Pros and cons of using **temporal derivatives** for **brain functional correlations**

J.K. Ochab
Marian Smoluchowski Institute of Physics
Jagiellonian University, Cracow

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Estimation of dynamic functional connectivity using Multiplication of Temporal Derivatives

James M. Shine [a,b,*], Oluwasanmi Koyejo [b], Peter T. Bell [a], Krzysztof J. Gorgolewski [b], Moran Gilat [a], Russell A. Poldrack [b]

[a] Parkinson's Disease Research Clinic, Brain and Mind Research Institute, The University of Sydney, NSW, Australia
[b] Department of Psychology, Stanford University, Stanford, CA, USA

Dynamic functional connectivity

- functional magnetic resonance imaging (fMRI)
- blood-oxygen level–dependent (BOLD) signal
- spatial topologies of interacting regions (functional "networks")
- strength of ……………… between pairs or sets of regions ("functional connectivity")
- absence of any task or stimuli ("resting-state")
- Dynamic = changing (time dependent)

Hutchison et al. (2013). *Dynamic functional connectivity: Promise, issues, and interpretations*, NeuroImage **80**, 360-378.
Chen, Rubinov, Chang (2017). *Methods and Considerations for Dynamic Analysis of Functional MR Imaging Data,* Neuroimaging Clinics of North America **27***,* 547-560

SMA of MTD vs Pearson correlation

$$ds_{it} = s_{it+1} - s_{it} \qquad \bar{ds}_i = \frac{1}{2w+1}\sum_{t'=t-w}^{t+w} ds_{it'} = \frac{s_{it-w} - s_{it+w}}{2w+1} \to 0 \tag{1}$$

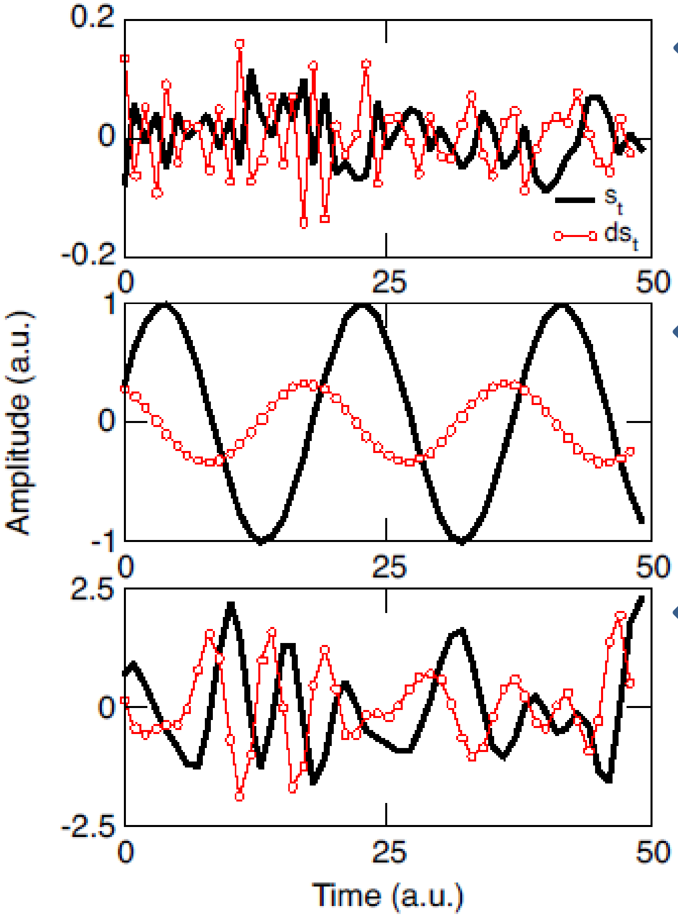
$$MTD_{ijt} = \frac{ds_{it} ds_{jt}}{\bar{\sigma}_i \bar{\sigma}_j} \tag{2}$$

$$SMA_{ijt} = \frac{1}{2w+1}\sum_{t'=t-w}^{t+w} MTD_{ijt'} = \frac{1}{2w+1}\sum_{t'=t-w}^{t+w} \frac{ds_{it'}}{\bar{\sigma}_i}\frac{ds_{jt'}}{\bar{\sigma}_j}, \tag{3}$$

$$r_{ij} = \frac{\sum_{t=1}^{T}(s_{it} - \bar{s}_i)(s_{jt} - \bar{s}_j)}{\sqrt{\sum_{t=1}^{T}(s_{it} - \bar{s}_i)^2}\sqrt{\sum_{t=1}^{T}(s_{jt} - \bar{s}_j)^2}} = \frac{1}{T-1}\sum_{t=1}^{T}\left(\frac{s_{it} - \bar{s}_i}{\sigma_i}\right)\left(\frac{s_{jt} - \bar{s}_j}{\sigma_j}\right)$$

$$r_{ijt} = \frac{\sum_{t'=t-w}^{t+w}(ds_{it'} - \bar{ds}_i)(ds_{jt'} - \bar{ds}_j)}{\sqrt{\sum_{t'=t-w}^{t+w}(ds_{it'} - \bar{ds}_i)^2}\sqrt{\sum_{t'=t-w}^{t+w}(ds_{jt'} - \bar{ds}_j)^2}} = \frac{1}{2w+1}\sum_{t'=t-w}^{t+w}\frac{ds_{it'} - \bar{ds}_i}{\sigma_i}\frac{ds_{jt'} - \bar{ds}_j}{\sigma_j}$$

1. What's the difference?
Simplest models



noise

oscillation

BOLD

2. What's the difference? Autoregressive model

$$x_{it} = \sum_{k=1}^{q} \sum_{j=1}^{N} A_{ij}^{(k)} x_{j,t-k} + \xi_{it}$$ ⇐ **VAR(q)**

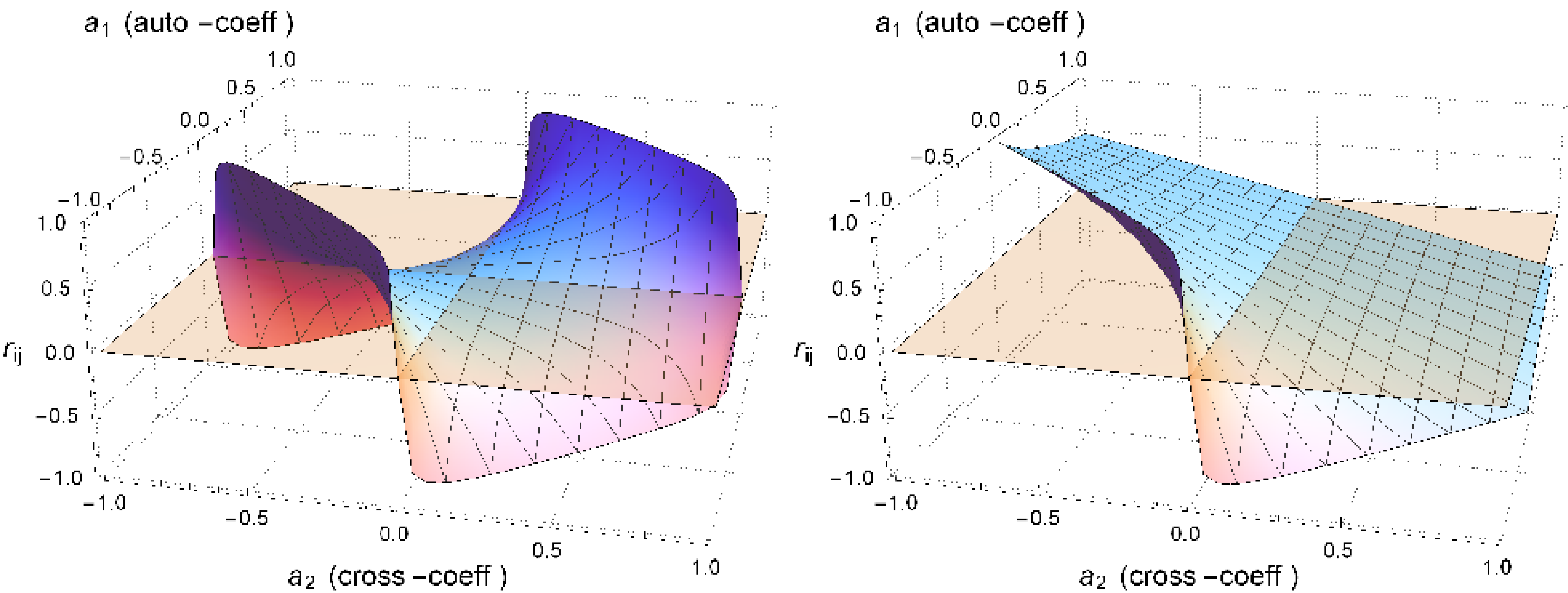
$$x_{1t} = a_1 x_{1,t-1} + a_2 x_{2t-1} + \xi_{1t}$$
$$x_{2t} = a_1 x_{2,t-1} + a_2 x_{1t-1} + \xi_{2t}$$ ⇐ **q=1, N=2**

We can analytically calculate correlations (for differences too)!

2. What's the difference? Autoregressive model

$$x_{it} = \sum_{k=1}^{q} \sum_{j=1}^{N} A_{ij}^{(k)} x_{j,t-k} + \xi_{it}$$ **VAR(q)**

$$x_{1t} = a_1 x_{1,t-1} + a_2 x_{2t-1} + \xi_{1t}$$
$$x_{2t} = a_1 x_{2,t-1} + a_2 x_{1t-1} + \xi_{2t}$$ **q=1, N=2**



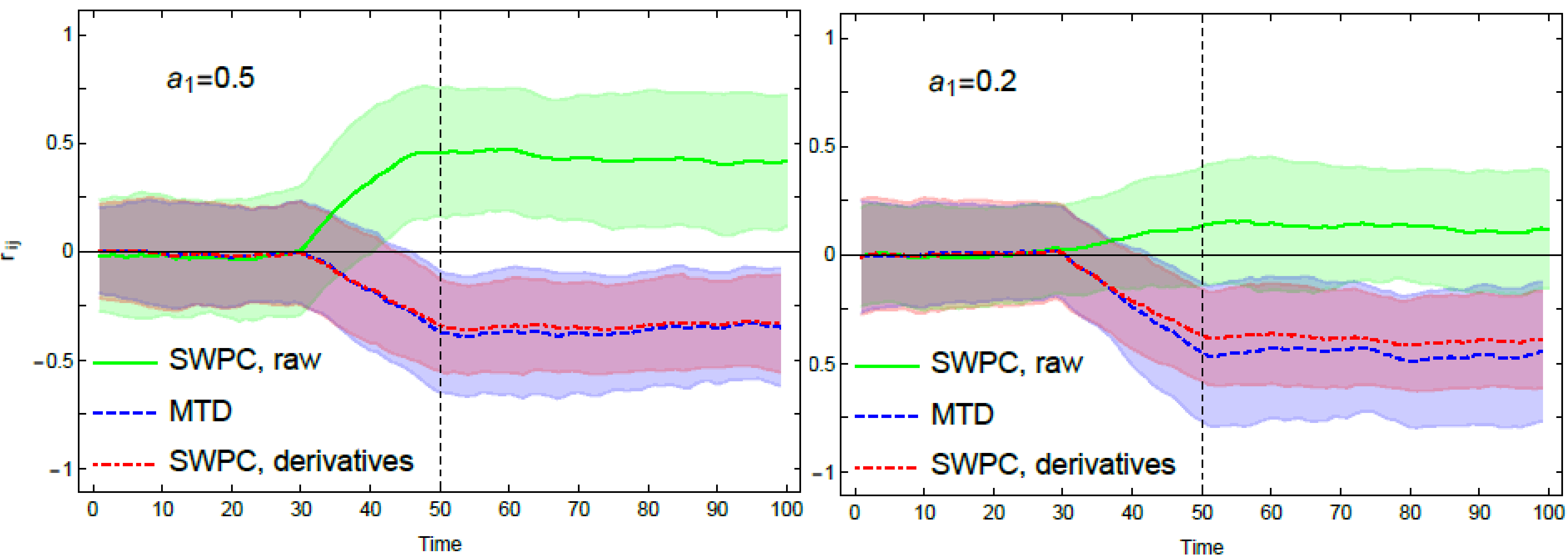
2. What's the difference?
Autoregressive model

$$x_{it} = \sum_{k=1}^{q} \sum_{j=1}^{N} A_{ij}^{(k)} x_{j,t-k} + \xi_{it}$$

VAR(q)

$$x_{1t} = a_1 x_{1,t-1} + a_2(t) x_{2t-1} + \xi_{1t}$$
$$x_{2t} = a_1 x_{2,t-1} + a_2(t) x_{1t-1} + \xi_{2t}$$

q=1, N=2

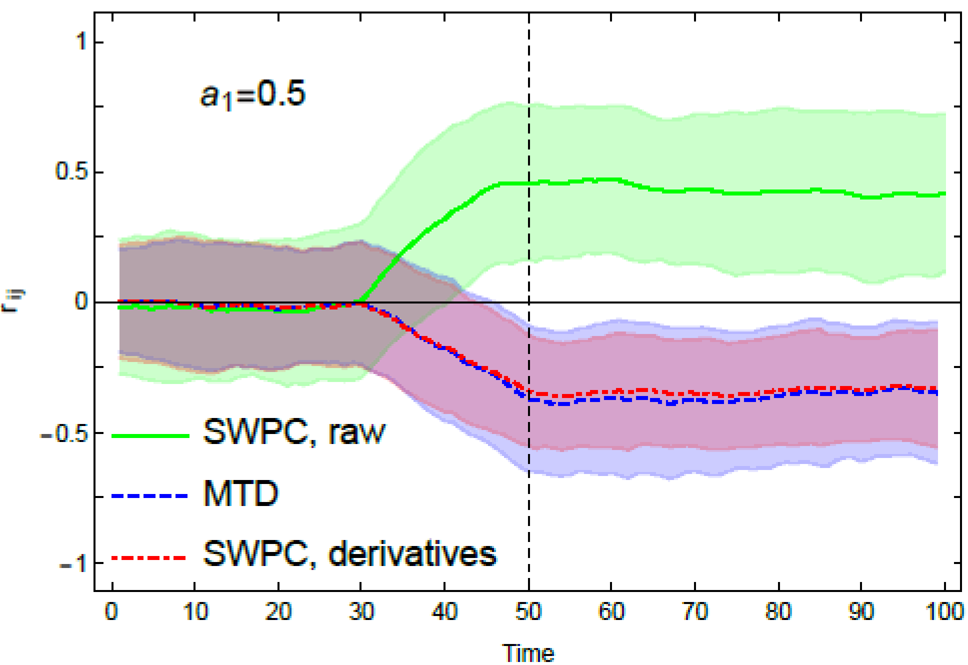


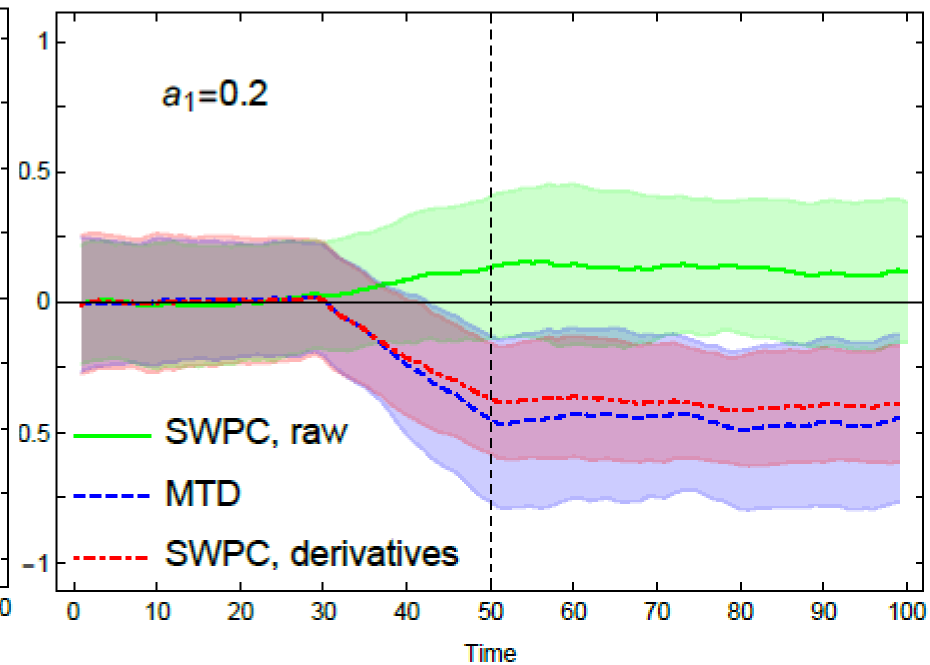
2. What's the difference? Autoregressive model

$$SMA_{ijt} = \frac{1}{2w+1}\sum_{t'=t-w}^{t+w} MTD_{ijt'} = \frac{1}{2w+1}\sum_{t'=t-w}^{t+w} \frac{ds_{it'}}{\bar{\sigma}_i}\frac{ds_{jt'}}{\bar{\sigma}_j}$$

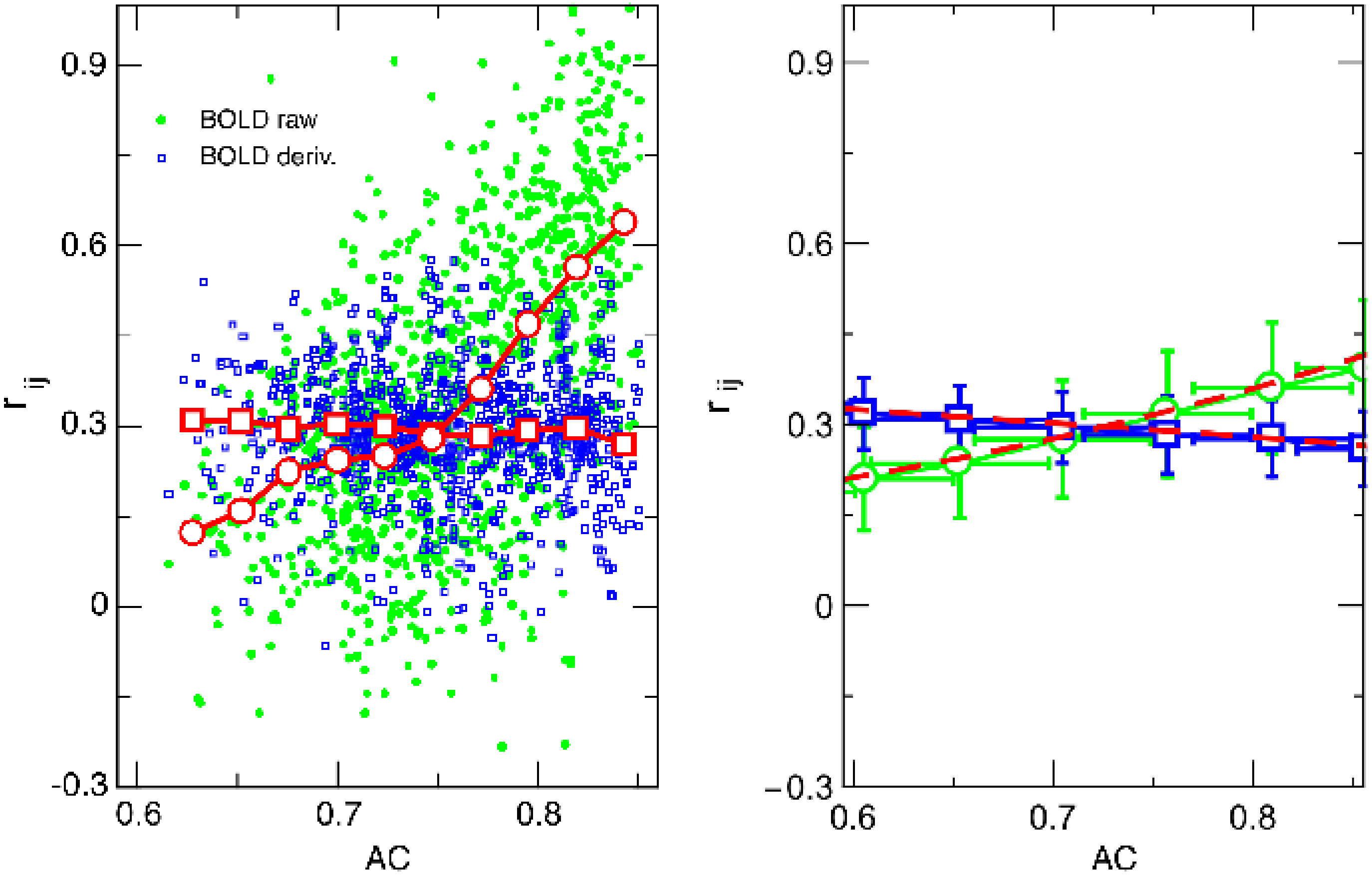
Remember?

$$\frac{1}{2w+1}\sum_{t'=t-w}^{t+w} \frac{ds_{it'} - \bar{ds}_i}{\sigma_i}\frac{ds_{jt'} - \bar{ds}_j}{\sigma_j}$$



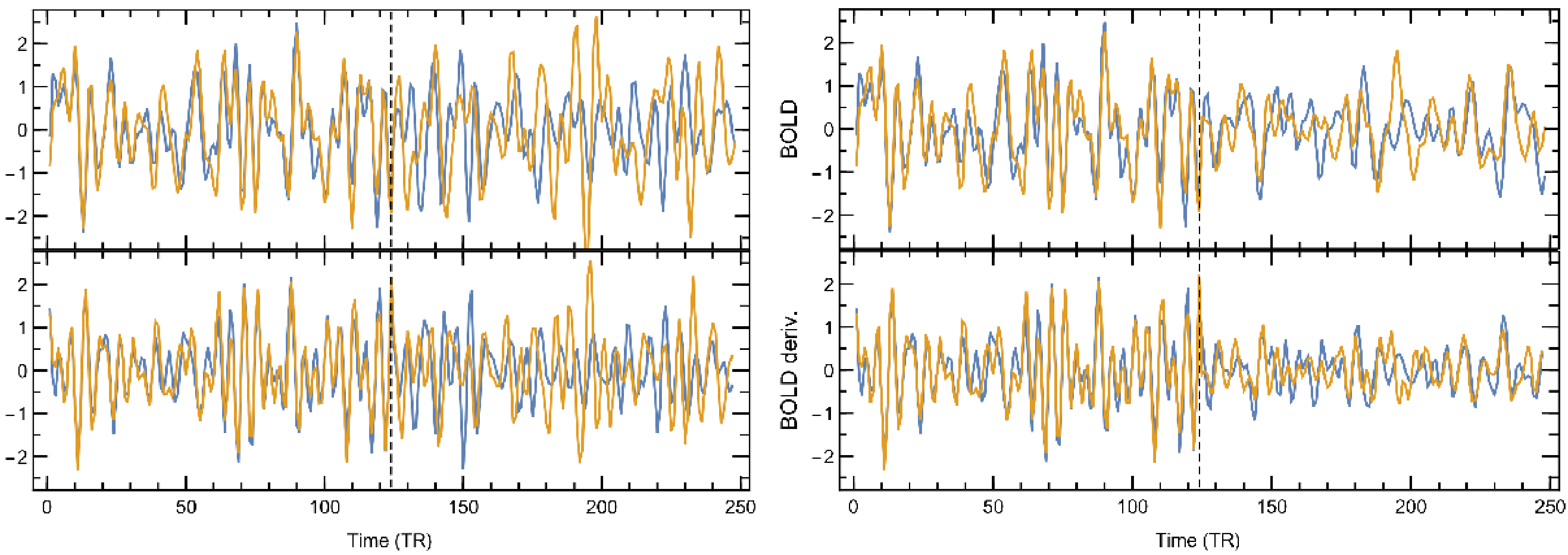


3. What's the difference? BOLD vs ARMA(1,1)x2



4. What's the difference? Surrogate BOLD

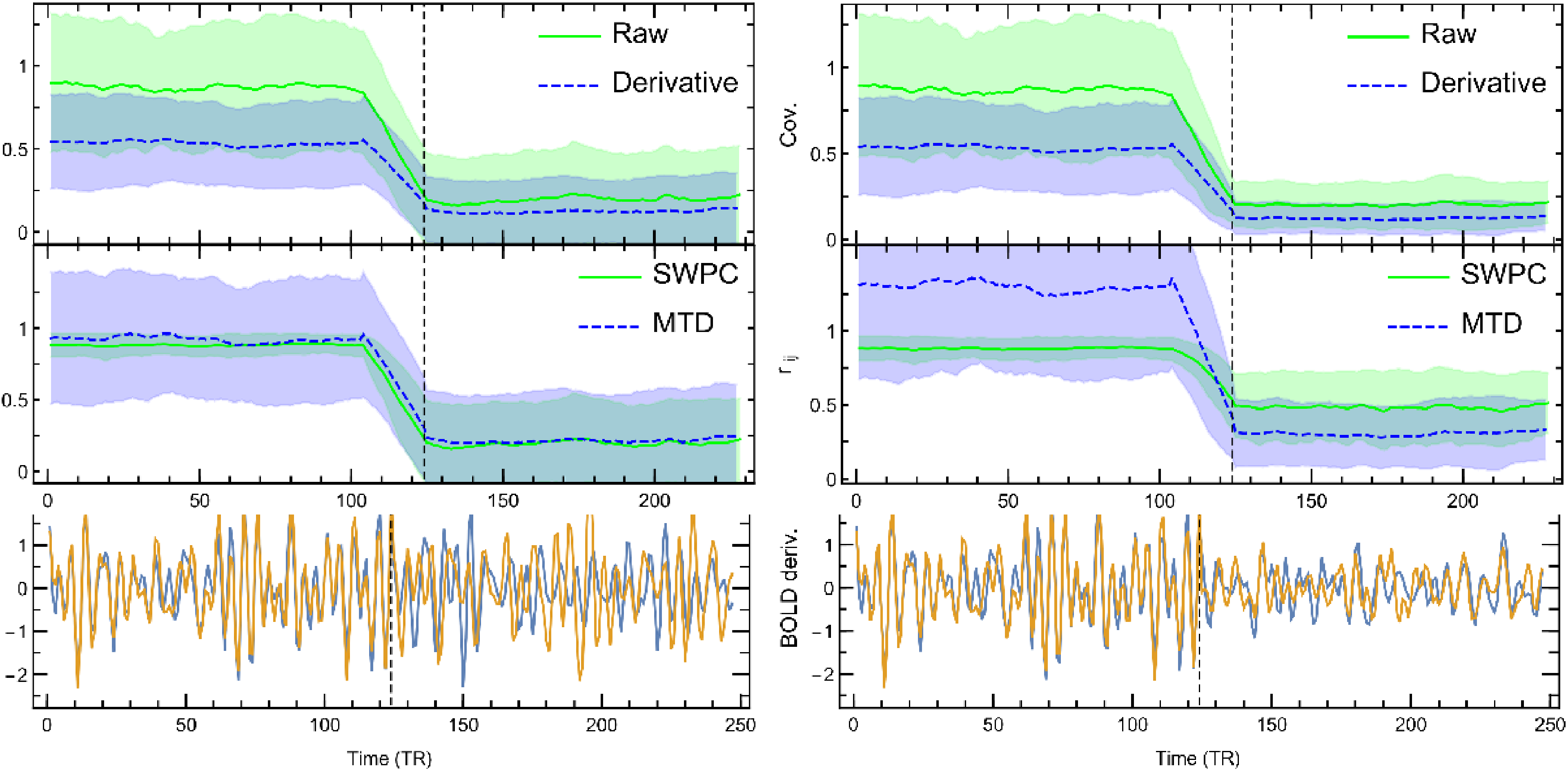
Laumann et al. (2016) *CerebralCortex,* 1–14



4. What's the difference? Surrogate BOLD

CC≈0.8 → **0.2** | **CC≈0.8** → **0.2**

AC≈1 → **1** | **AC≈1** → **0.4**



Conclusions

What's the difference? It depends.

- centering and windowed standardization decrease uncertainty of correlations
- differences: decrease signal-to-noise ratio
- differences: enhance stationarity, not affected by low frequency drifts
- differences: have lower sensitivity to autocorrelations
  (but worse than raw series for high autocorr.)

Based on:

- JK Ochab, W Tarnowski, MA Nowak, DR Chialvo,
  On the pros and cons of using temporal derivatives to assess brain functional connectivity, submitted (2018)**; arXiv:1803.05048 [q-bio.NC]**

Thank you

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