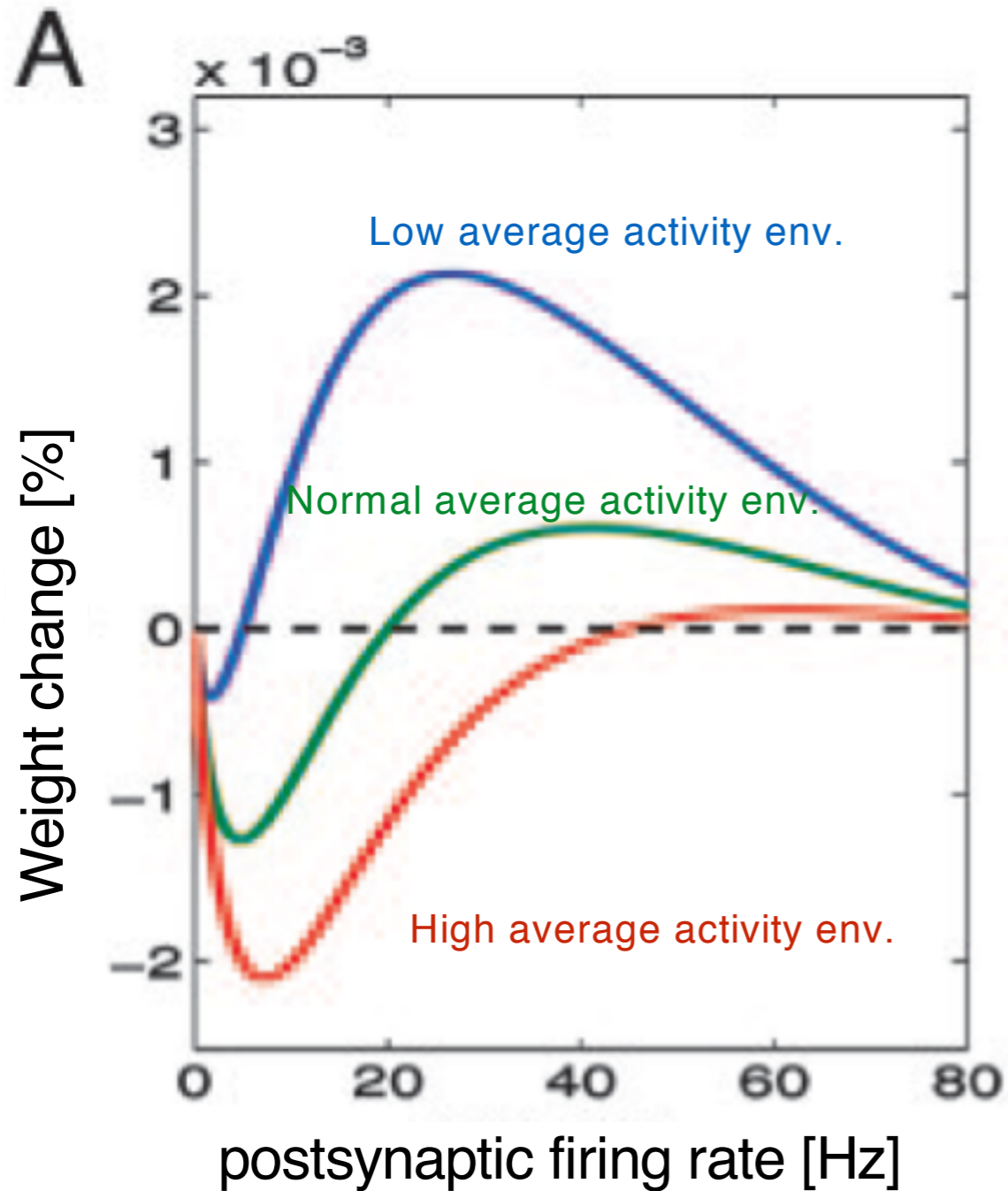
(Linsker 1988)



Firing rate dependency and meta-plasticity

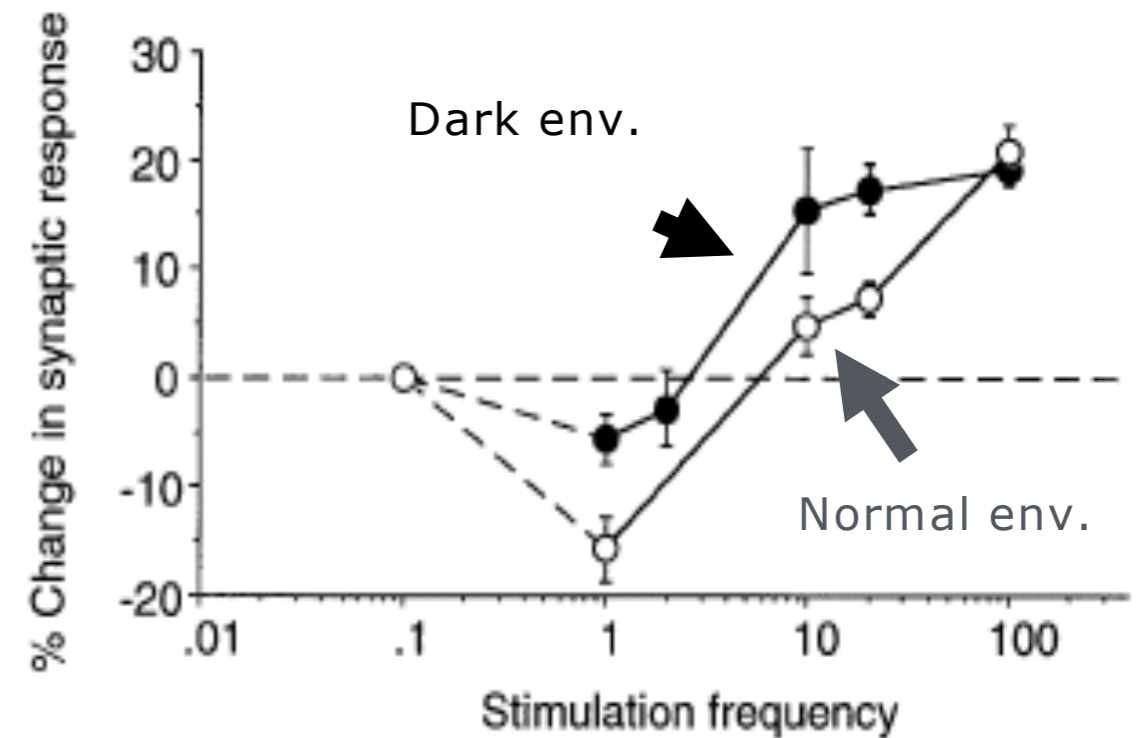
With an energy constraint....

Theory



Toyoizumi et al. 2005

Experiment

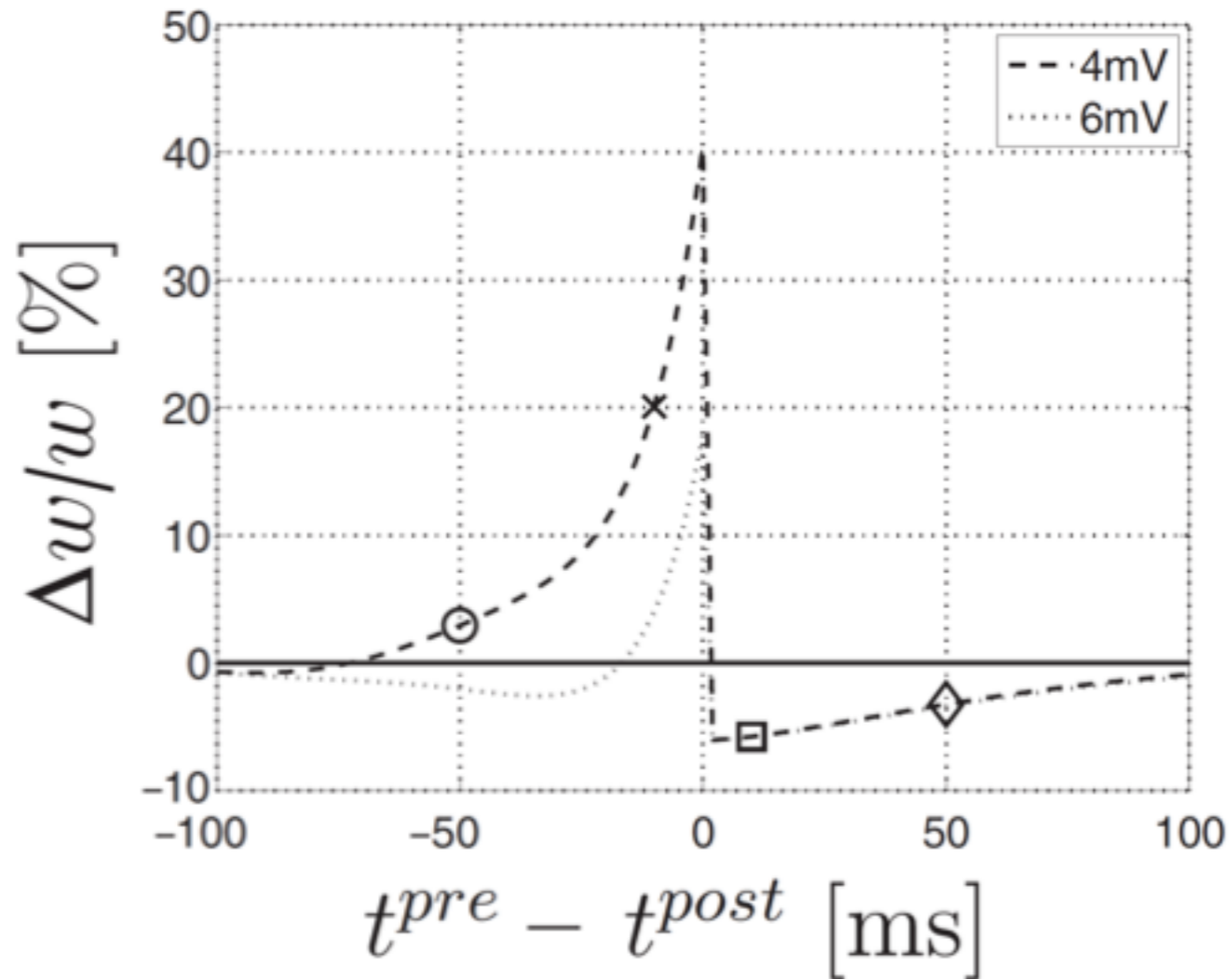


Kirkwood et al. 1996

STDP and weight-dependency

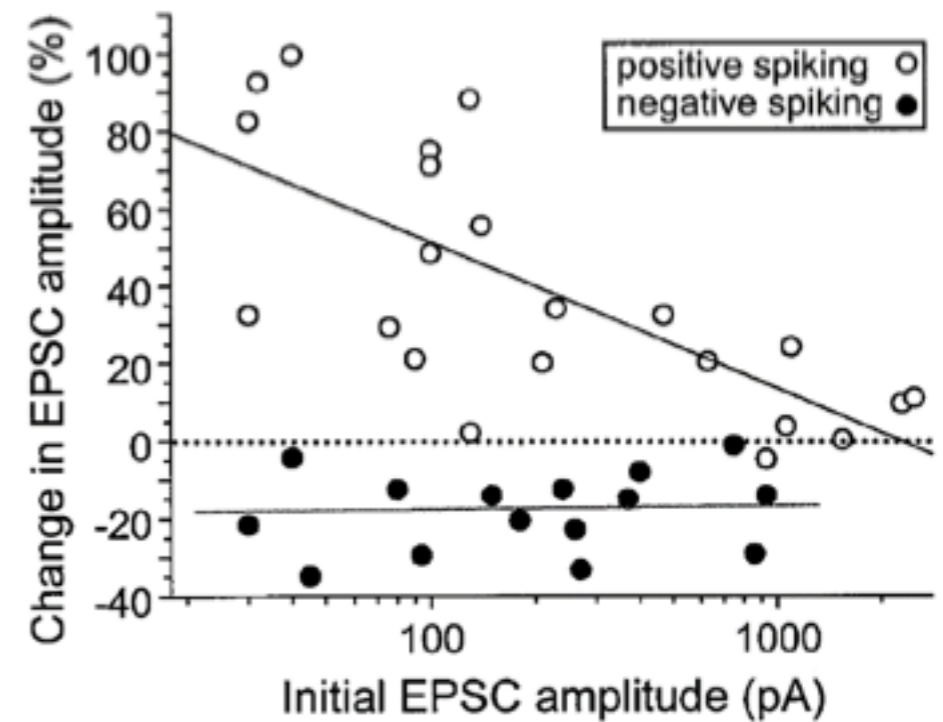
With a cost of maintaining synapses....

Theory



Toyoizumi et al. 2007

Experiment



Bi and Poo 1998

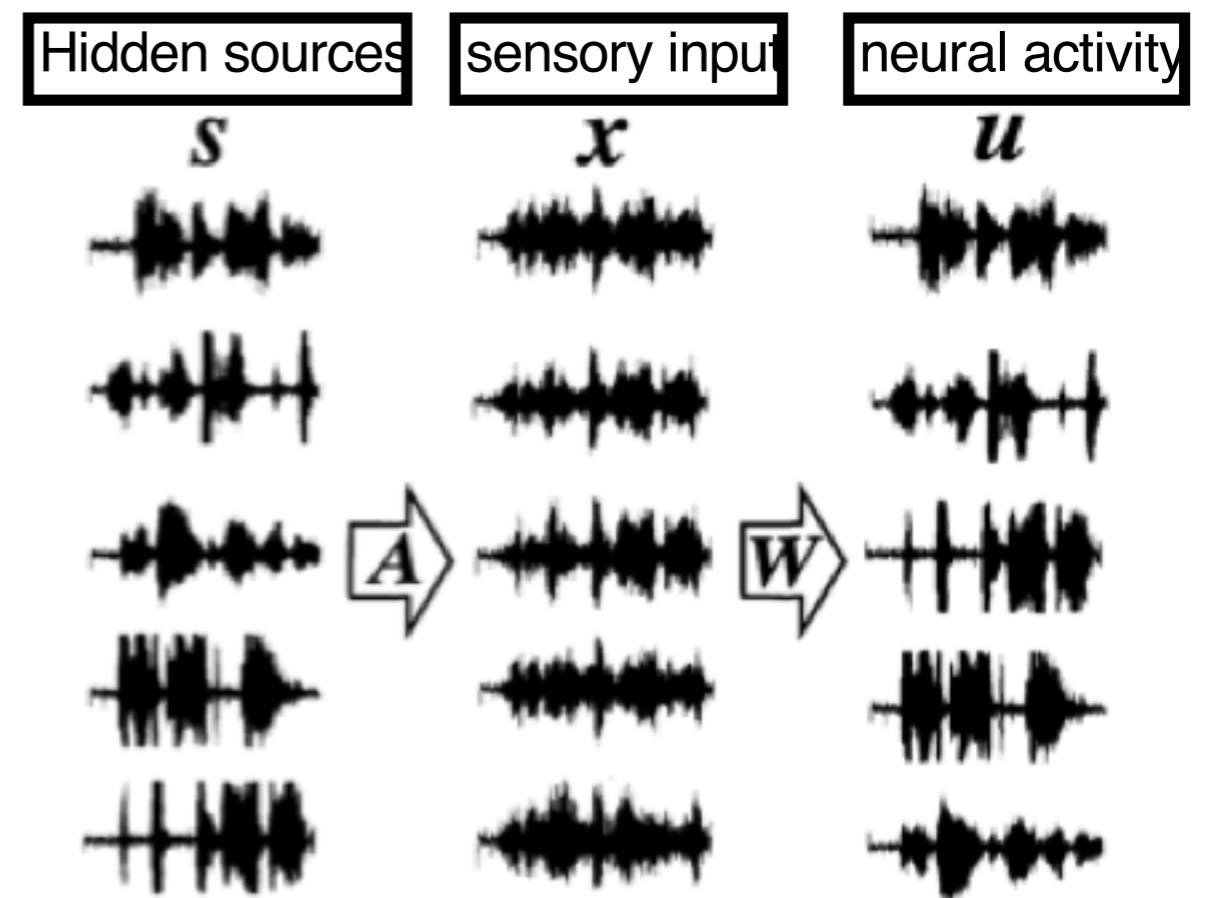
A Local Learning Rule for ICA and PCA

Information maximization using multiple output neurons



Bell and Sejnowski 1995

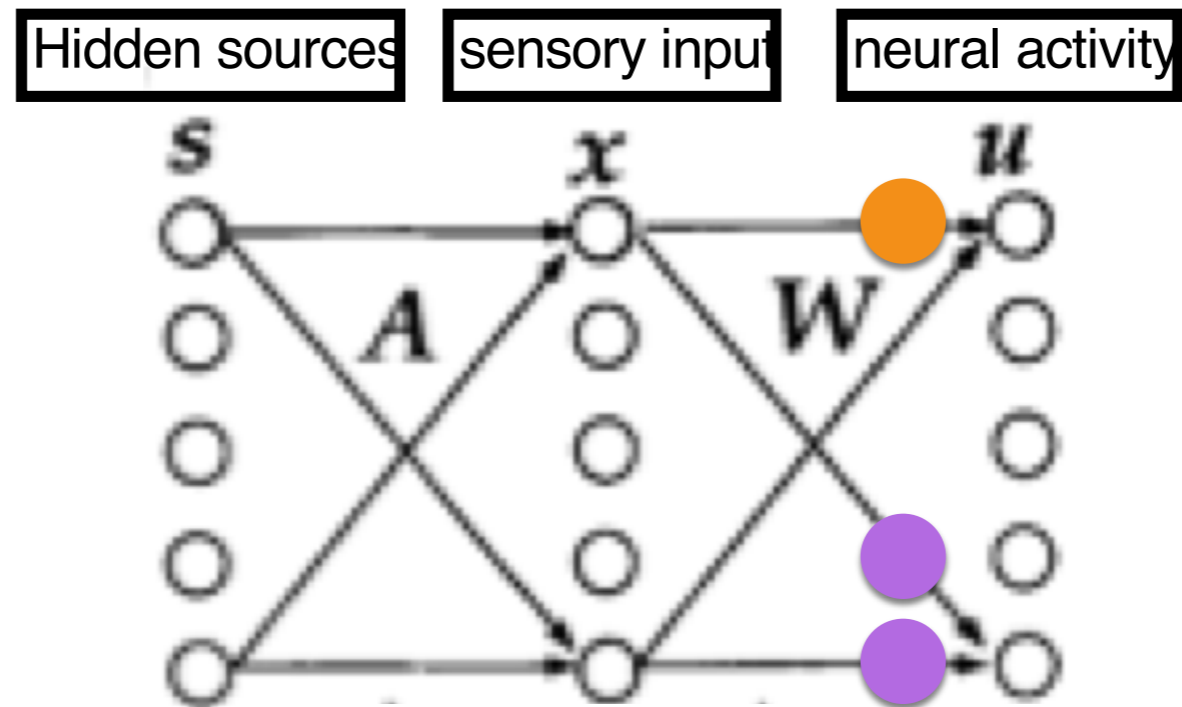
Independent component analysis (ICA)



$$\dot{W}_{ij} \propto (W^{-T})_{ij} - g(u_i)x_j$$

Non-local engineering rule

A typical setup for ICA



$$\mathbf{x} = \mathbf{A}\mathbf{s}$$
$$\mathbf{u} = \mathbf{W}\mathbf{x}$$

$$g(u_i) = -(\log p_s(u_i))'$$

Bell and Sejnowski 1995

$$\dot{W}_{ij} \propto (W^{-T})_{ij} - g(u_i)x_j$$

Non-local learning rule

Error-gated Hebbian rule (EGHR)

Bell and Sejnowski 1995

$$\dot{W}_{ij} \propto (W^{-T})_{ij} - g(u_i)x_j$$



$$\langle F(u)g(u) \rangle \approx \langle F'(u) \rangle$$

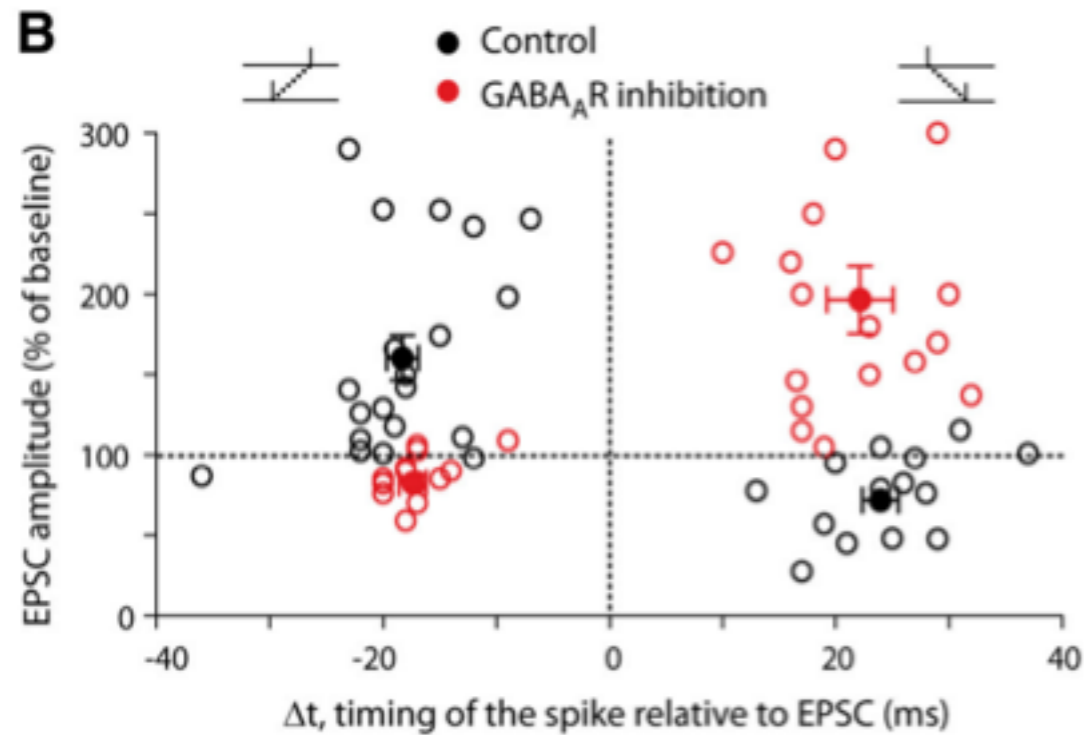
holds around the ICA solution.

Isomura and Toyozumi 2016

$$E(u) \equiv E_0 - \sum_i \int^{u_i} g(z) dz$$

$$\dot{W}_{ij} \propto E(u)g(u_i)x_j$$

A local ICA rule: Modulation of Hebb's rule by a third factor



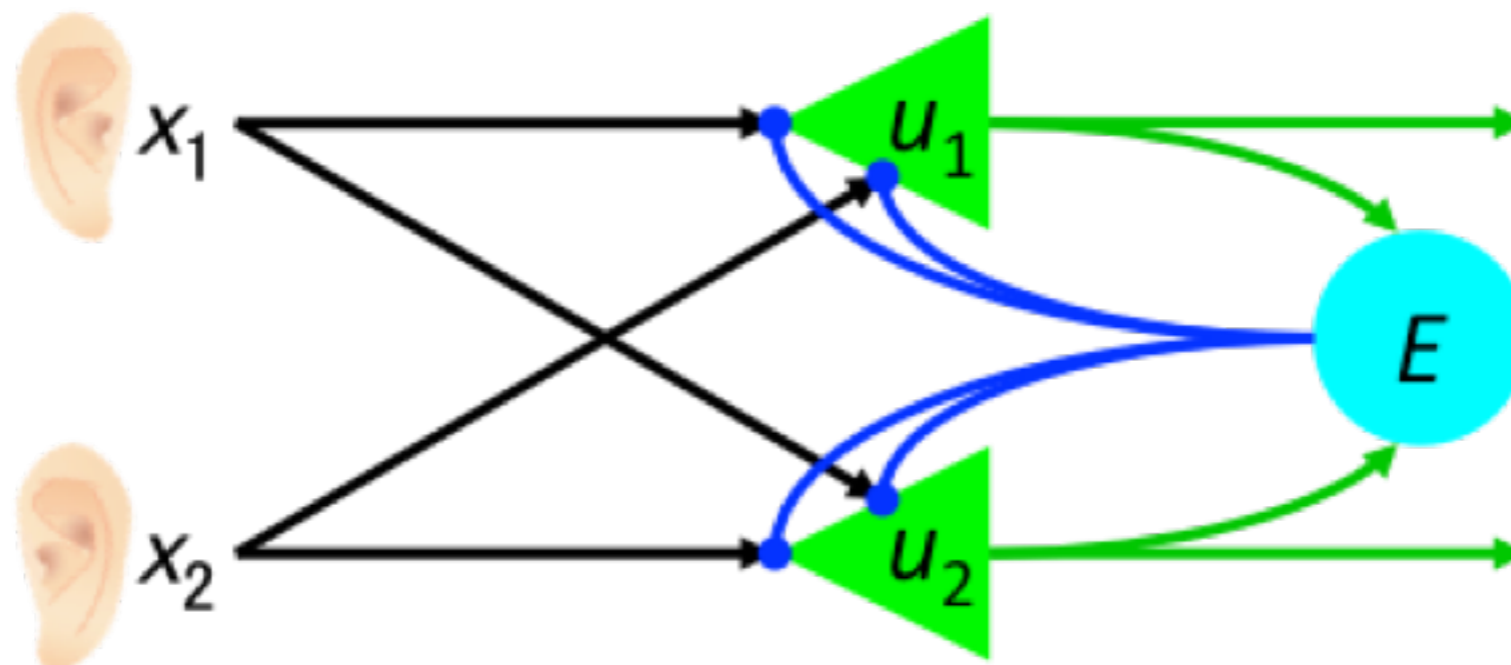
$$\dot{W}_{ij} \propto E(u)g(u_i)x_j$$

global third-factor
(e.g. GABA, neuromodulator, glial factor, etc.)

postsynaptic

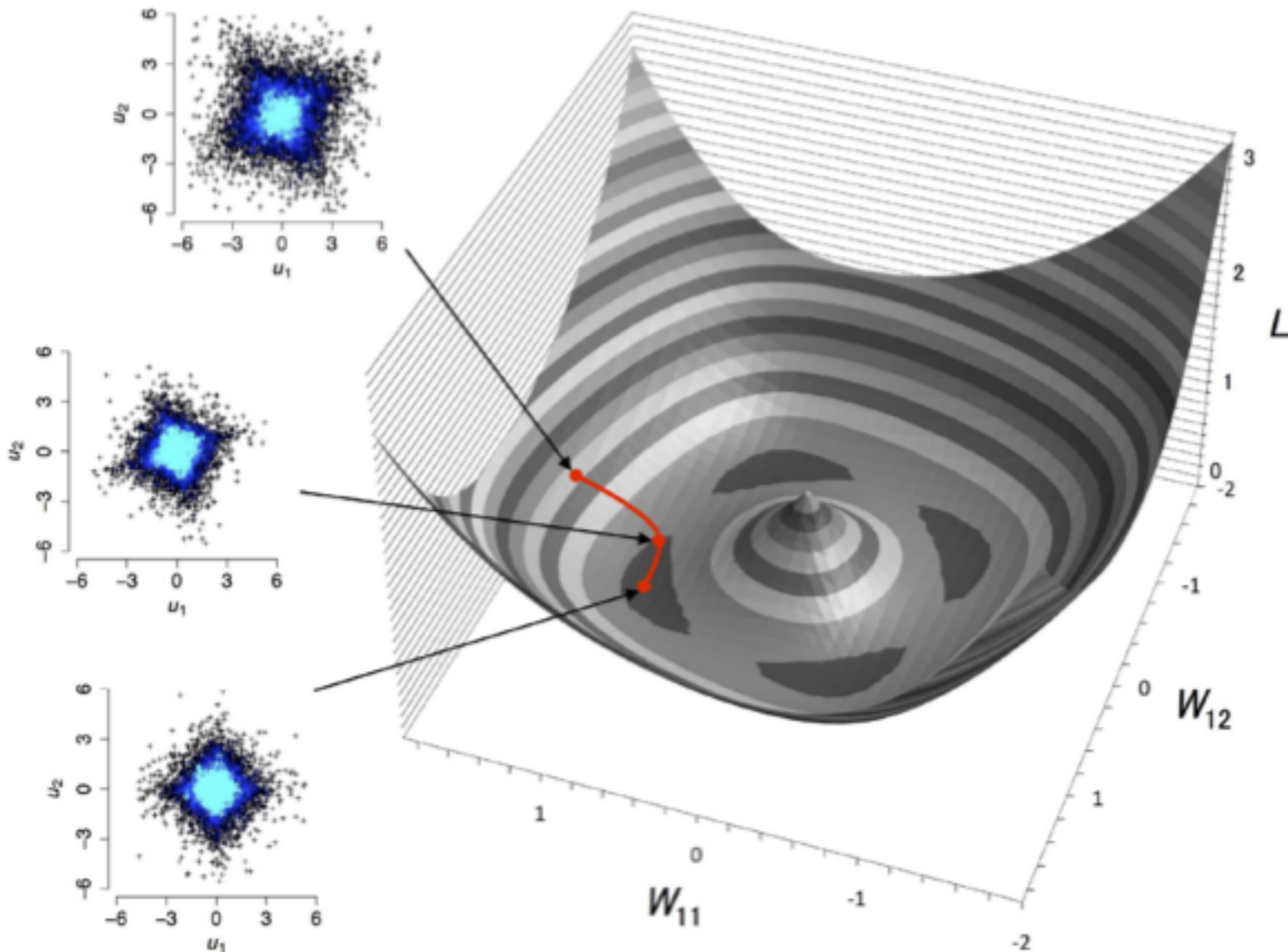
presynaptic

Paille et al. 2013



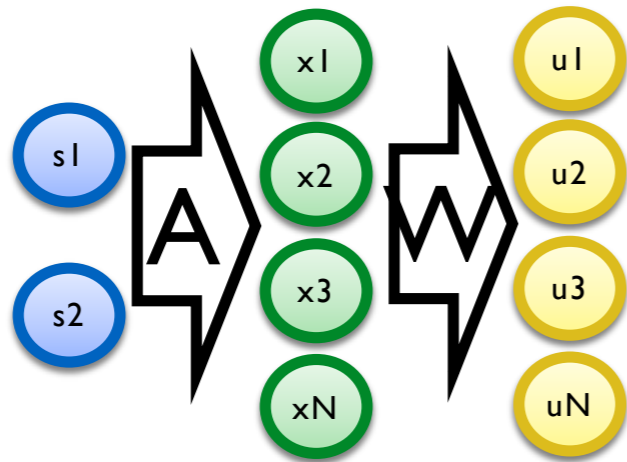
Dynamics of EGHR

$$\dot{W}_{ij} \propto \frac{\partial \langle E(u)^2 \rangle}{\partial W_{ij}}$$

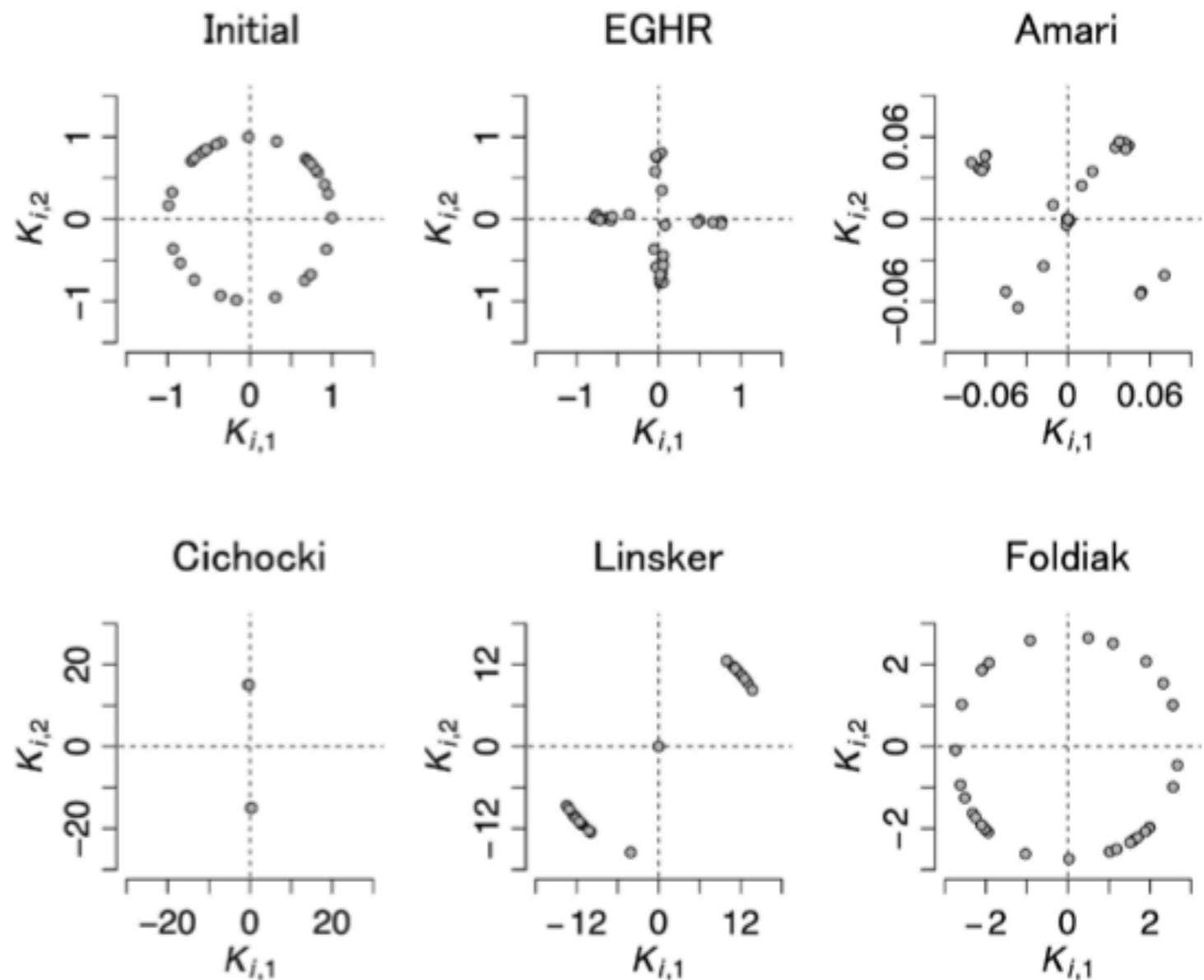


- EGHR is an approximation of the Infomax rule around the solution. Stability of the solution is roughly the same.
- No spurious solutions if the source distribution is near Gaussian.
- Unlike some heuristic rules, no preprocessing (e.g. whitening) is necessary.

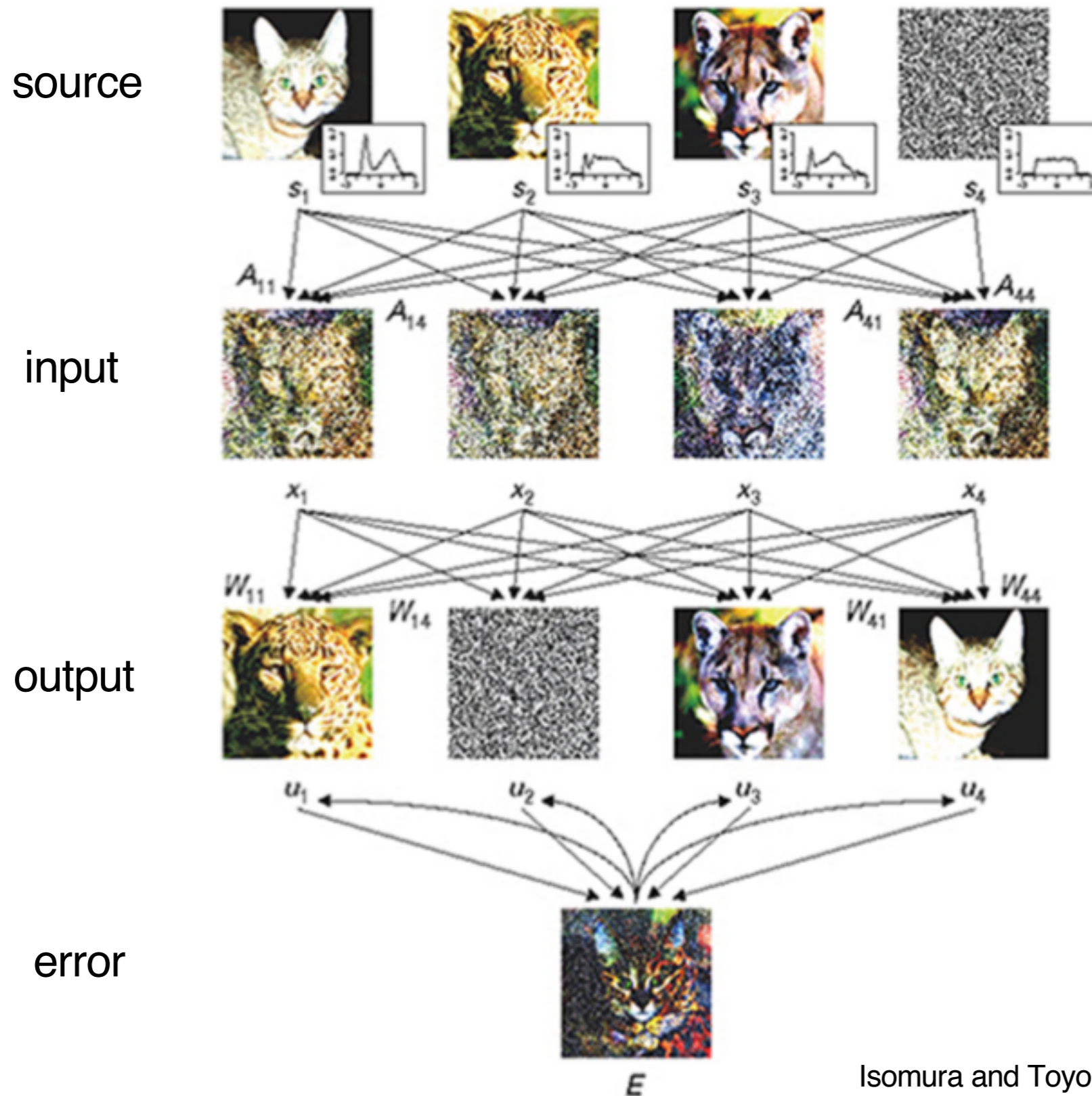
EGHR is robust to the number of sources



Each output neuron describes a source if a marker is located on one of the axes.



Separating mixtures of natural images



Separating movie clips

Source



Mixed input

Output

Not only separating
sources but also
extracting major features
by PCA

A local learning rule for PCA

Oja 1989

$$\dot{W}_{ij} \propto u_i(x_j - \sum_k u_k W_{kj})$$



$$\langle F(u)g(u) \rangle \approx \langle F'(u) \rangle$$

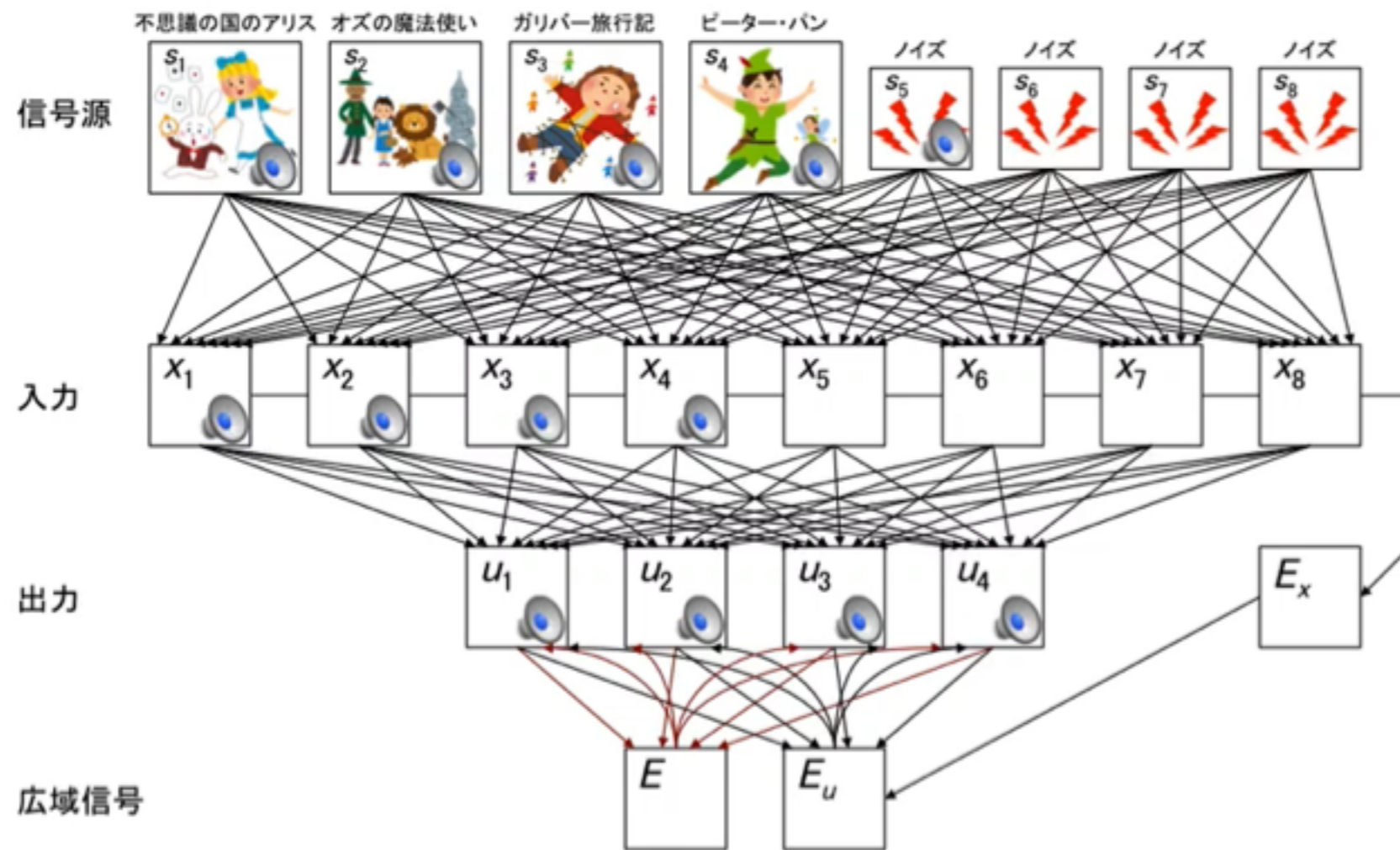
Isomura and Toyozumi, *submitted*

$$E_{PCA}(x, u) \equiv \sum_j (x_j^2 - \langle x_j^2 \rangle) - \sum_i (u_i^2 - \langle u_i^2 \rangle)$$

$$\dot{W}_{ij} \propto E_{PCA}(x, u)u_i x_j$$

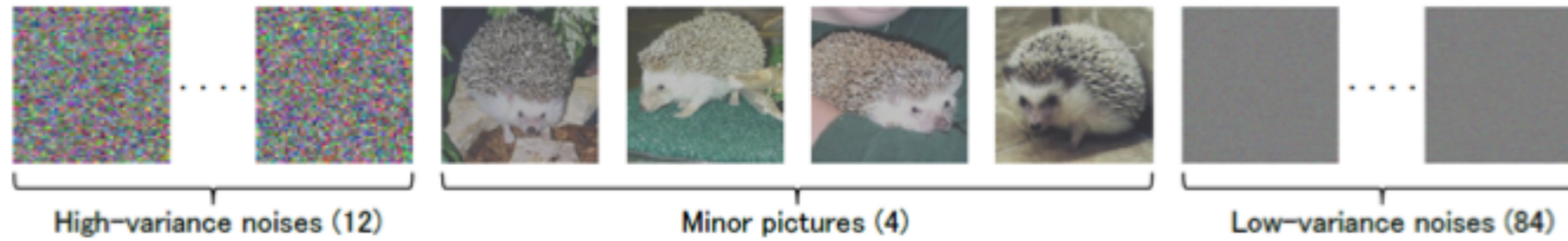
PCA + ICA

$$\dot{W}_{ij} \propto (1 - \beta)E(u)g(u_i)x_j + \beta E_{PCA}(x, u)u_ix_j$$

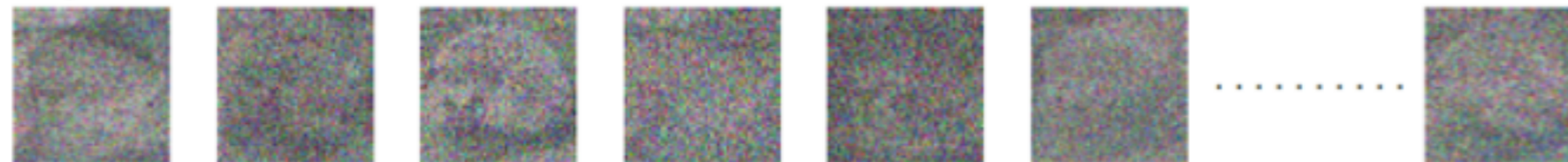


EGHR vs a cascade of PCA & ICA

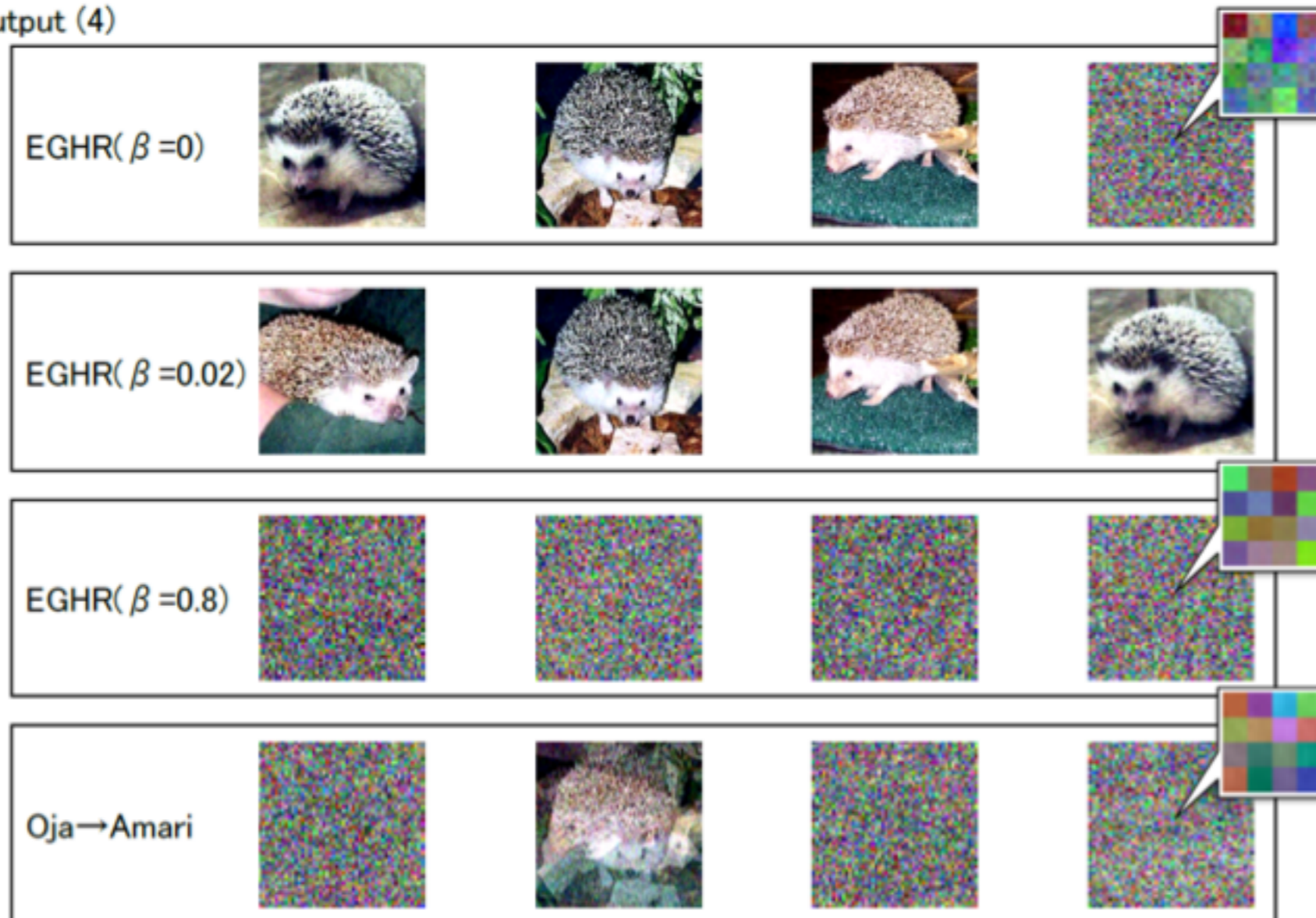
A. Source (100)



B. Input (100)

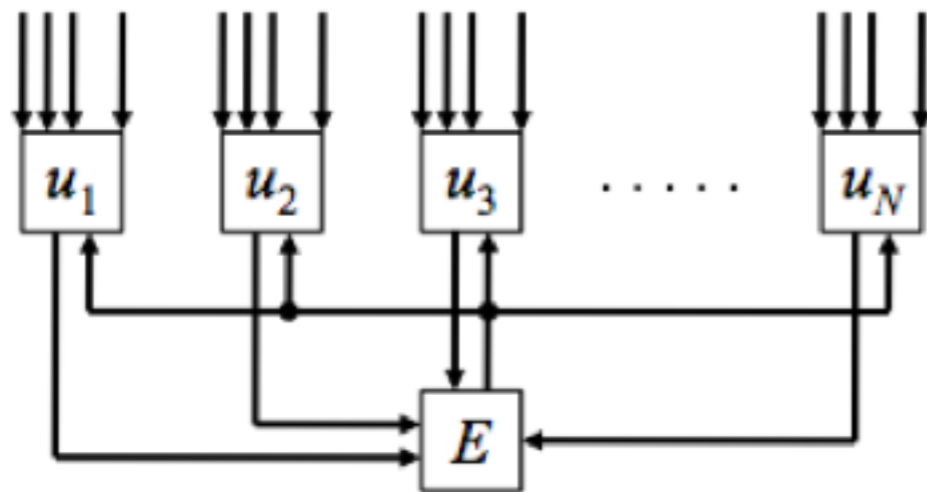


C. Output (4)

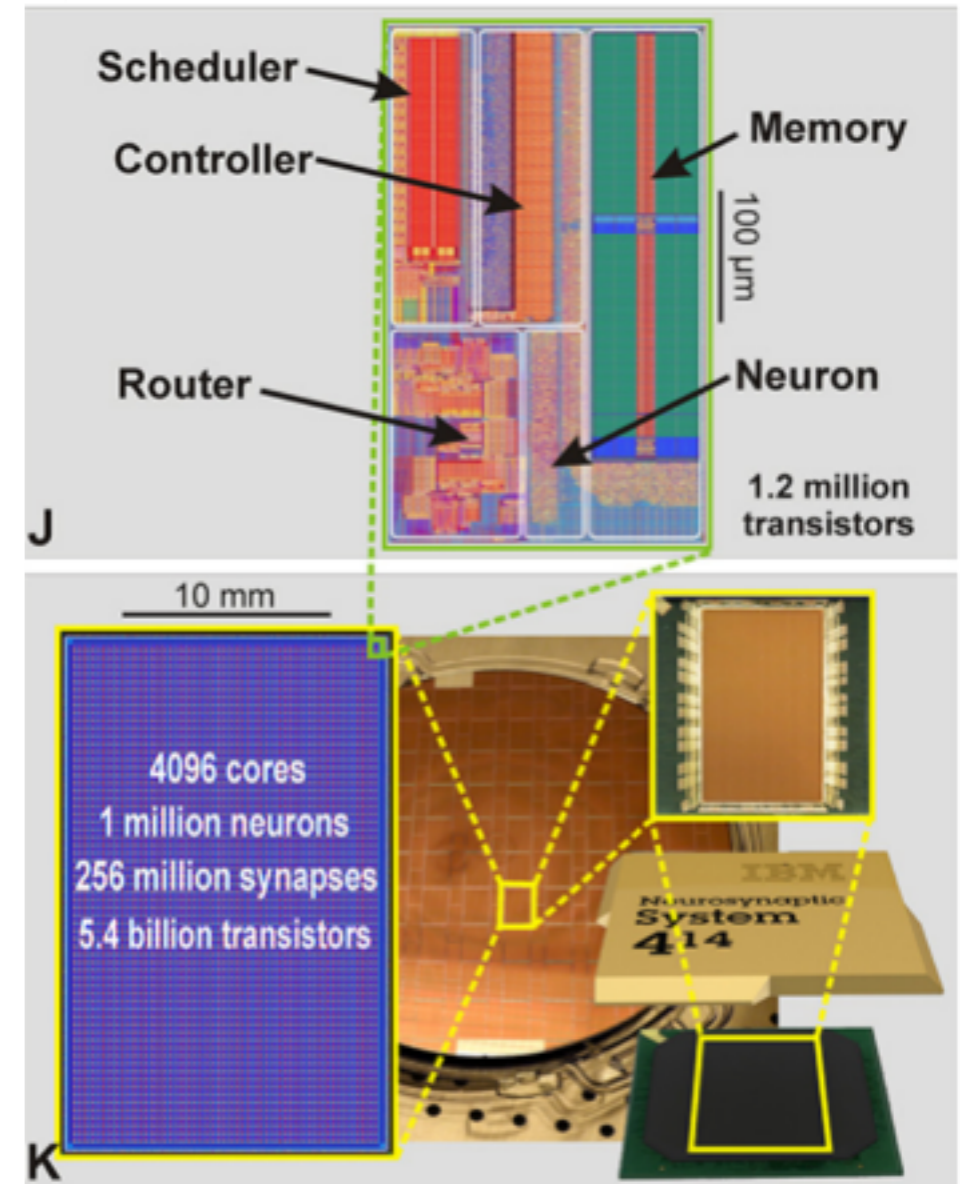
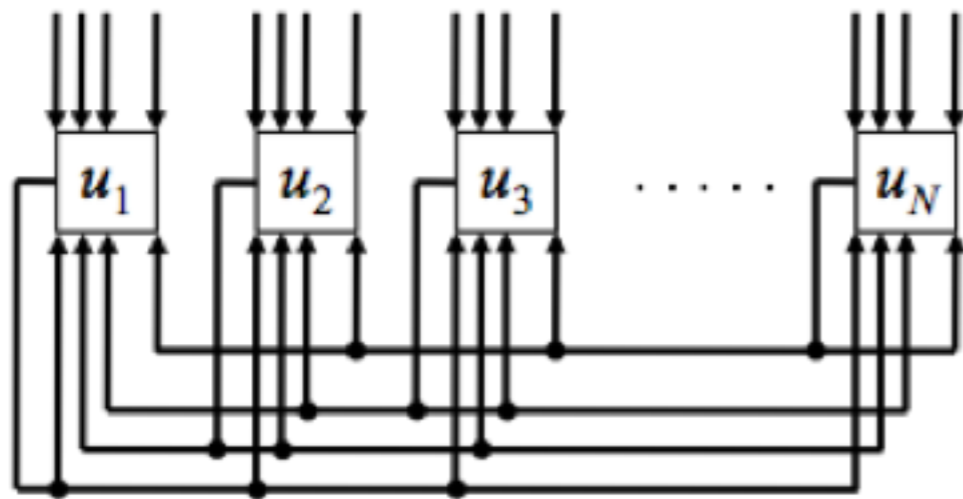


Large-scale and parallel neuromorphic computation

Proposed rule (parallel)



Conventional rules (serial)



Summary

- We have developed a local ICA&PCA algorithm, EGHR, that requires only global information sharing by a third plasticity factor.
- EGHR is more robust than conventional local ICA rules.
- EGHR is suitable for parallel blind-source separation and feature extraction by neuromorphic hardware.
- Generally, information decoding is hard if neural activities are correlated. The independent coding by EGHR can work with a simple decoding scheme.

Collaborator

- Lukasz Kusmierz (RIKEN)
- Takuya Isomura (RIKEN)



Funding



Brain/MINDS



Thank you