



INSTYTUT FIZYKI JĄDROWEJ  
IM. HENRYKA NIEWODNICZAŃSKIEGO  
POLSKIEJ AKADEMII NAUK

# More non-linear circuits: integrated implementation, versatile motor pattern generation, criticality

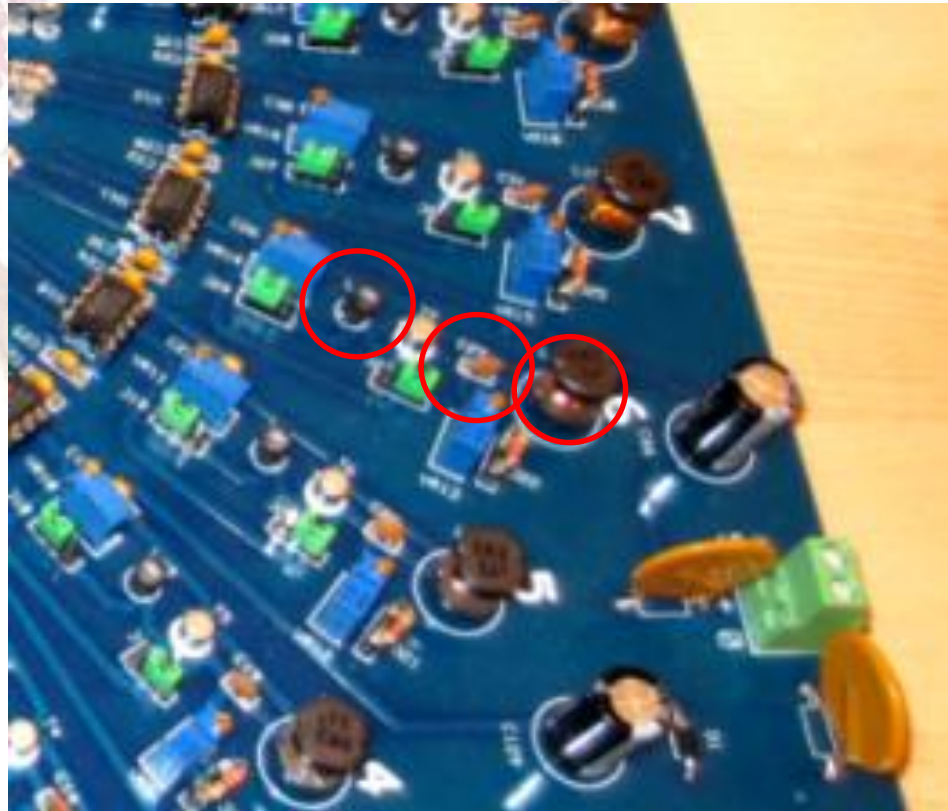
*Ludovico Minati*

Institute of Nuclear Physics - Polish Academy of Sciences (IFJ-PAN), Kraków, Poland  
Tokyo Institute of Technology, Tokyo, Japan  
University of Trento, Trento, Italy



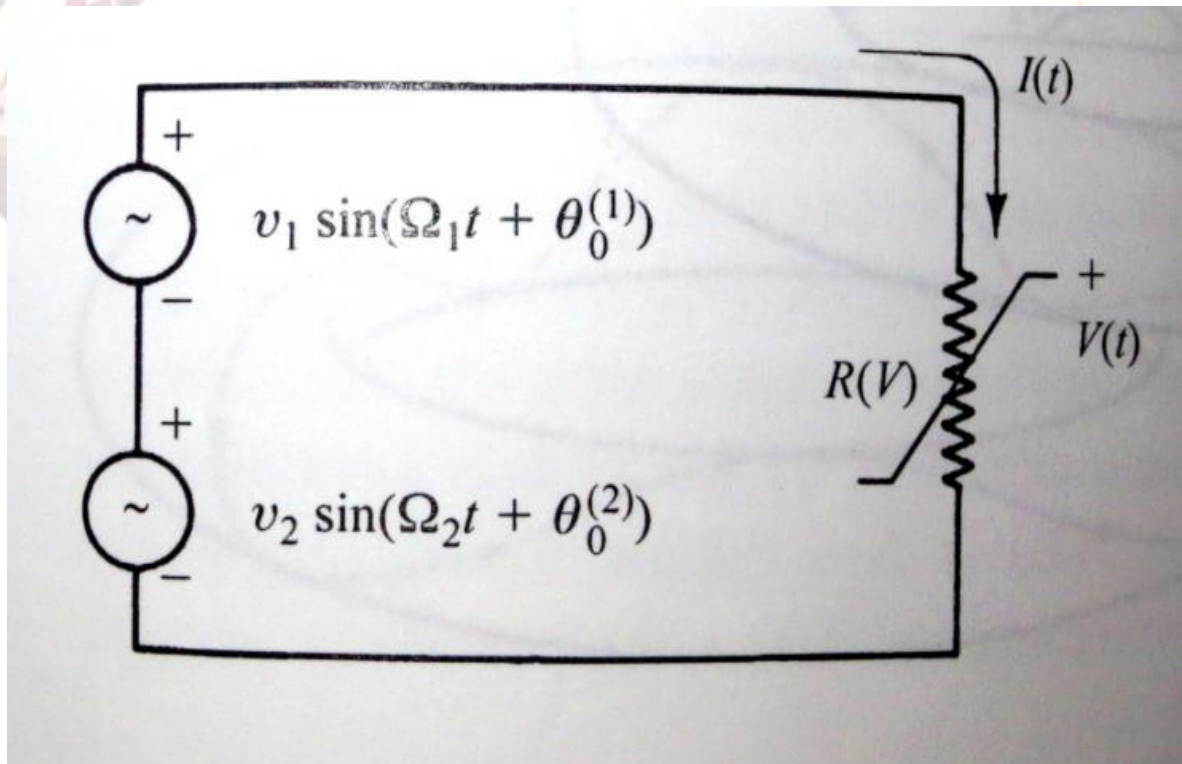
# Integrated circuit implementation

Problem: inductors and capacitors not suitable for CMOS realization



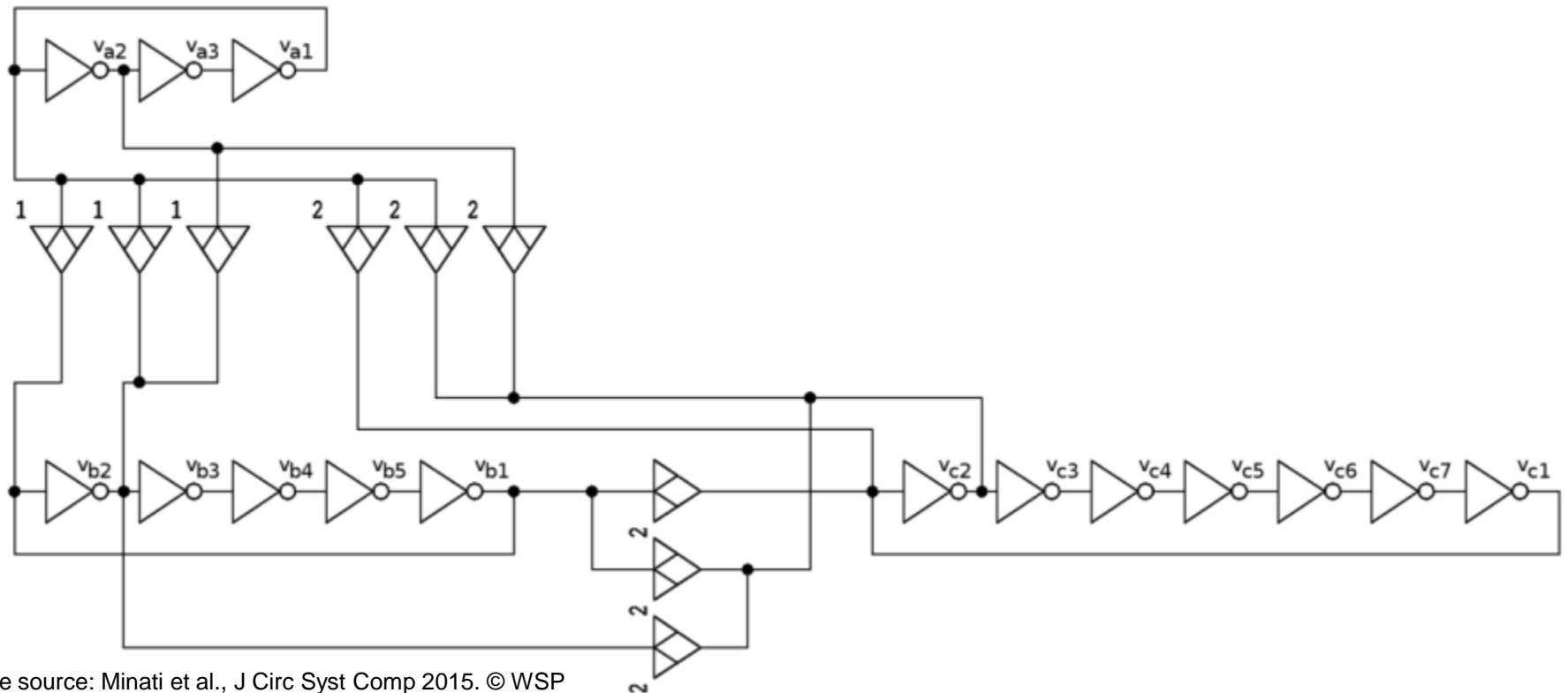
# Integrated circuit implementation

## Quasi-periodicity route to chaos



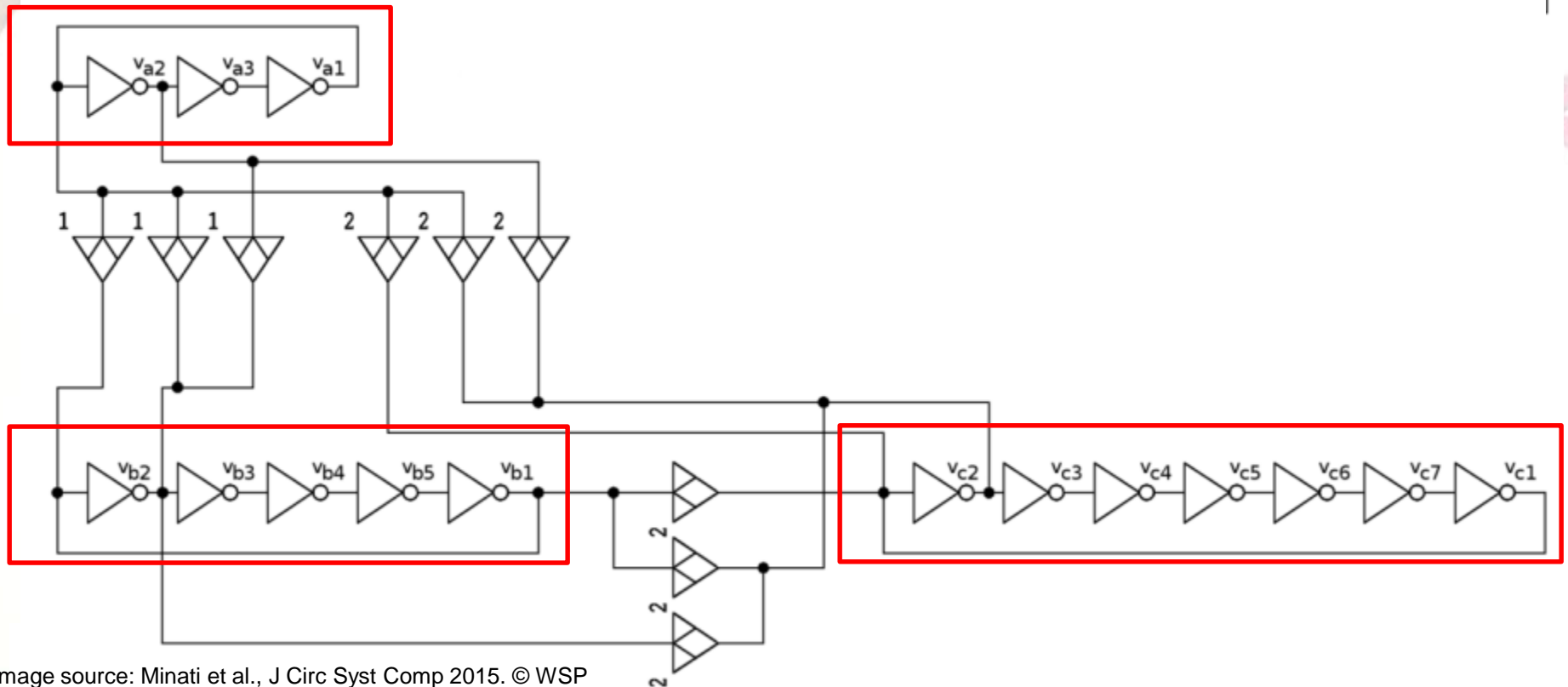
# Integrated circuit implementation

Coupled inverter rings having length equal to smallest prime numbers



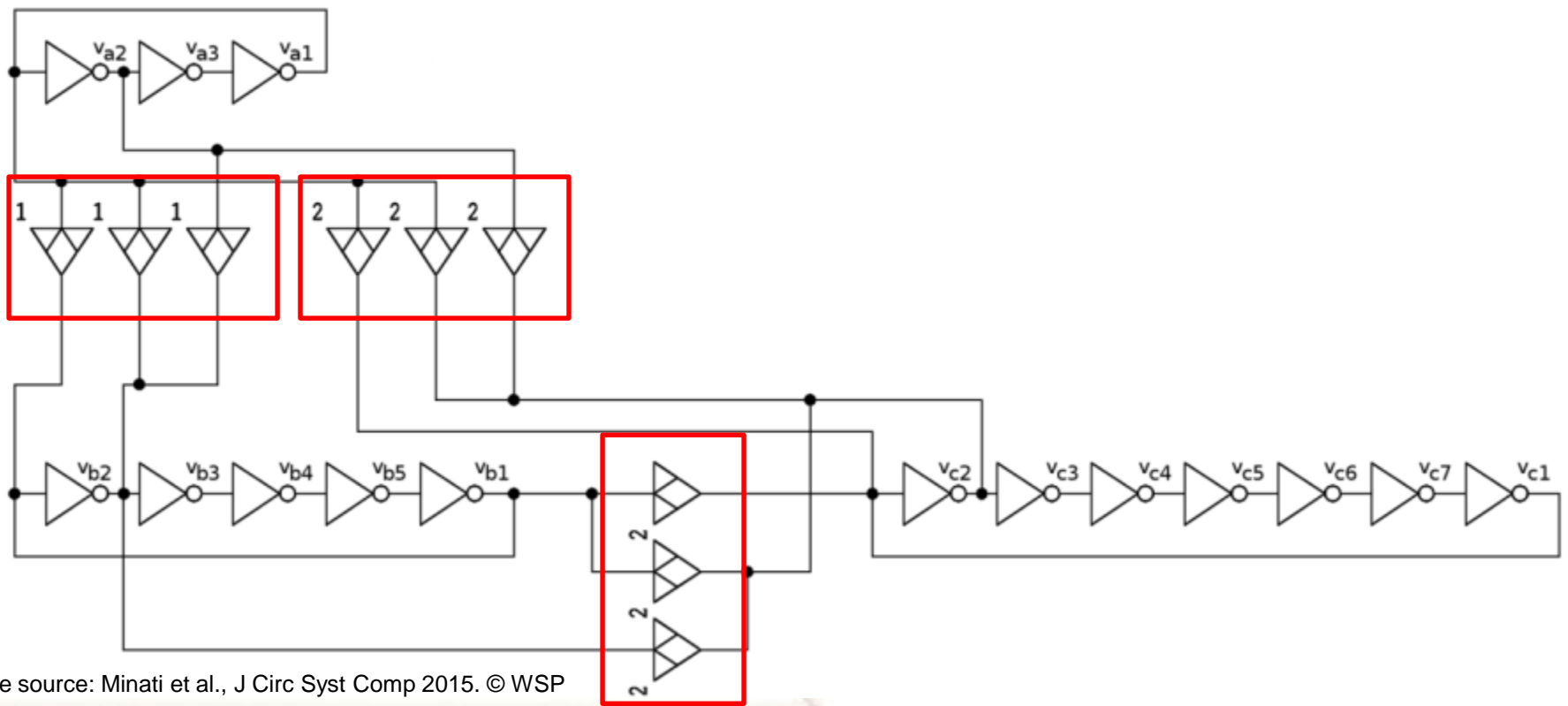
# Integrated circuit implementation

## Coupled inverter rings



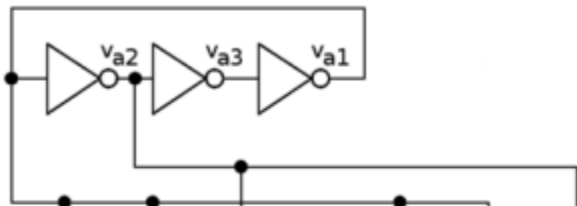
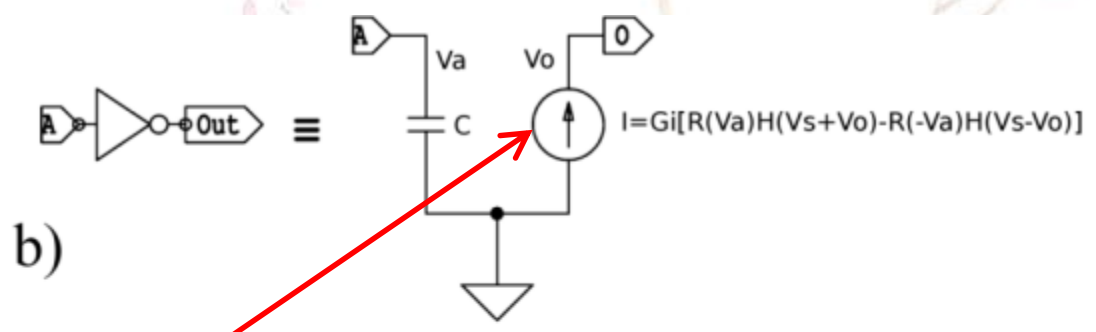
# Integrated circuit implementation

## Coupling elements

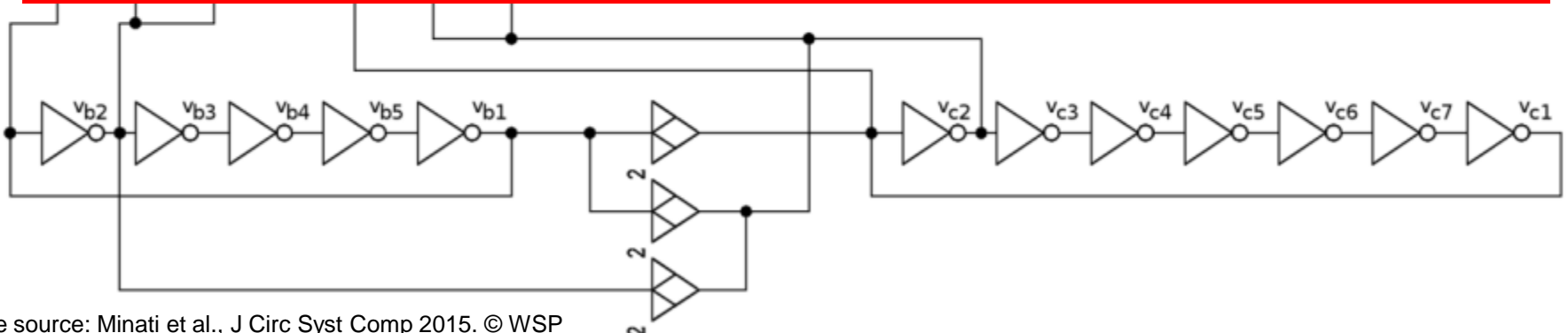


# Integrated circuit implementation

## Simplified numerical model

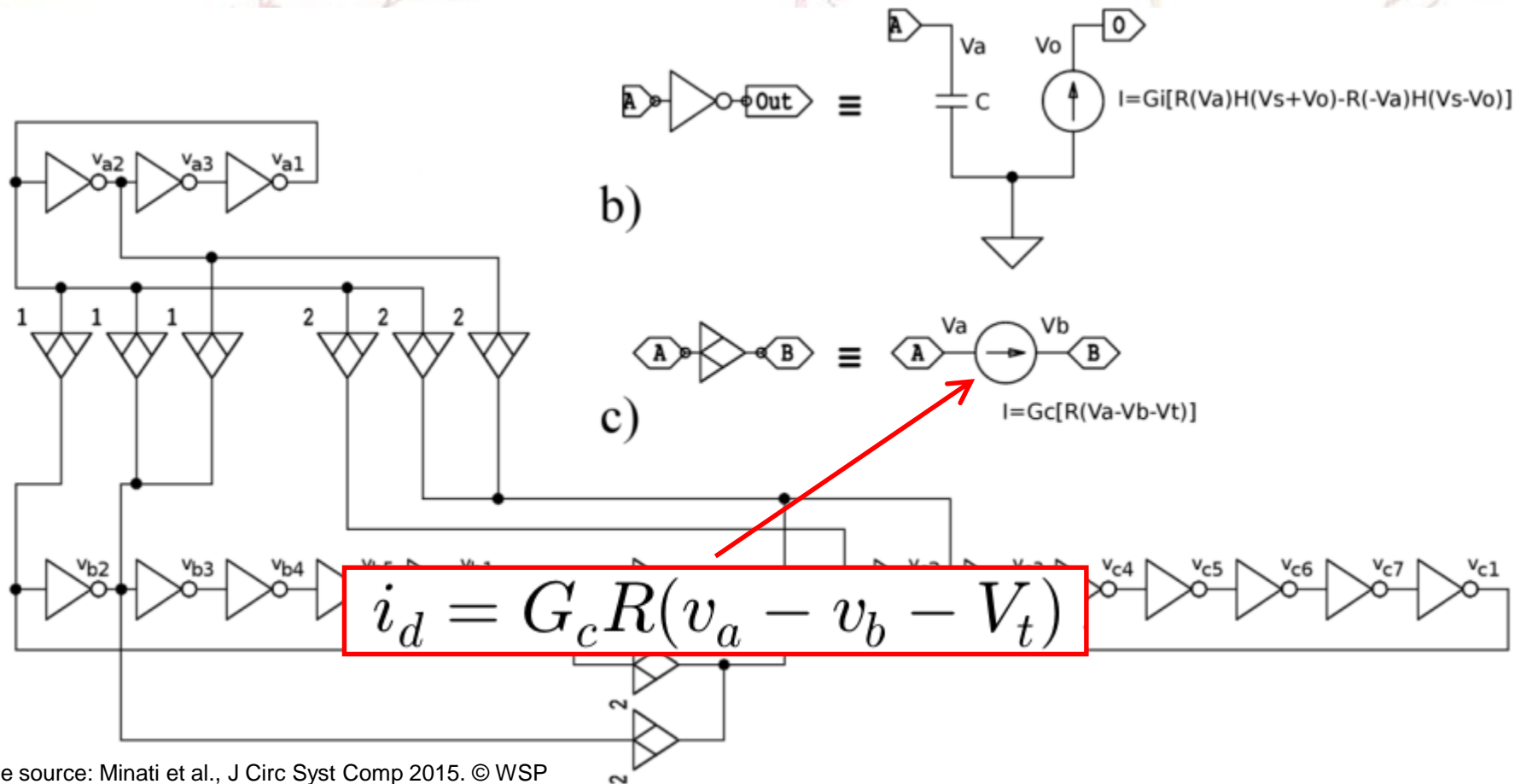


$$i_o = G_i [R(v_a)H(V_s + v_o) - R(-v_a)H(V_s - v_o)]$$



# Integrated circuit implementation

## Simplified numerical model

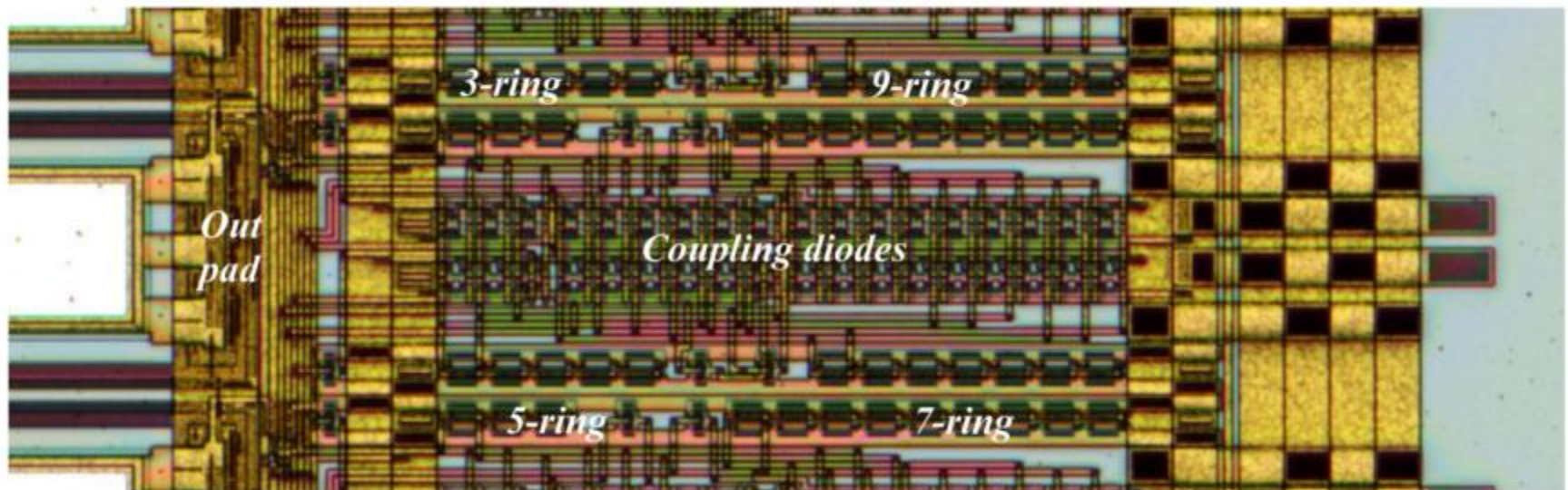






# Integrated circuit implementation

CMOS realization – single cell and ring

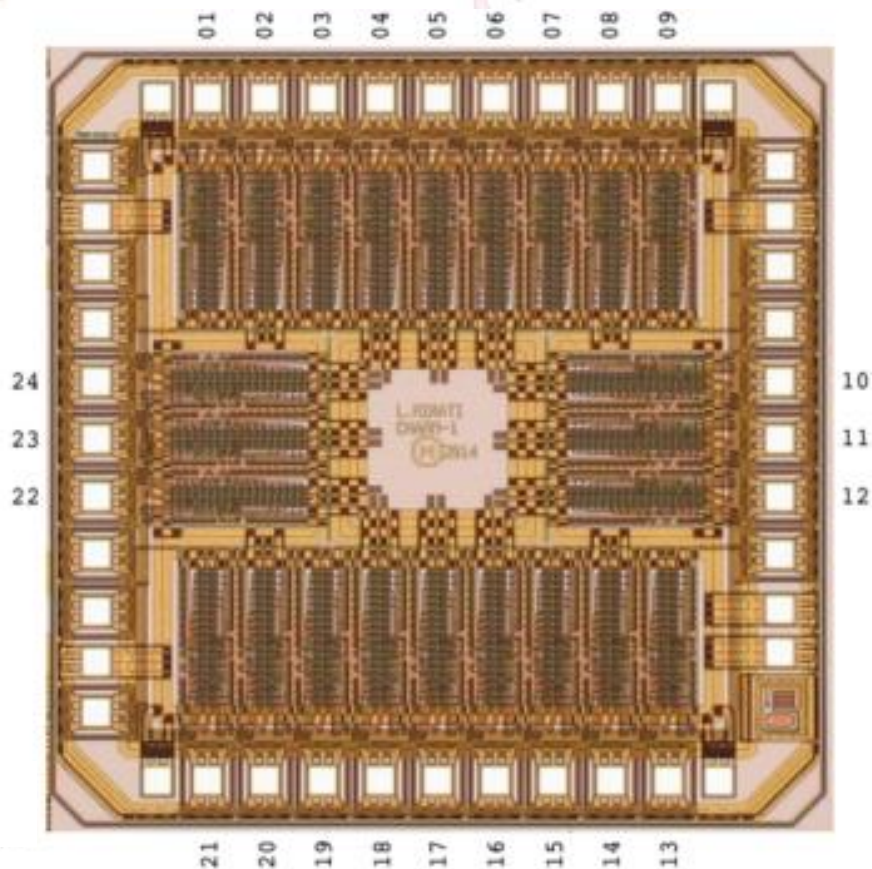




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# Integrated circuit implementation

CMOS realization – single cell and ring



Realized through





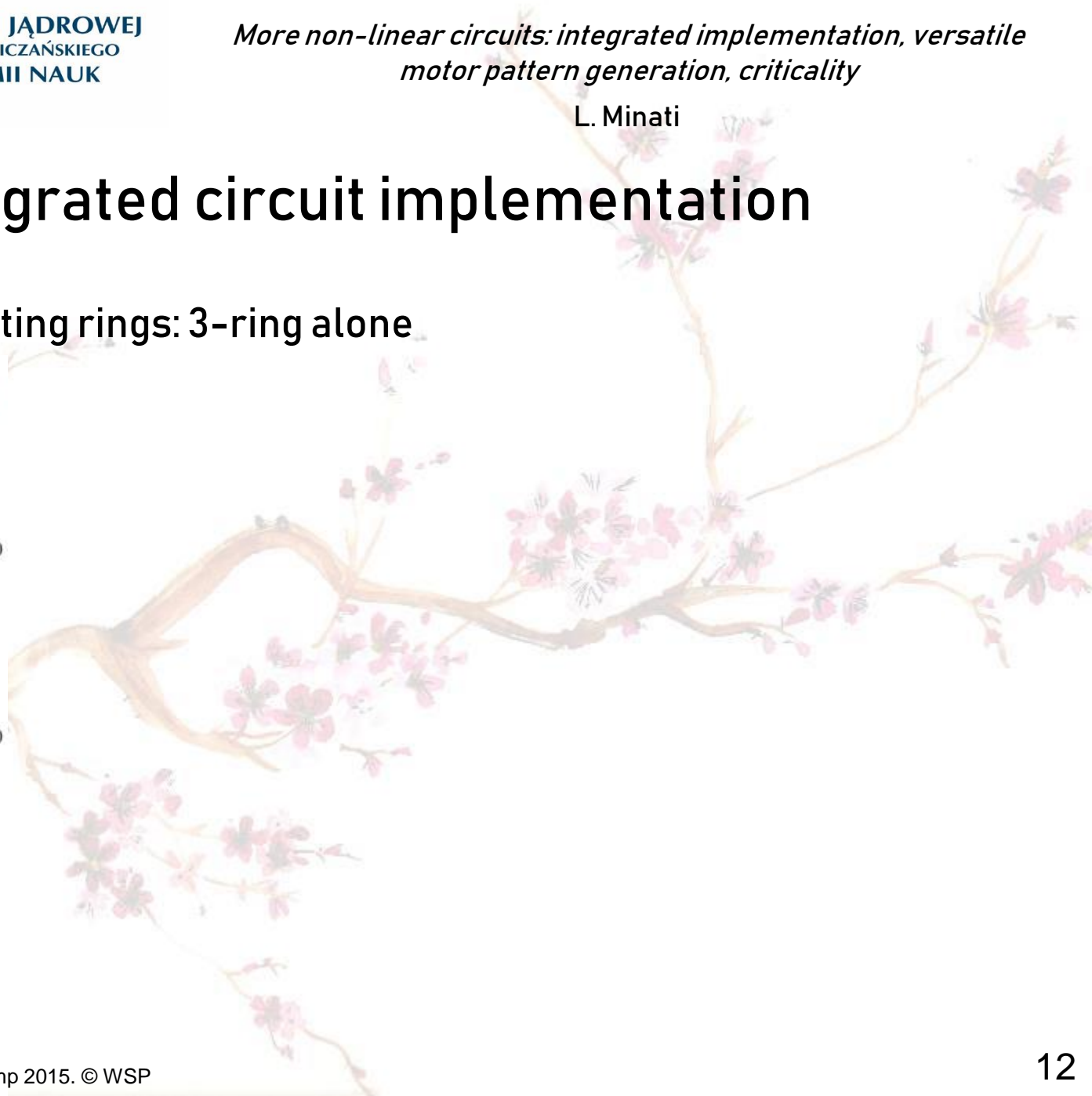
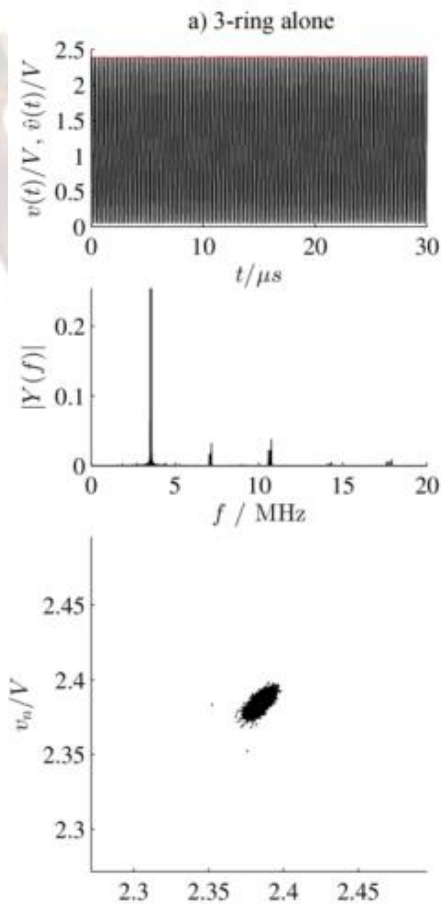
# Integrated circuit implementation

CMOS realization – test board



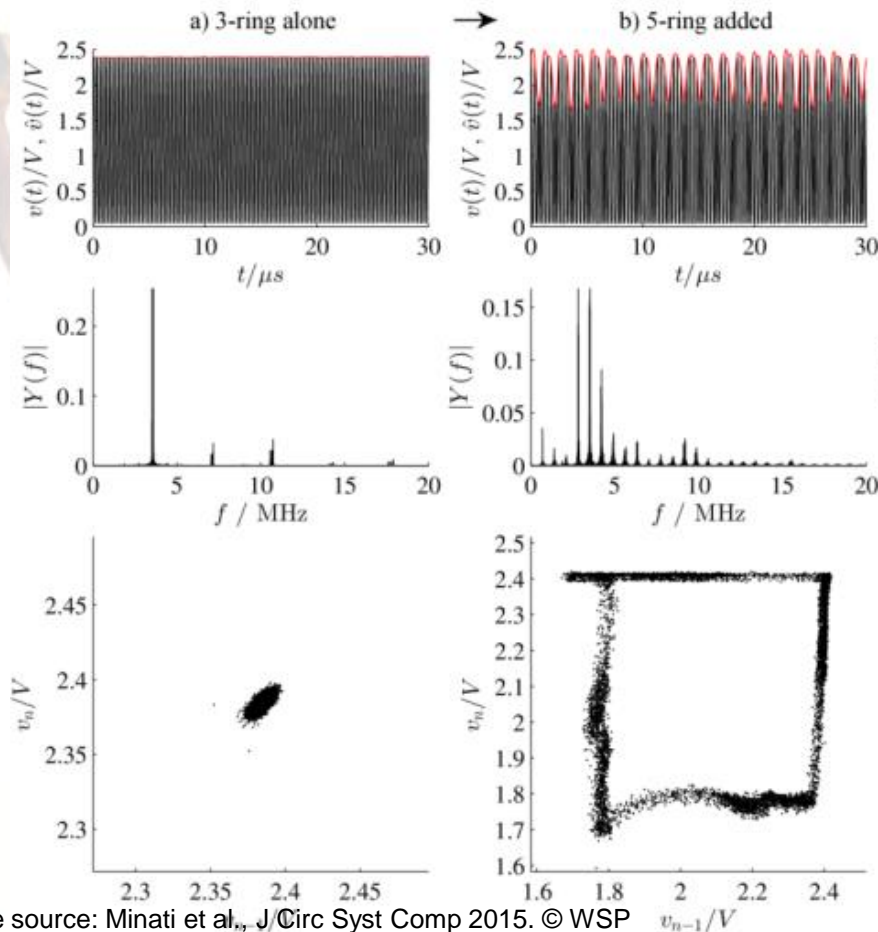
# Integrated circuit implementation

## Effect of connecting rings: 3-ring alone



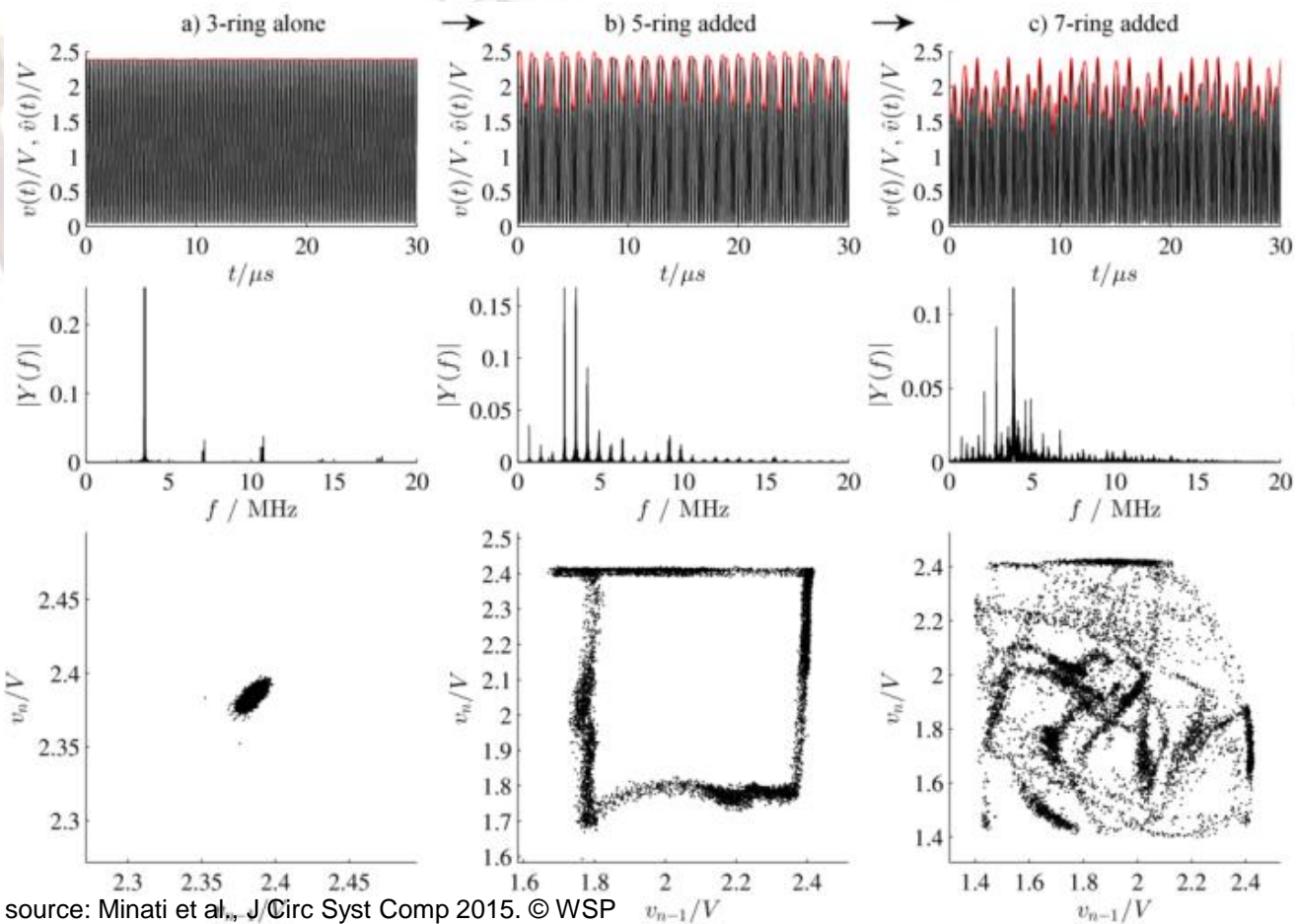
# Integrated circuit implementation

## Effect of connecting rings: 3-ring + 5-ring



# Integrated circuit implementation

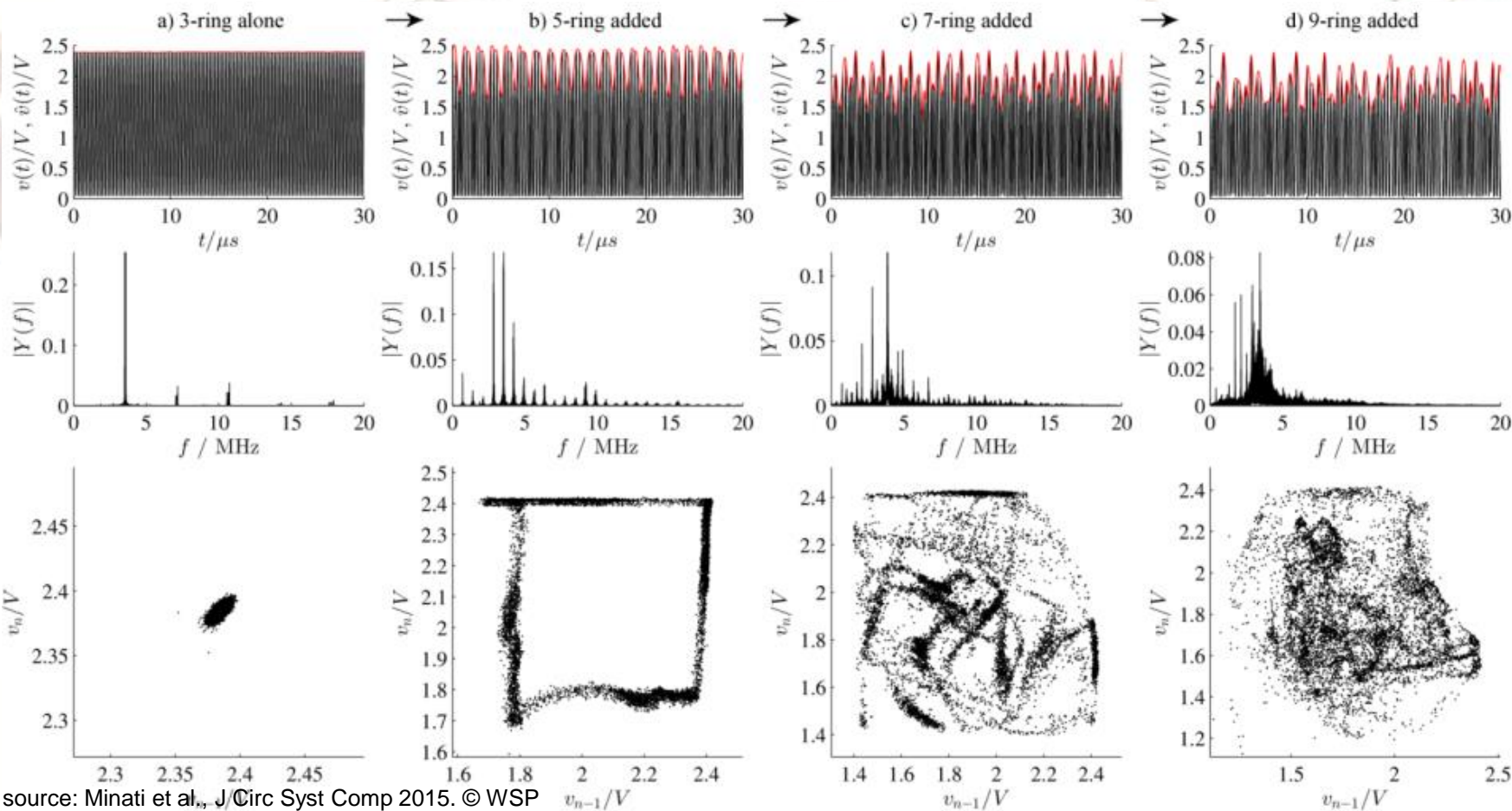
## Effect of connecting rings: 3-ring + 5,7-rings





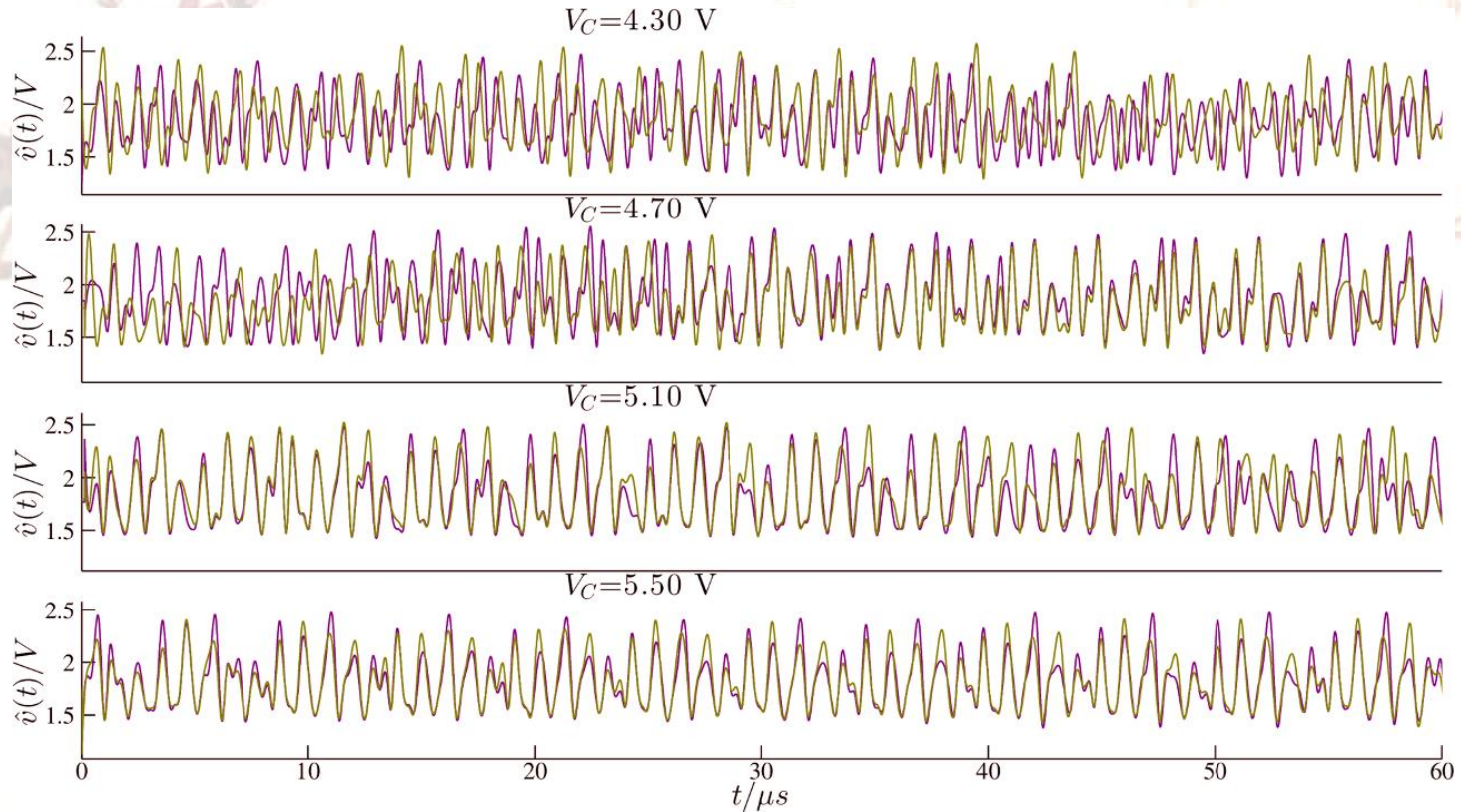
# Integrated circuit implementation

## Effect of connecting rings: 3-ring + 5,7,9-rings



# Integrated circuit implementation

## Effect of increasing coupling strength

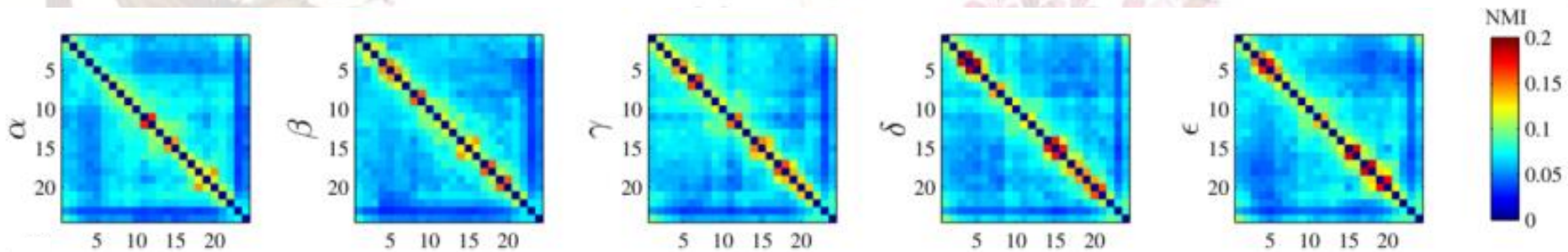






# Integrated circuit implementation

## Cluster synchronization: formation of communities





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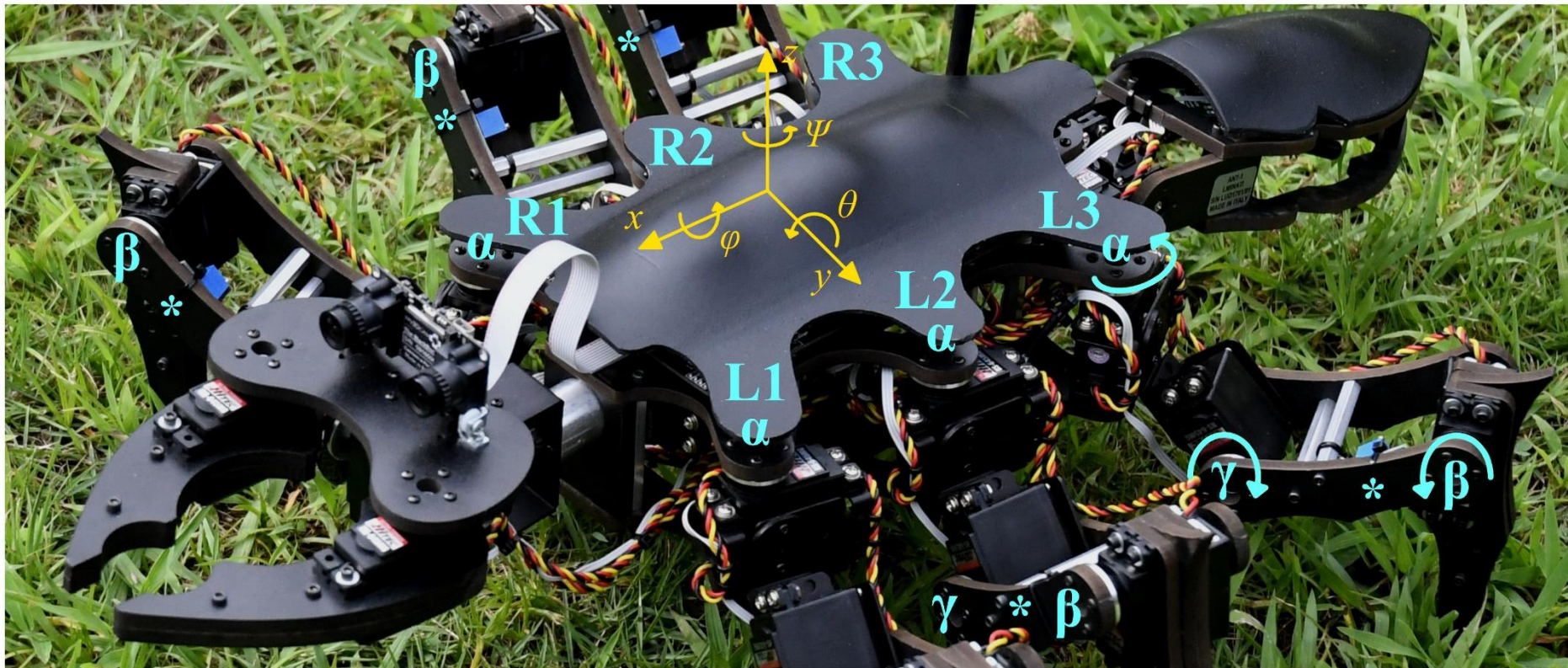
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*More non-linear circuits: integrated implementation, versatile  
motor pattern generation, criticality*

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# Versatile motor pattern generation

Generation of viable walking gaits for the 非線形蟻-1 robot

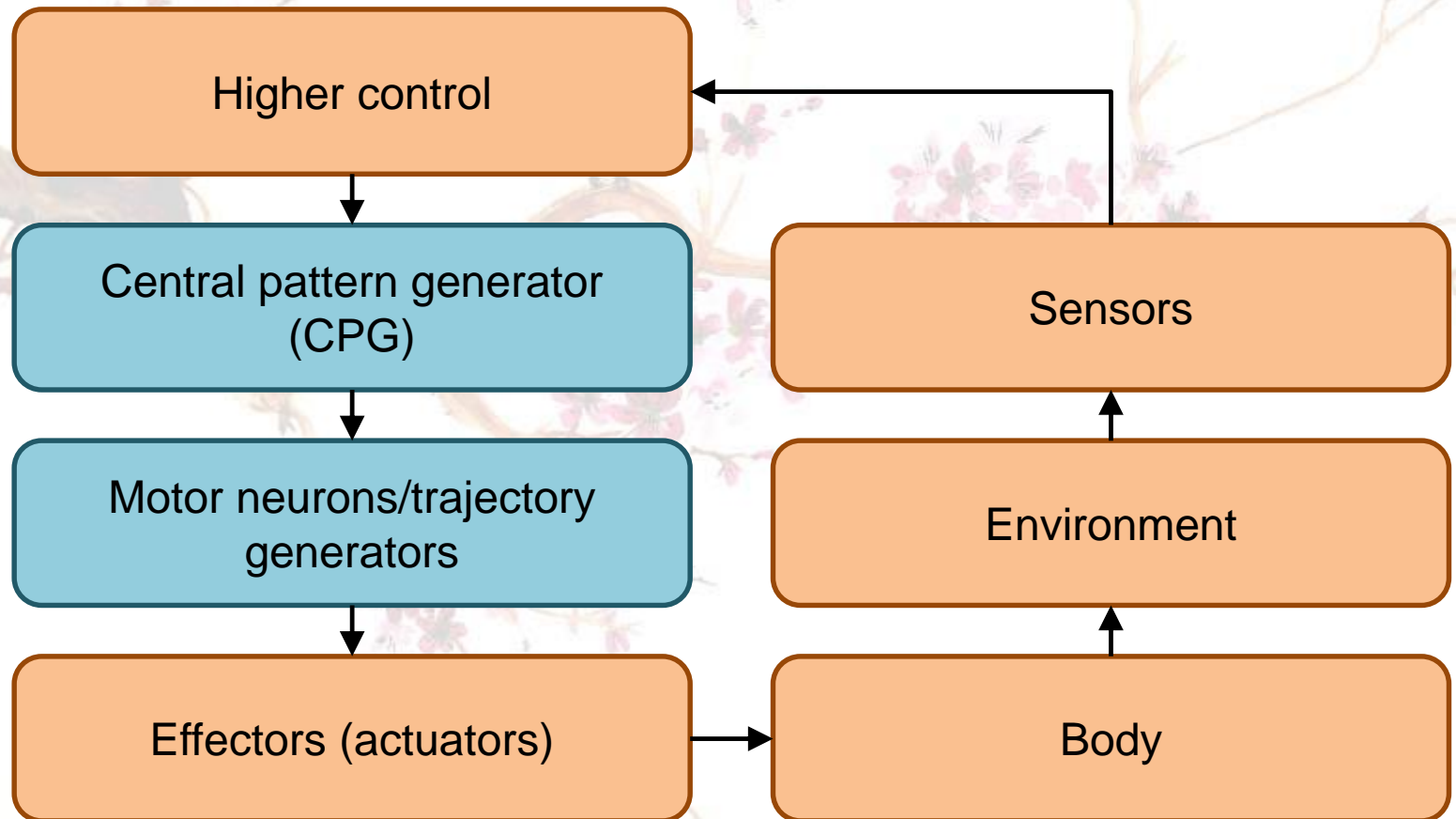


Original mechanics design: A-Pod by Kåre Halvorsen (a.k.a. "Zenta")



# Versatile motor pattern generation

## Central pattern generation (CPG)





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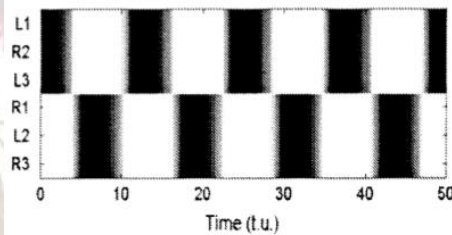
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# Versatile motor pattern generation

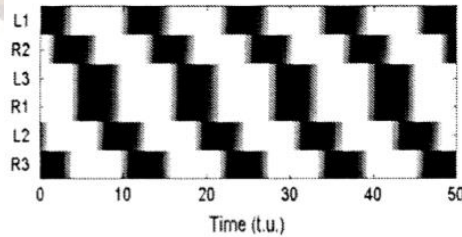
## Canonical gaits and postures in insects

### Gaits

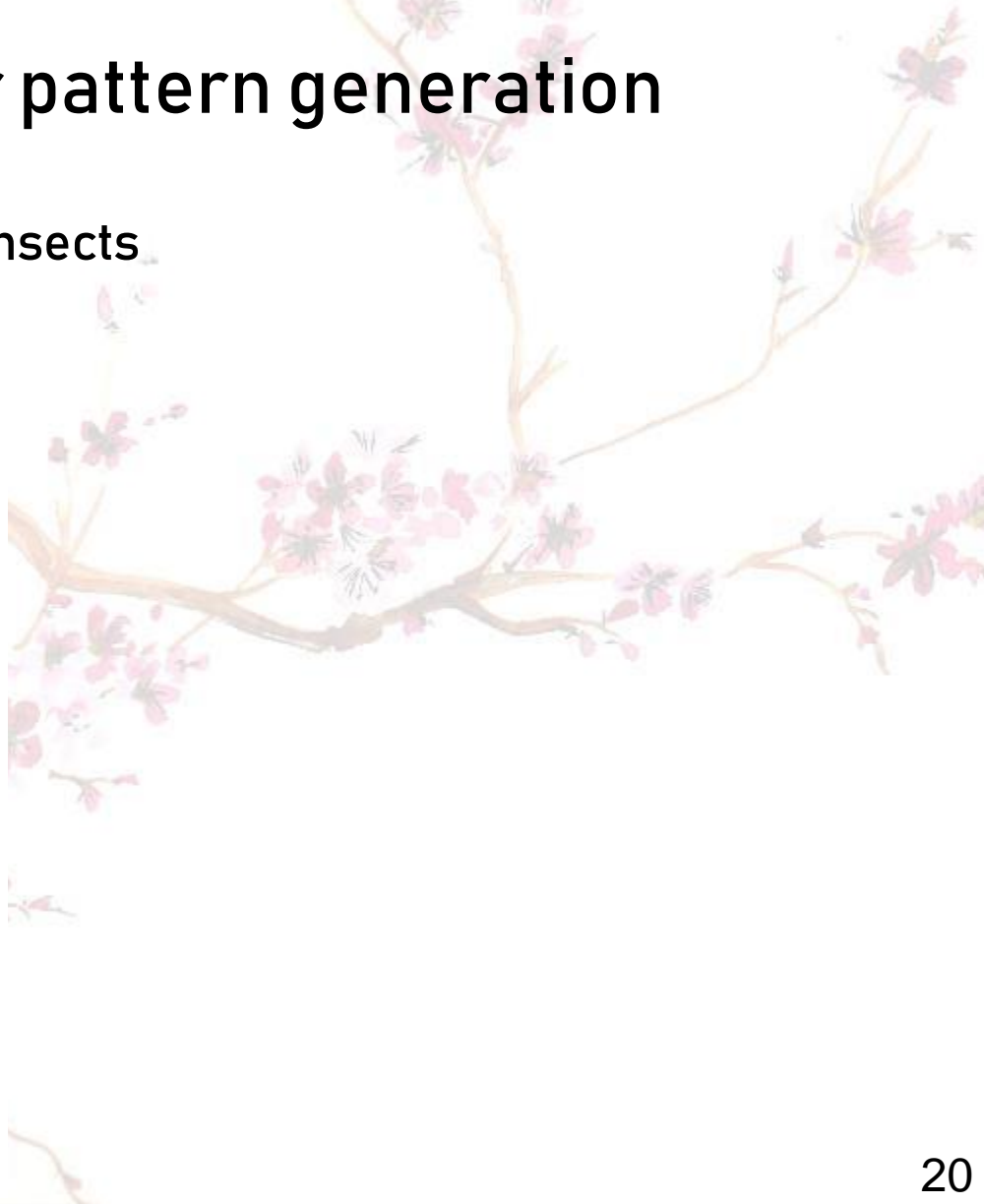
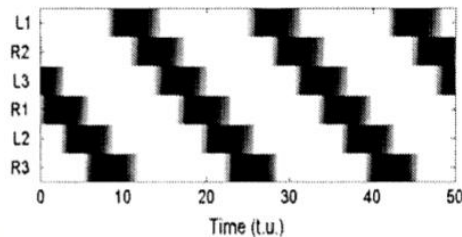
Alternating tripod



Metachronal



Wave





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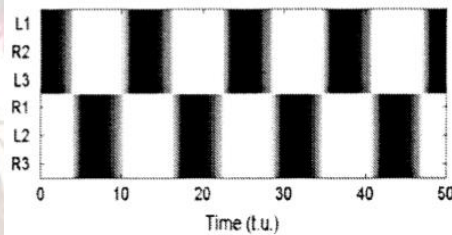
# Versatile motor pattern generation

## Canonical gaits and postures in insects

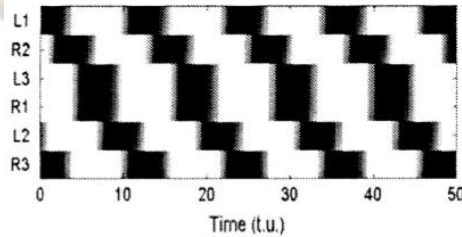
Gaits

Postures

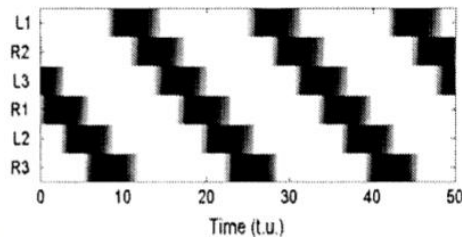
Alternating tripod



Metachronal



Wave



Ant



Cockroach



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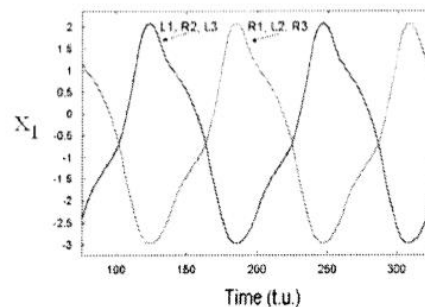
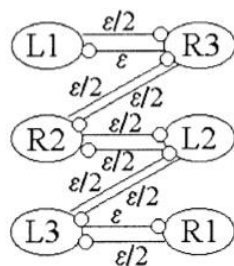
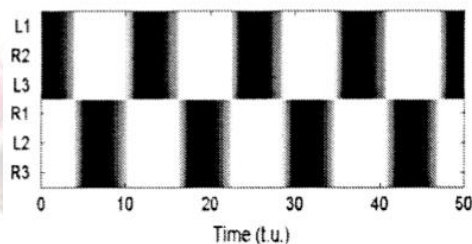
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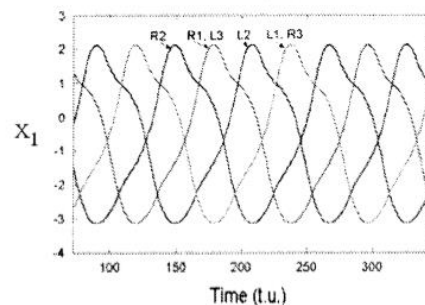
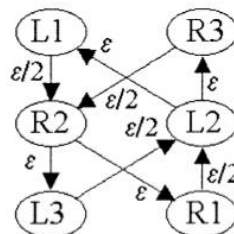
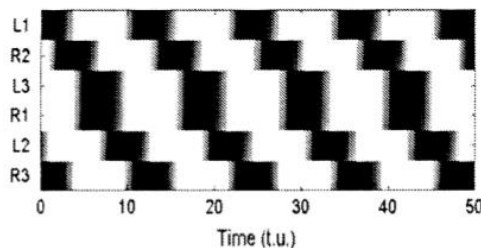
# Versatile motor pattern generation

## The connectionist approach

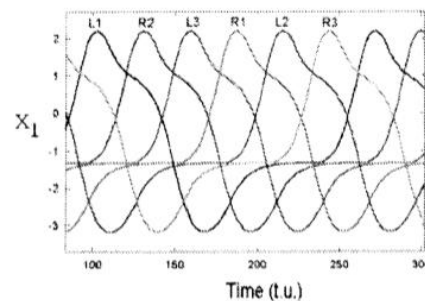
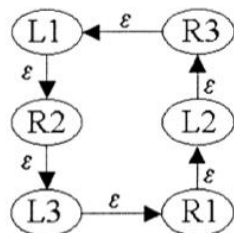
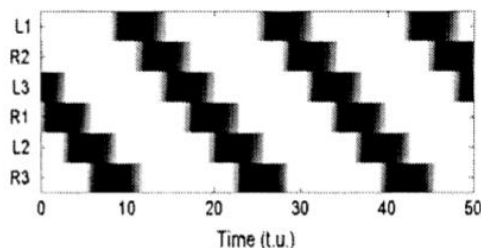
Alternating tripod



Metachronal



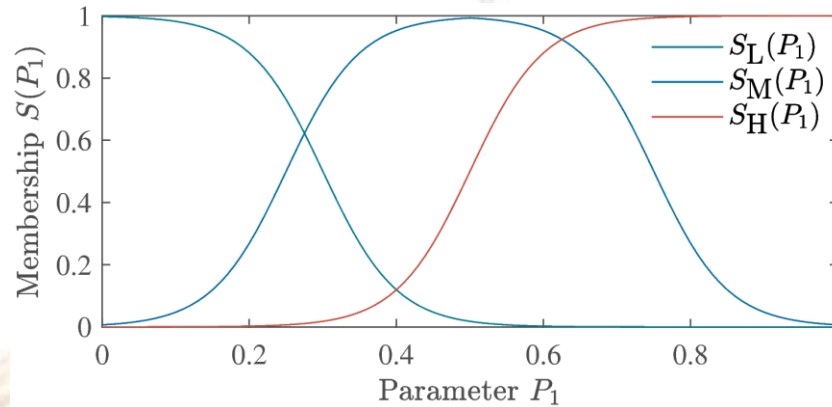
Wave





# Versatile motor pattern generation

## Generalization via fuzzy membership functions



$$\begin{cases} S_L(P_1) = 1 - \frac{1}{1 + e^{A_L(P_1 + C_L)}} \\ S_M(P_1) = 1 - \frac{1}{1 + e^{A_M(|P_1 + C_M| + C_M/2)}} \\ S_H(P_1) = \frac{1}{1 + e^{A_H(P_1 + C_H)}} \end{cases}$$

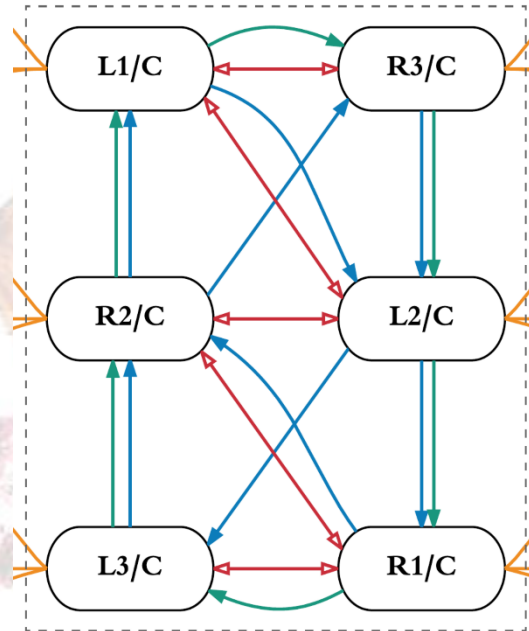


# Versatile motor pattern generation

Hierarchical approach (high level: gait, lower level: posture)

## Central Pattern Generator (CPG)

Control parameters:  $P_1$  and  $P_2$ .



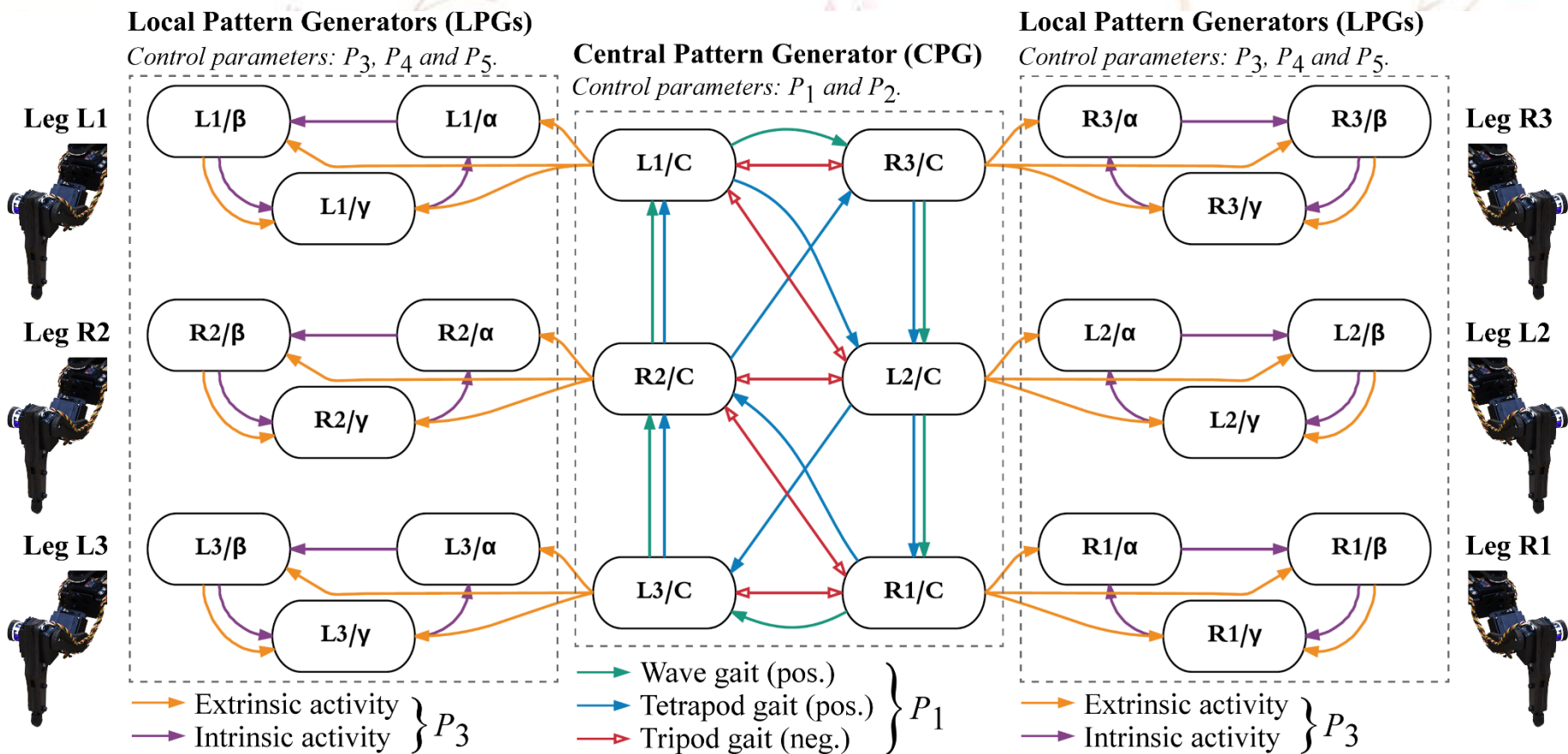
- Wave gait (pos.)
  - Tetrapod gait (pos.)
  - Tripod gait (neg.)
- }  $P_1$





# Versatile motor pattern generation

## Hierarchical approach (high level: gait, lower level: posture)





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# Versatile motor pattern generation

Problem: connectionist approach → huge number of parameters

$$A = 2 \begin{bmatrix} G_{1,L3 \rightarrow R3} & G_{1,R3 \rightarrow L1} & G_{1,L1 \rightarrow L2} & G_{1,L2 \rightarrow R2} & G_{1,L2 \rightarrow R1} & G_{1,L2 \rightarrow L3} \\ G_{2,R2 \rightarrow R3} & G_{2,L2 \rightarrow L1} & G_{2,R3 \rightarrow L2} & G_{2,R1 \rightarrow R2} & G_{2,R2 \rightarrow R1} & G_{2,R1 \rightarrow L3} \\ G_{3,L1 \rightarrow R3} & G_{3,R2 \rightarrow L1} & G_{3,R2 \rightarrow L2} & G_{3,L3 \rightarrow R2} & G_{3,L3 \rightarrow R1} & G_{3,R3 \rightarrow L3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -S_H/2 & (S_M - S_H)/2 & -S_H/2 & S_M + S_L & S_M \\ S_M & -S_H/2 & S_M/2 + S_L & (S_M - S_H)/2 & -S_H/2 & S_L - S_H \\ S_L - S_H & S_M + S_L & -S_H/2 & S_M/2 + S_L & -S_H/2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} G_{1,L1,\gamma \rightarrow \alpha} & G_{2,L1,C \rightarrow \alpha} & G_{3,L1,\alpha} & G_{1,L1,\alpha \rightarrow \beta} & G_{2,L1,C \rightarrow \beta} & G_{3,L1,\alpha} & G_{1,L1,\beta \rightarrow \gamma} & G_{2,L1,C \rightarrow \gamma} & G_{3,L1,\alpha} \\ G_{1,R1,\gamma \rightarrow \alpha} & G_{2,R1,C \rightarrow \alpha} & G_{3,R1,\alpha} & G_{1,R1,\alpha \rightarrow \beta} & G_{2,R1,C \rightarrow \beta} & G_{3,R1,\alpha} & G_{1,R1,\beta \rightarrow \gamma} & G_{2,R1,C \rightarrow \gamma} & G_{3,R1,\alpha} \\ G_{1,L2,\gamma \rightarrow \alpha} & G_{2,L2,C \rightarrow \alpha} & G_{3,L2,\alpha} & G_{1,L2,\alpha \rightarrow \beta} & G_{2,L2,C \rightarrow \beta} & G_{3,L2,\alpha} & G_{1,L2,\beta \rightarrow \gamma} & G_{2,L2,C \rightarrow \gamma} & G_{3,L2,\alpha} \\ G_{1,R2,\gamma \rightarrow \alpha} & G_{2,R2,C \rightarrow \alpha} & G_{3,R2,\alpha} & G_{1,R2,\alpha \rightarrow \beta} & G_{2,R2,C \rightarrow \beta} & G_{3,R2,\alpha} & G_{1,R2,\beta \rightarrow \gamma} & G_{2,R2,C \rightarrow \gamma} & G_{3,R2,\alpha} \\ G_{1,L3,\gamma \rightarrow \alpha} & G_{2,L3,C \rightarrow \alpha} & G_{3,L3,\alpha} & G_{1,L3,\alpha \rightarrow \beta} & G_{2,L3,C \rightarrow \beta} & G_{3,L3,\alpha} & G_{1,L3,\beta \rightarrow \gamma} & G_{2,L3,C \rightarrow \gamma} & G_{3,L3,\alpha} \\ G_{1,R3,\gamma \rightarrow \alpha} & G_{2,R3,C \rightarrow \alpha} & G_{3,R3,\alpha} & G_{1,R3,\alpha \rightarrow \beta} & G_{2,R3,C \rightarrow \beta} & G_{3,R3,\alpha} & G_{1,R3,\beta \rightarrow \gamma} & G_{2,R3,C \rightarrow \gamma} & G_{3,R3,\alpha} \end{bmatrix}$$

$$= \begin{bmatrix} B_1 P_3' & P_3 & B_2 P_3' + B_3 P_3 P_4' & -P_3' & P_3(1 + B_4 P_4) & B_5 P_3 P_4 & B_6 P_3' + B_7 P_3 P_4' & -B_8 P_3 P_4 & B_9 P_3 P_4' \\ B_1 P_3' & P_3 & B_2 P_3' + B_3 P_3 P_4' & -P_3' & P_3(1 + B_4 P_4) & B_5 P_3 P_4 & B_6 P_3' + B_7 P_3 P_4' & -B_8 P_3 P_4 & B_9 P_3 P_4' \\ B_1 P_3' & P_3 & B_2 P_3' + B_3 P_3 & -P_3' & P_3 & 0 & B_6 P_3' + B_7 P_3 & 0 & B_9 P_3 \\ B_1 P_3' & P_3 & B_2 P_3' + B_3 P_3 & -P_3' & P_3 & 0 & B_6 P_3' + B_7 P_3 & 0 & B_9 P_3 \end{bmatrix}$$

$$C = 5 \begin{bmatrix} G_{4,L1,\alpha} & G_{5,L1,\alpha} & G_{4,L1,\beta} & G_{5,L1,\beta} & G_{4,L1,\gamma} & G_{5,L1,\gamma} \\ G_{4,R1,\alpha} & G_{5,R1,\alpha} & G_{4,R1,\beta} & G_{5,R1,\beta} & G_{4,R1,\gamma} & G_{5,R1,\gamma} \\ G_{4,L2,\alpha} & G_{5,L2,\alpha} & G_{4,L2,\beta} & G_{5,L2,\beta} & G_{4,L2,\gamma} & G_{5,L2,\gamma} \\ G_{4,R2,\alpha} & G_{5,R2,\alpha} & G_{4,R2,\beta} & G_{5,R2,\beta} & G_{4,R2,\gamma} & G_{5,R2,\gamma} \\ G_{4,L3,\alpha} & G_{5,L3,\alpha} & G_{4,L3,\beta} & G_{5,L3,\beta} & G_{4,L3,\gamma} & G_{5,L3,\gamma} \\ G_{4,R3,\alpha} & G_{5,R3,\alpha} & G_{4,R3,\beta} & G_{5,R3,\beta} & G_{4,R3,\gamma} & G_{5,R3,\gamma} \end{bmatrix} =$$

$$\begin{bmatrix} C_1 P_3 P_4 & C_2 P_3' + C_3 P_3 P_4 P_5' & C_4 P_3' + C_5 P_3 P_4' & C_6 P_3' + C_7 P_3 P_4 P_5' & C_8 P_3' + C_9 P_3 P_4' + C_{10} P_3 P_4 & C_{11} P_3' \\ C_1 P_3 P_4 & C_2 P_3' + C_3 P_3 P_4 P_5'' & C_4 P_3' + C_5 P_3 P_4' & C_6 P_3' + C_7 P_3 P_4 P_5'' & C_8 P_3' + C_9 P_3 P_4' + C_{10} P_3 P_4 & C_{11} P_3' \\ 0 & C_{12} P_3' + C_{13} P_3 P_5' & C_{14} & C_{15} P_3' & C_{16} P_3' + C_{17} P_3 & C_{18} P_3' \\ 0 & C_{12} P_3' + C_{13} P_3 P_5'' & C_{14} & C_{15} P_3' & C_{16} P_3' + C_{17} P_3 & C_{18} P_3' \\ C_{19} P_3 P_4 & C_{20} P_3' + C_{21} P_3 P_4 P_5' & C_{22} P_3' + C_{23} P_3 P_4' & C_{24} P_3' + C_{25} P_3 P_4 P_5' & C_{26} P_3' + C_{27} P_3 P_4' + C_{28} P_3 P_4 & C_{29} P_3' \\ C_{19} P_3 P_4 & C_{20} P_3' + C_{21} P_3 P_4 P_5'' & C_{22} P_3' + C_{23} P_3 P_4' & C_{24} P_3' + C_{25} P_3 P_4 P_5'' & C_{26} P_3' + C_{27} P_3 P_4' + C_{28} P_3 P_4 & C_{29} P_3' \end{bmatrix}$$



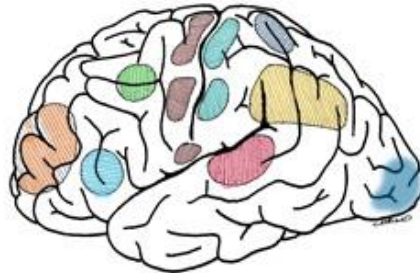
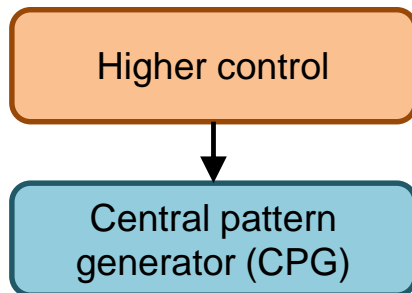
B747-100



# Versatile motor pattern generation

Answer: declutter by collapsing into few higher-level parameters

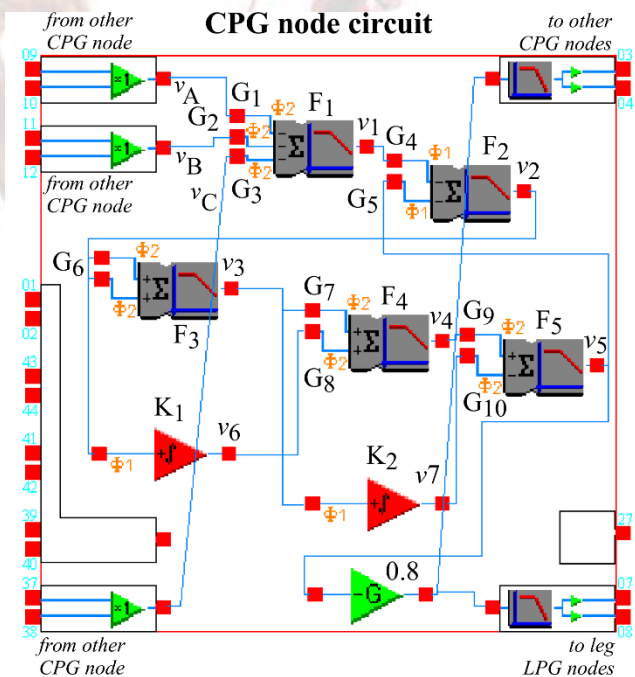
Parameter	Range	Target	Denomination	Function
$P_1$	[0, 1]	CPG	Gait selection parameter	Determines the phase relationships between legs, e.g. allows choosing between wave (slow), tetrapod (metachronal, medium speed) and tripod (fast) gaits
$P_2$	[-1, 1]	CPG	Activation parameter	Allows inhibiting and approximately reversing the CPG activity, i.e. walking
$P_3$	[0, 1]	LPG	CPG → LPG coupling strength parameter	Determines the level of synchronization between the CPG and the LPGs (i.e., movement coordination)
$P_4$	[0, 1]	LPG	Posture parameter	Allows choosing between the ant-like and cockroach-like postures
$P_5$	[-1, 1]	LPG	Steering parameter	Allows steering the robot trajectory sideways





# Versatile motor pattern generation

Node oscillator – almost same as in experiments on remote sync.



For node	$G_1$	$G_2$	$G_3$
L1/C	$-S_H/2$	$-S_H/2$	$S_M + S_L$
R2/C	$-S_H/2$	$(S_M - S_H)/2$	$S_M/2 + S_L$
L3/C	$S_M$	$S_L - S_H$	0
R1/C	$S_M + S_L$	$-S_H/2$	$-S_H/2$
L2/C	$(S_M - S_H)/2$	$S_M/2 + S_L$	$-S_H/2$
R3/C	0	$S_M$	$S_L - S_H$





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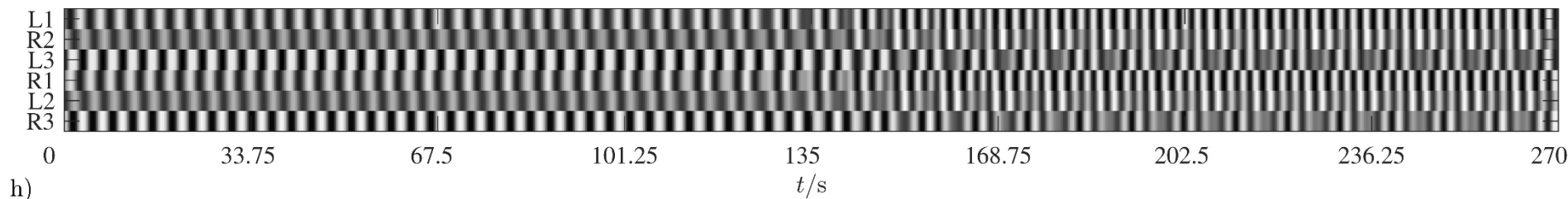
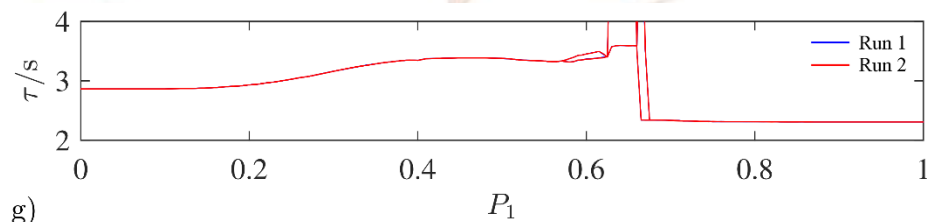
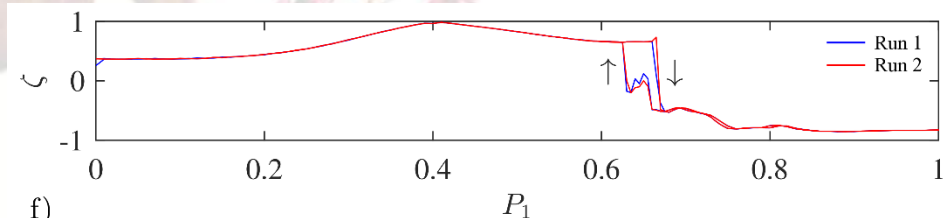
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# Versatile motor pattern generation

## Emergent hysteresis and metastability, as in living insects!





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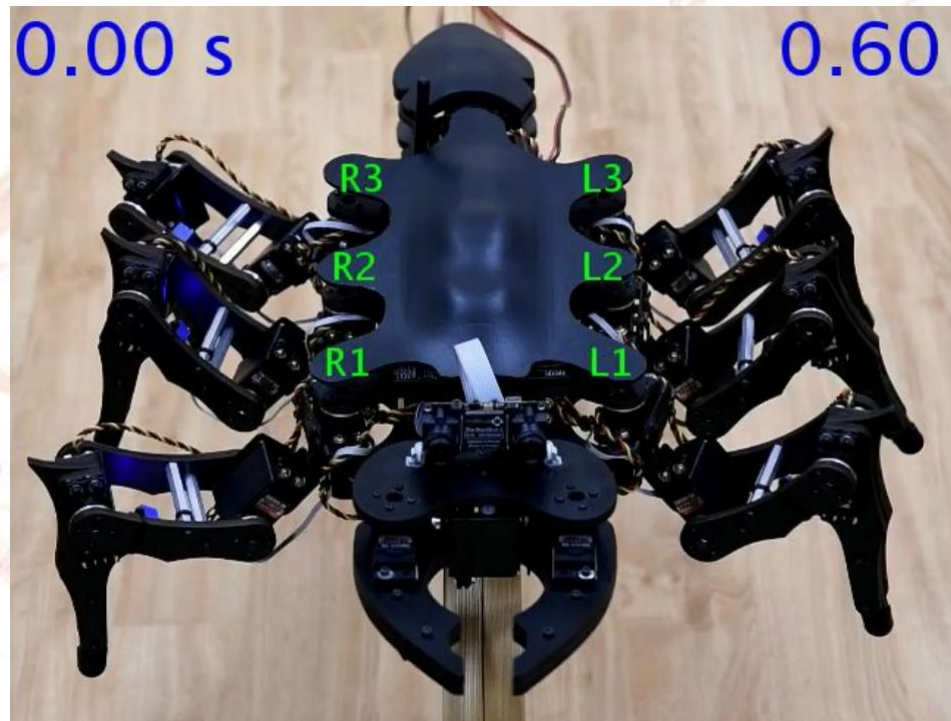
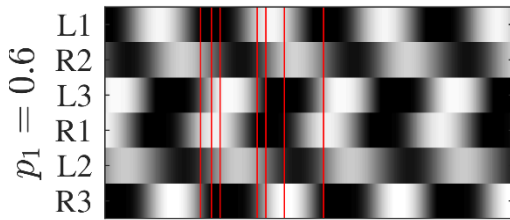
*More non-linear circuits: integrated implementation, versatile  
motor pattern generation, criticality*

L. Minati

# Versatile motor pattern generation

Emergent intermediate patterns, turn out to be viable

## Paradoxical gait





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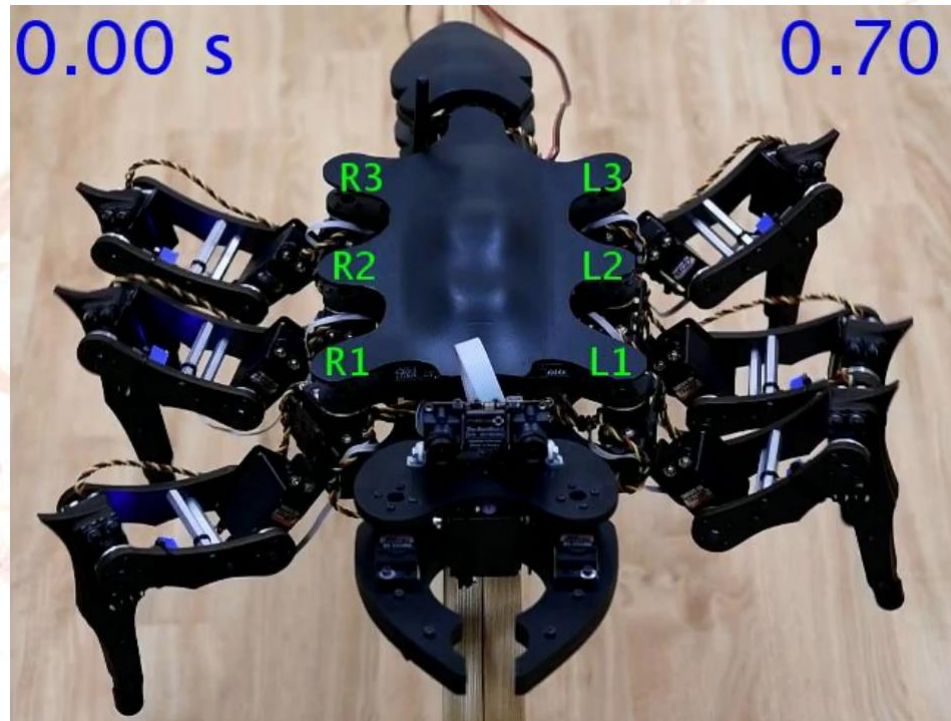
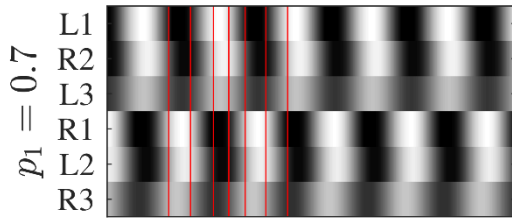
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# Versatile motor pattern generation

Emergent intermediate patterns, turn out to be viable

Unstable gait







# Versatile motor pattern generation

Posture and leg kinematics - Different power stroke delivery

Ant



All: coxa-body

Cockroach



Front, hind: femur-tibia  
Middle: coxa-body



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# Versatile motor pattern generation

Posture and leg kinematics - Different power stroke delivery





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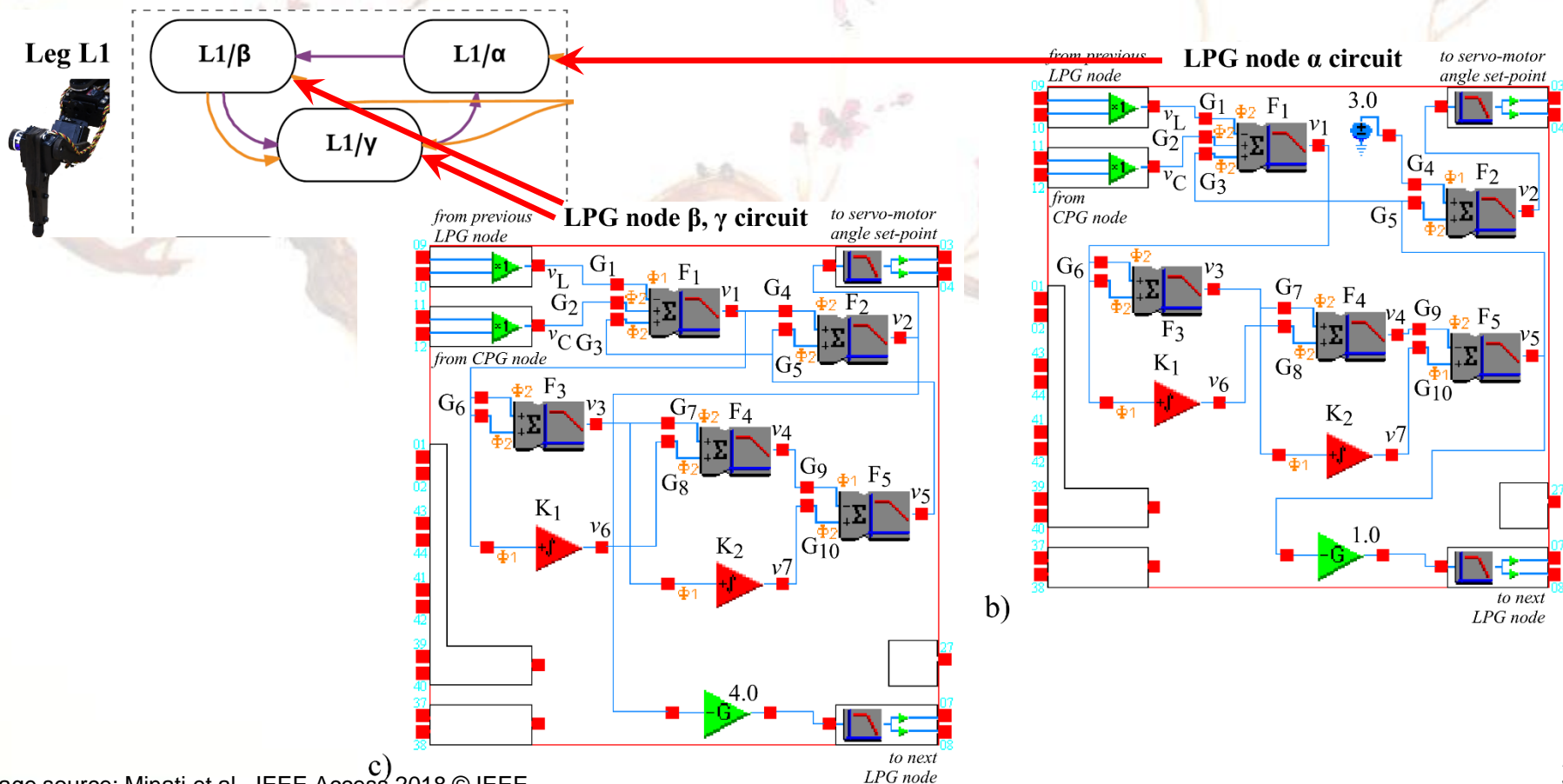
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# Versatile motor pattern generation

## No explicit kinematic model, but dedicated oscillator variants





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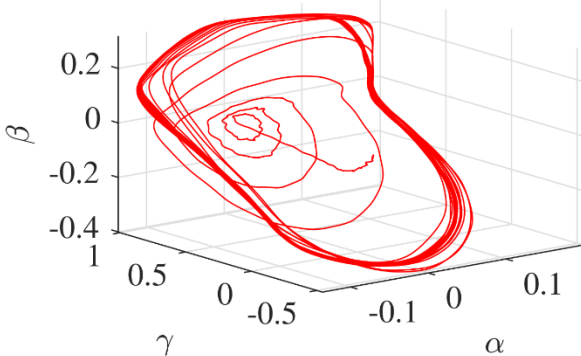
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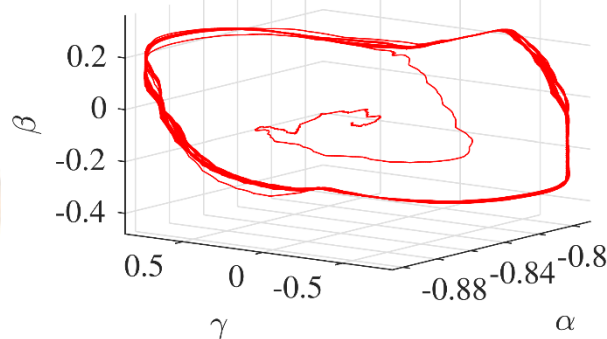
# Versatile motor pattern generation

## Emergence of limit cycle

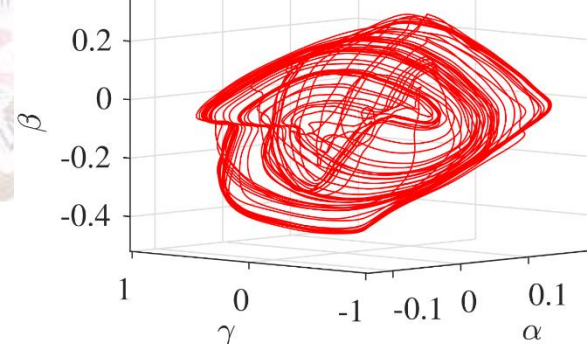
Ant



Cockroach



Deafferentated





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# Versatile motor pattern generation

Emergence of motor pattern - ant





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# Versatile motor pattern generation

Generation of motor pattern - ant





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# Versatile motor pattern generation

## Generation of motor pattern - cockroach





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# Versatile motor pattern generation

## Generation of motor pattern - deafferented



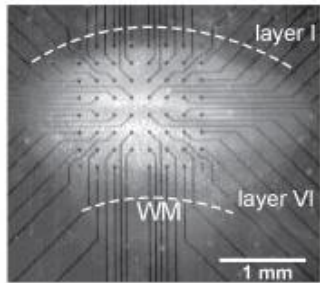




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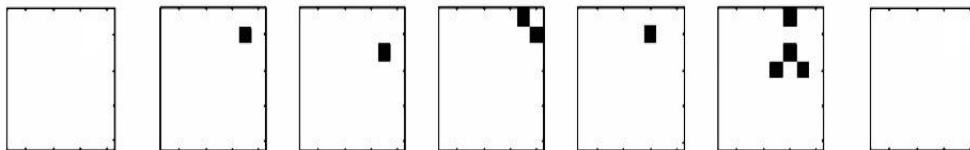
# Critical phenomena

## Neural avalanches recorded in-vitro and in-vivo

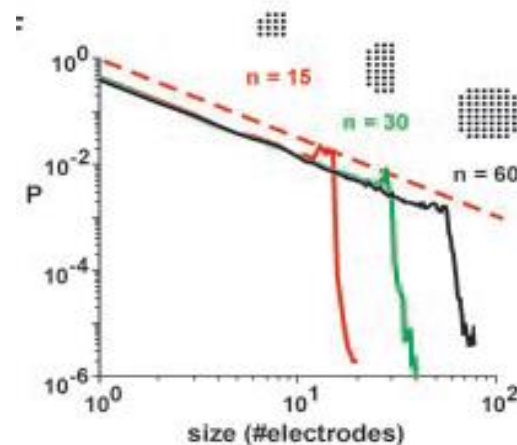
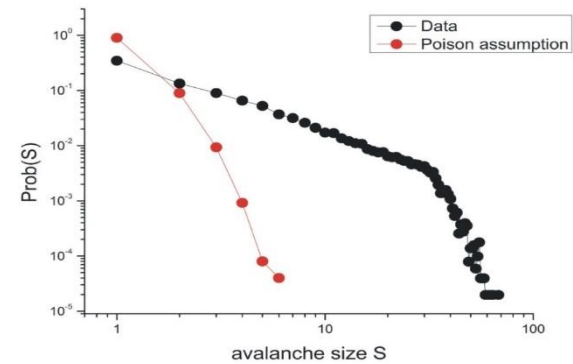


Cortical slice on array

### Example of avalanche:

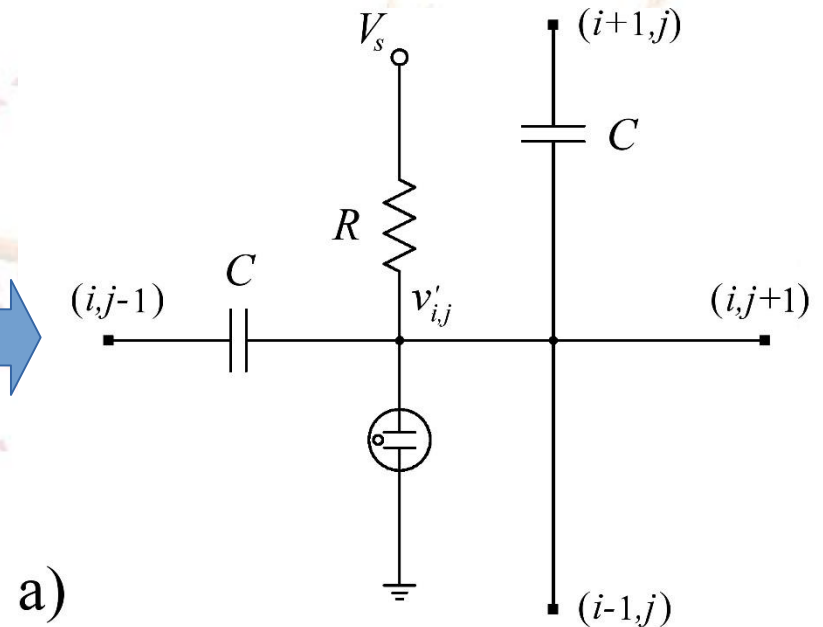
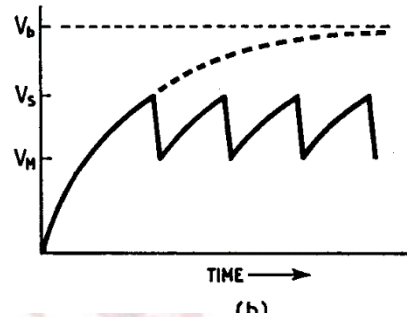
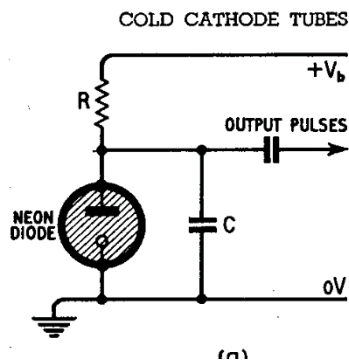


time (4 ms frames) →



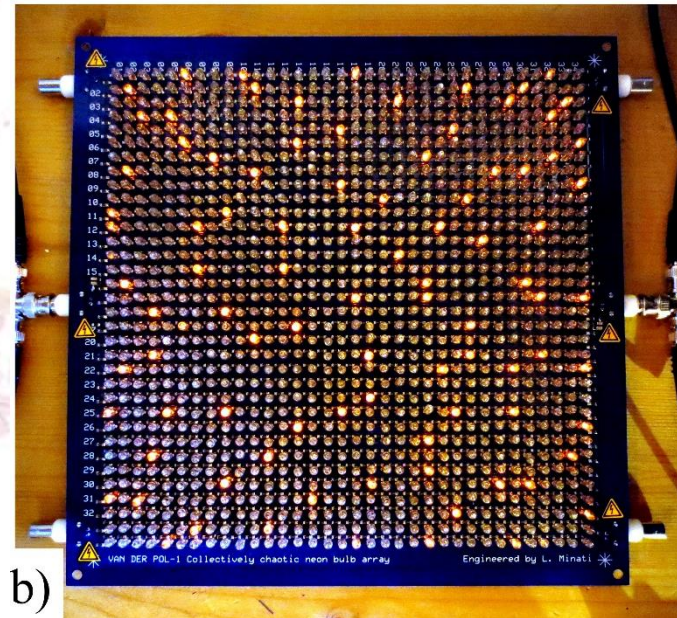
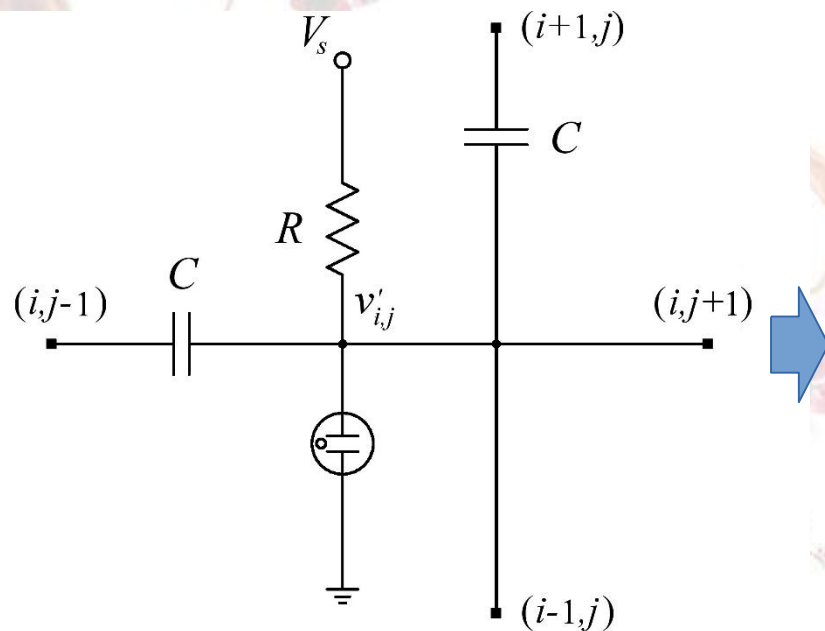
# Critical phenomena

In-silico “replica” of critical dynamics in a 2D lattice



# Critical phenomena

In-silico “replica” of critical dynamics in a 2D lattice



Lattice size: 34 x 34

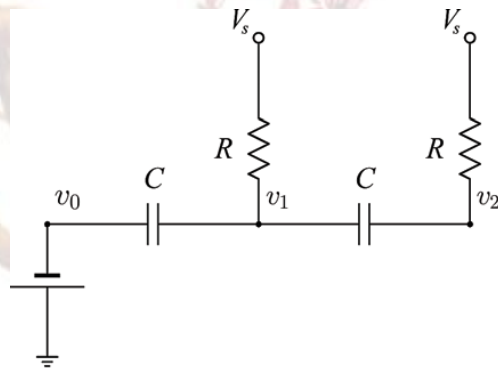
$V_{\text{strike}} = 76.3 \pm 0.8 \text{ V}$ ,  $V_{\text{extinction}} = 61.4 \pm 0.6 \text{ V}$

$R = 2.2 \text{ M}\Omega$ ,  $C = 220 \text{ nF}$

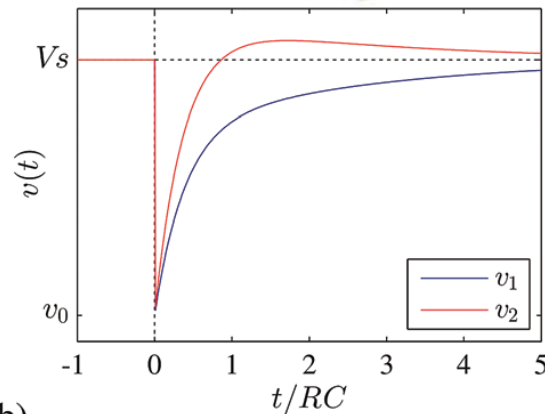
2 CCD cameras, 1 photodiode

# Critical phenomena

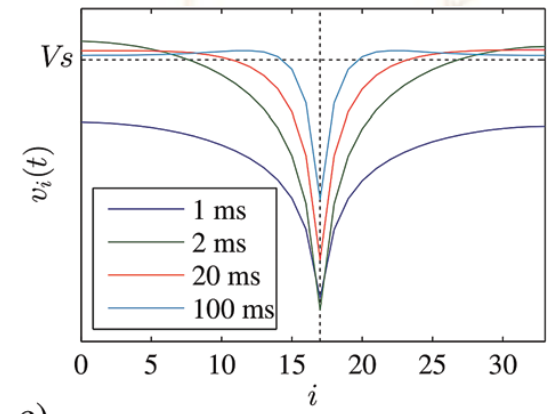
## Activity propagation



a)



b)



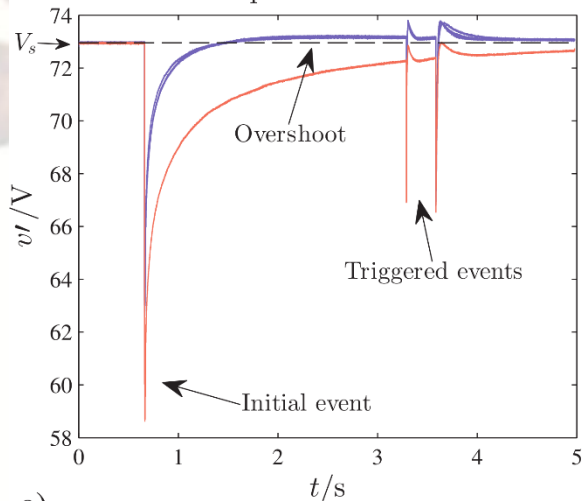
c)

**Despite short-range structural coupling, interactions are effectively long-range!**

# Critical phenomena

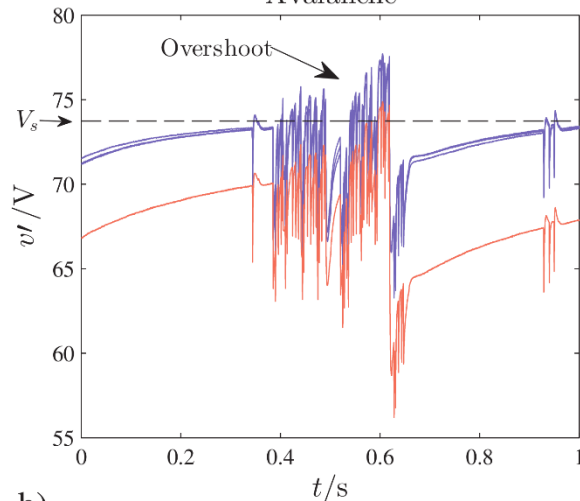
## Activity propagation

Sporadic event



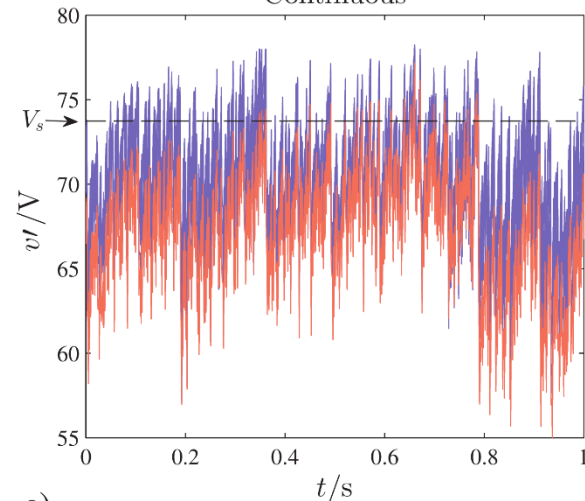
a)

Avalanche



b)

Continuous



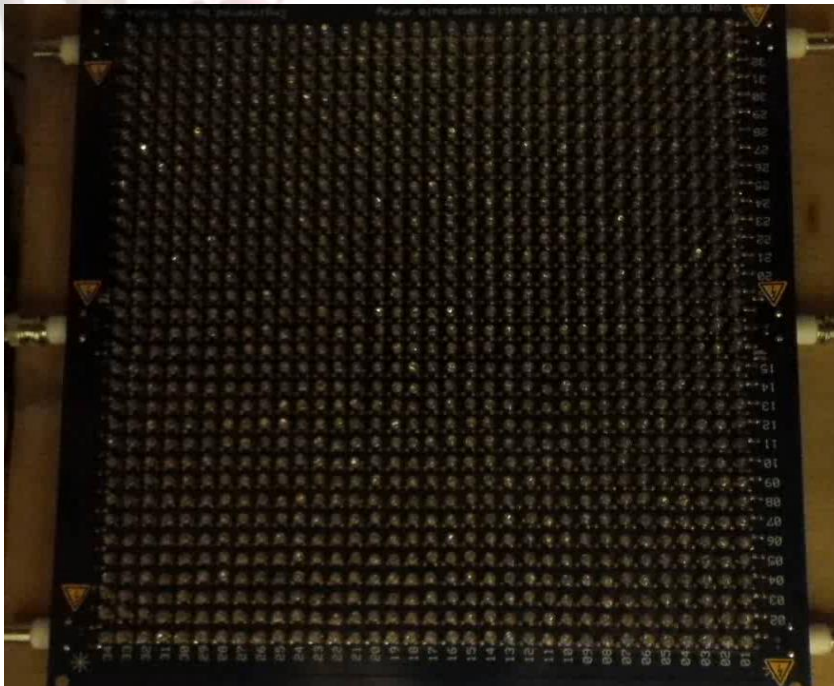
c)

Increasing the supply voltage, activity eventually can propagate and self-sustain.

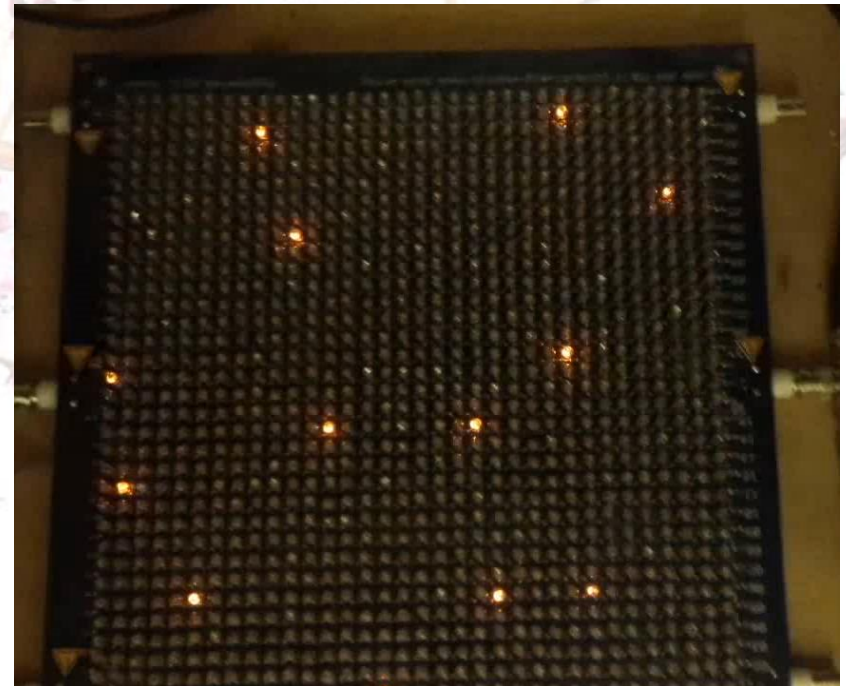
# Critical phenomena

## Phases I and II

Phase I (low rate, disordered, glass-like)



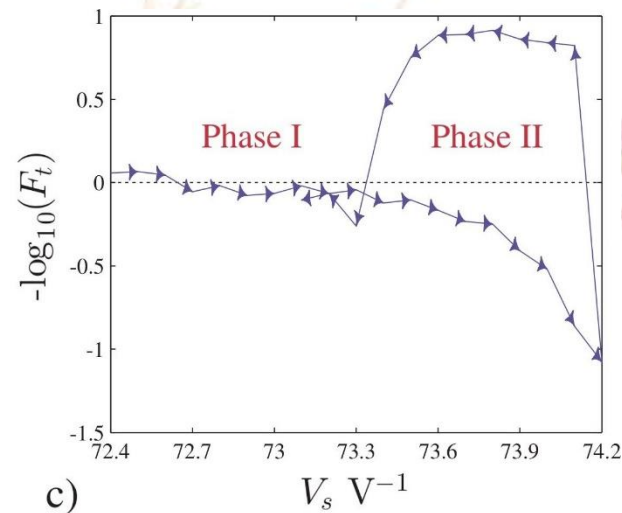
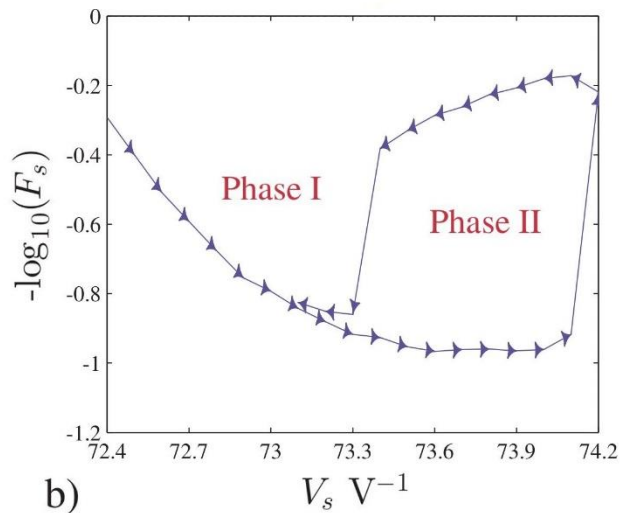
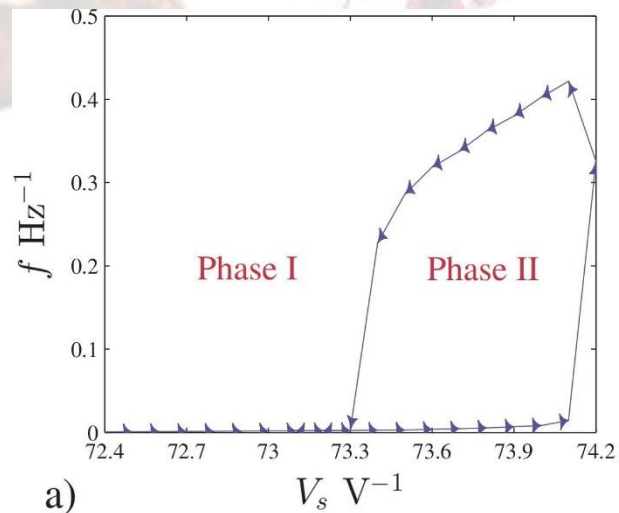
Phase II (high rate, ordered, crystal-like)





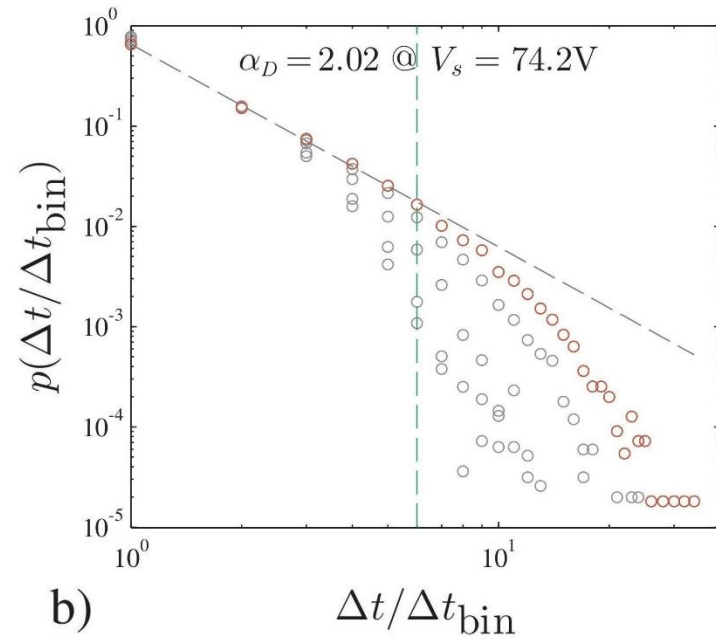
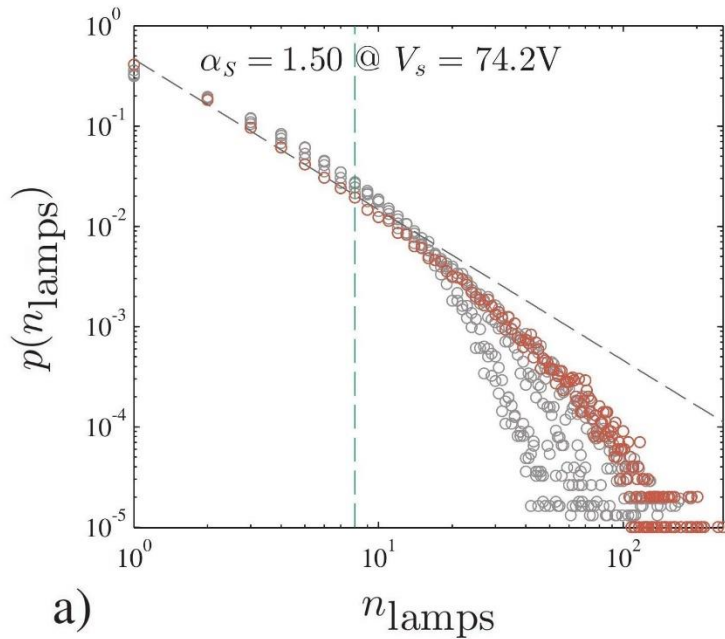
# Critical phenomena

## Transition between phases I and II



# Critical phenomena

## Avalanching







## Conclusions

- 1) Integrated circuit-friendly (area efficient) realization of these circuits is feasible
- 2) Demonstration of application to gait and motor pattern generation. What about sensory processing, or other control functions in homeostasis?
- 3) Possibility to replicate critical phenomena even in elementary configuration (but not self-organized)



# Thank you for your attention

## **References:**

1. Minati L. Experimental Implementation of Networked Chaotic Oscillators Based on Cross-Coupled Inverter Rings in a CMOS Integrated Circuit. *J Circuit Syst Comp* 2015; 24:1550144
2. Minati L, de Candia A, Scarpetta S. Critical phenomena at a first-order phase transition in a lattice of glow lamps: Experimental findings and analogy to neural activity. *Chaos*. 2016; 26(7):073103
3. Minati L, Frasca M, Yoshimura N, Koike Y. Versatile locomotion control of a hexapod robot using a hierarchical network of non-linear oscillator circuits. *IEEE Access* 2018; 99:2799145

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