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Remote synchronization: detailed account of a peculiar pattern-formation mechanism

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Context

A “rewarding” experiment about relationship(s) between structural connectivity and synchronization in an electronic network

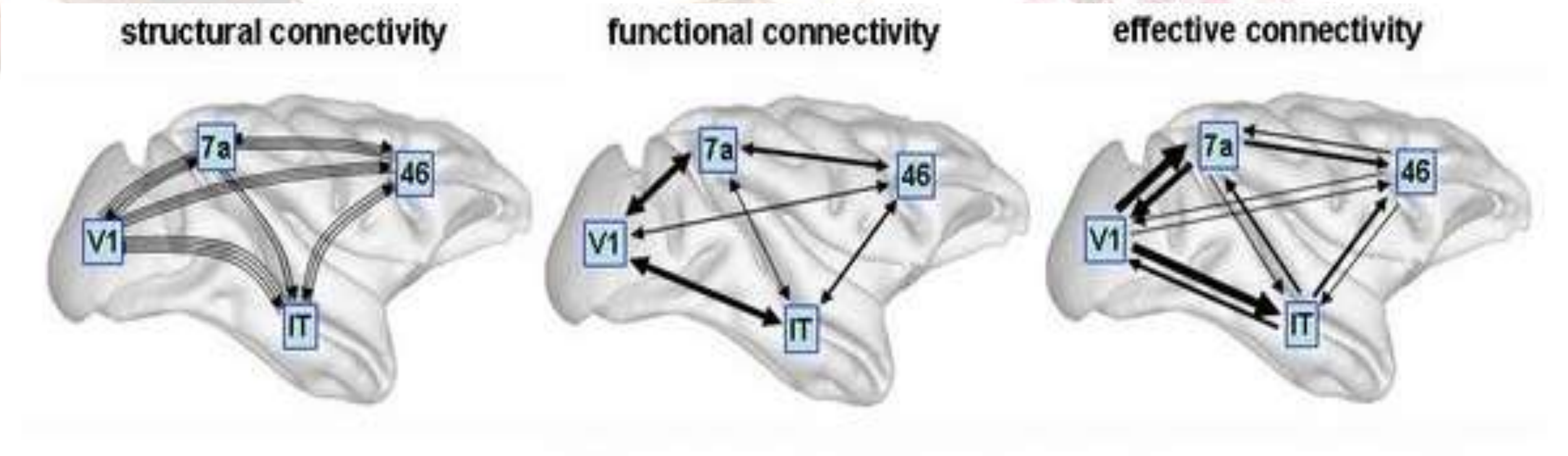
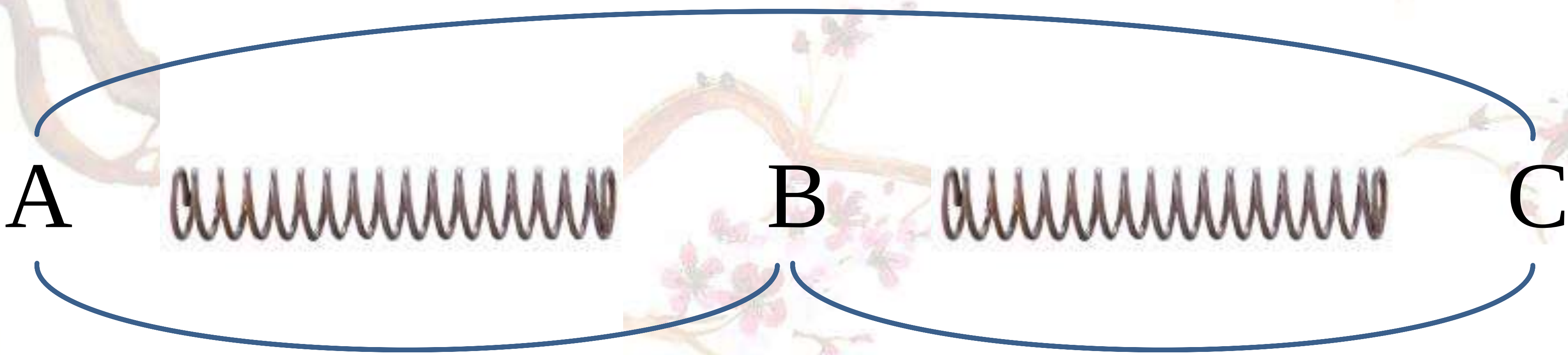


Image credit: O. Sporns, SPM Course

What is remote synchronization?

Synchronised



Non-synchronised

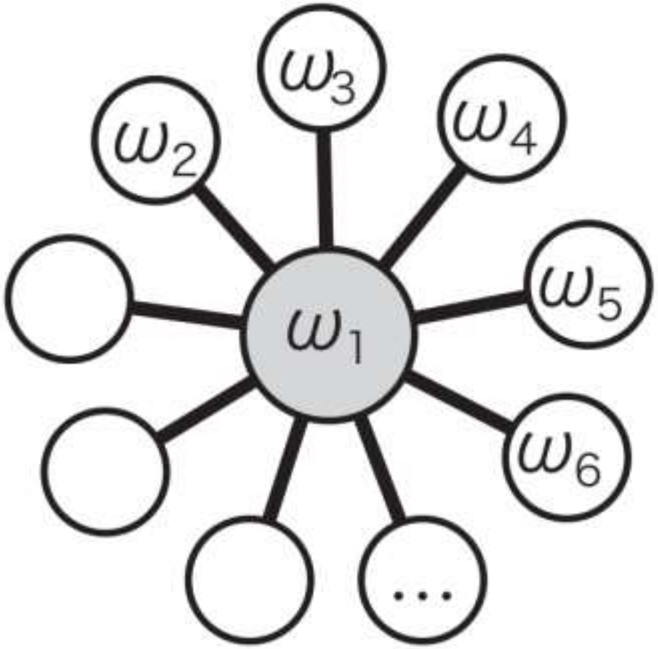
Non-synchronised

Remote synchronization from mismatches

PHYSICAL REVIEW E **85**, 026208 (2012)

Remote synchronization in star networks

A. Bergner,[1,3] M. Frasca,[2] G. Sciuto,[2] A. Buscarino,[2] E. J. Ngamga,[3] L. Fortuna,[2] and J. Kurths[3,4,5]



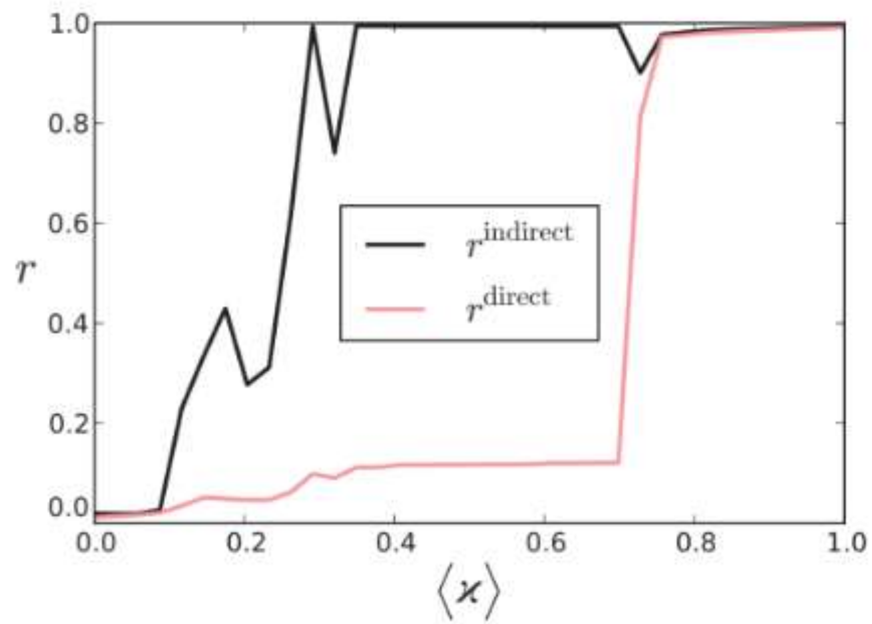
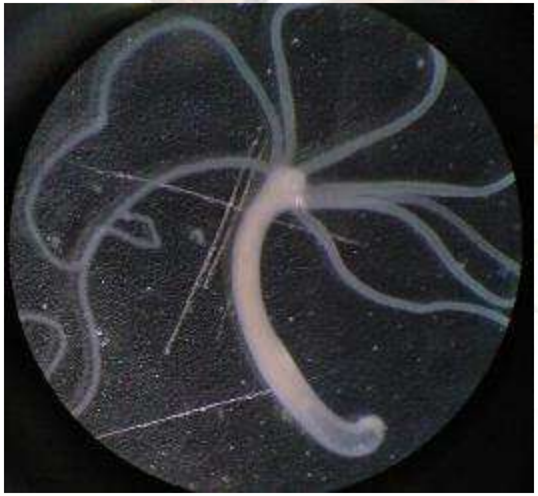


Image credit: cited study, © APS

Remote synchronization as morphogenesis



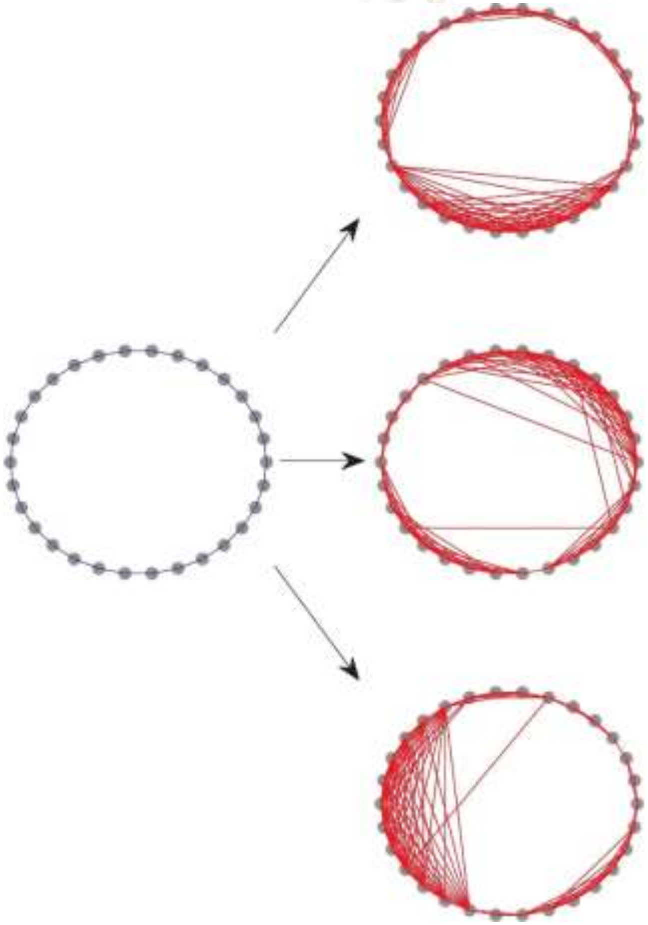
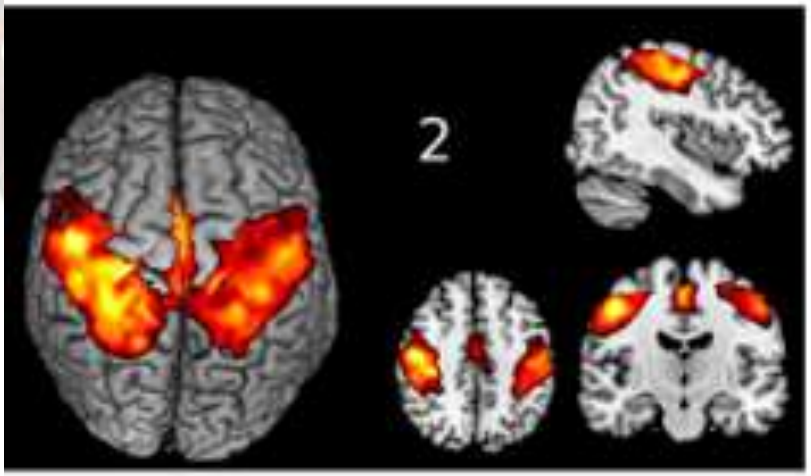


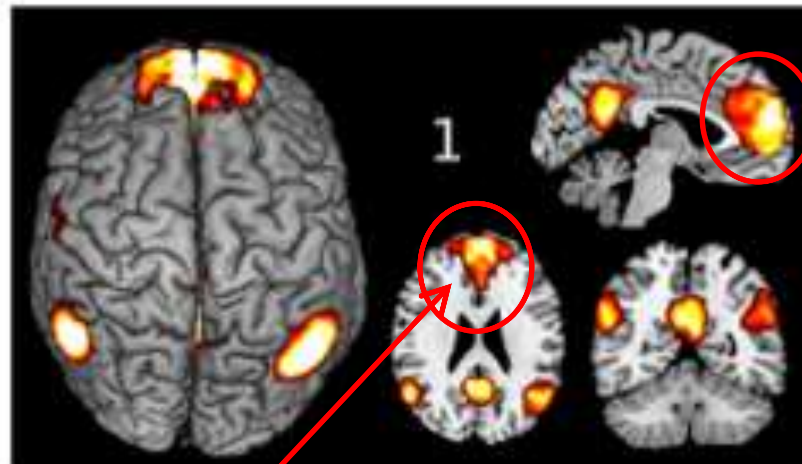
Image credit: Wikipedia, others

Remote synchronization in brain networks?

Sensory-motor network: directly wired

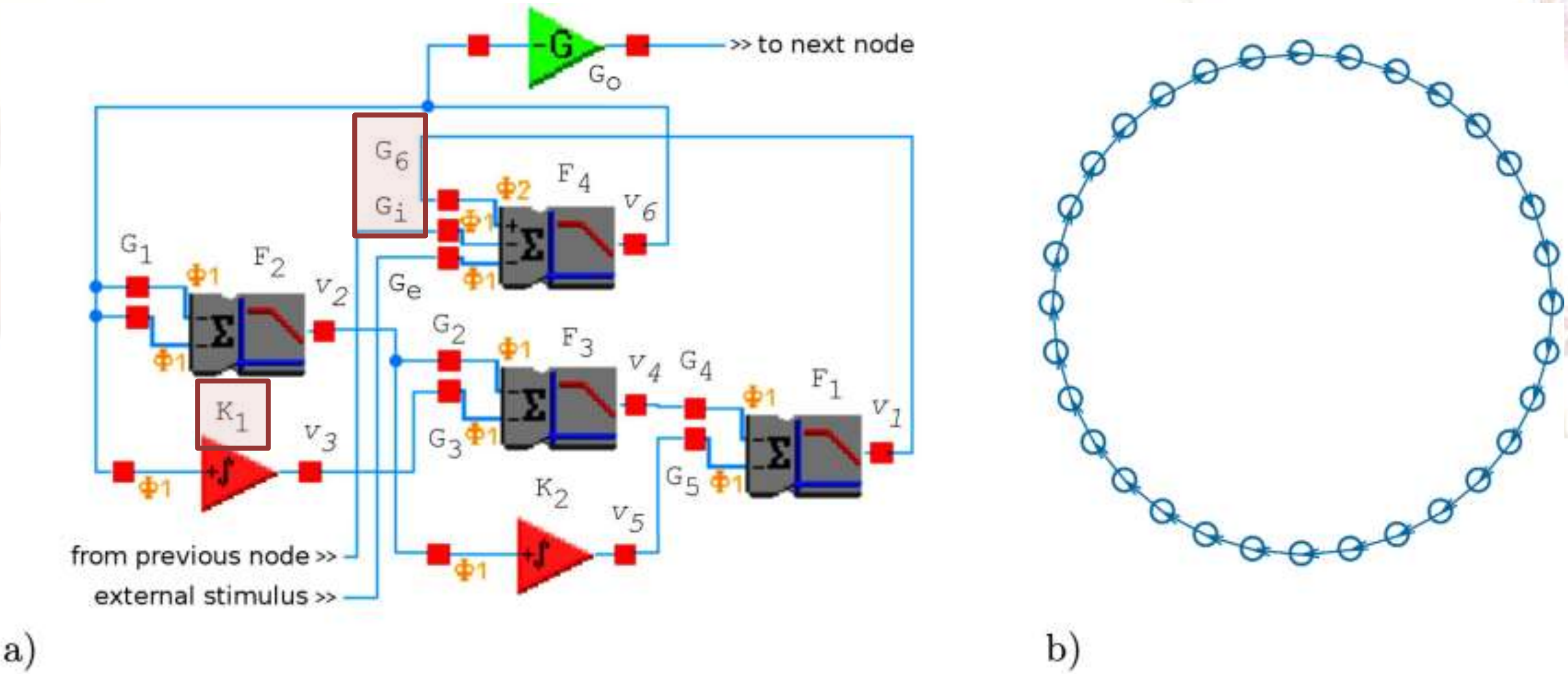


Default-mode network: emergent



No direct anatomical link to posterior areas.
Remotely synchronized?

A simple, reconfigurable non-linear network



a)

b)

Oscillator equations

$$\begin{cases} \dfrac{\mathrm{d}v_1}{\mathrm{d}t} = \Gamma\Big(2\pi F_1(G_4 v_4 + G_5 v_5 - v_1), v_1\Big) \\ \dfrac{\mathrm{d}v_2}{\mathrm{d}t} = \Gamma\Big(2\pi F_2(G_1 v_6 - v_2), v_2\Big) \\ \dfrac{\mathrm{d}v_3}{\mathrm{d}t} = \Gamma\Big(\boxed{K_1} v_6, v_3\Big) \\ \dfrac{\mathrm{d}v_4}{\mathrm{d}t} = \Gamma\Big(2\pi F_3(G_2 v_2 + G_3 v_3 - v_4), v_4\Big) \\ \dfrac{\mathrm{d}v_5}{\mathrm{d}t} = \Gamma\Big(K_2 v_2, v_5\Big) \\ \dfrac{\mathrm{d}v_6}{\mathrm{d}t} = \Gamma\Big(2\pi F_4(\boxed{G_6} v_1 + \boxed{G_\mathrm{i}} v_\mathrm{i} + G_\mathrm{e} v_\mathrm{e} - v_6), v_6\Big) \end{cases}$$

$$\Gamma(x, y) = \mathrm{R}(x)\,\mathrm{H}(V_\mathrm{s} - y) - \mathrm{R}(-x)\,\mathrm{H}(V_\mathrm{s} + y)$$

Parametric mismatch
~0.5% in physical system

Applications in versatile pattern generation

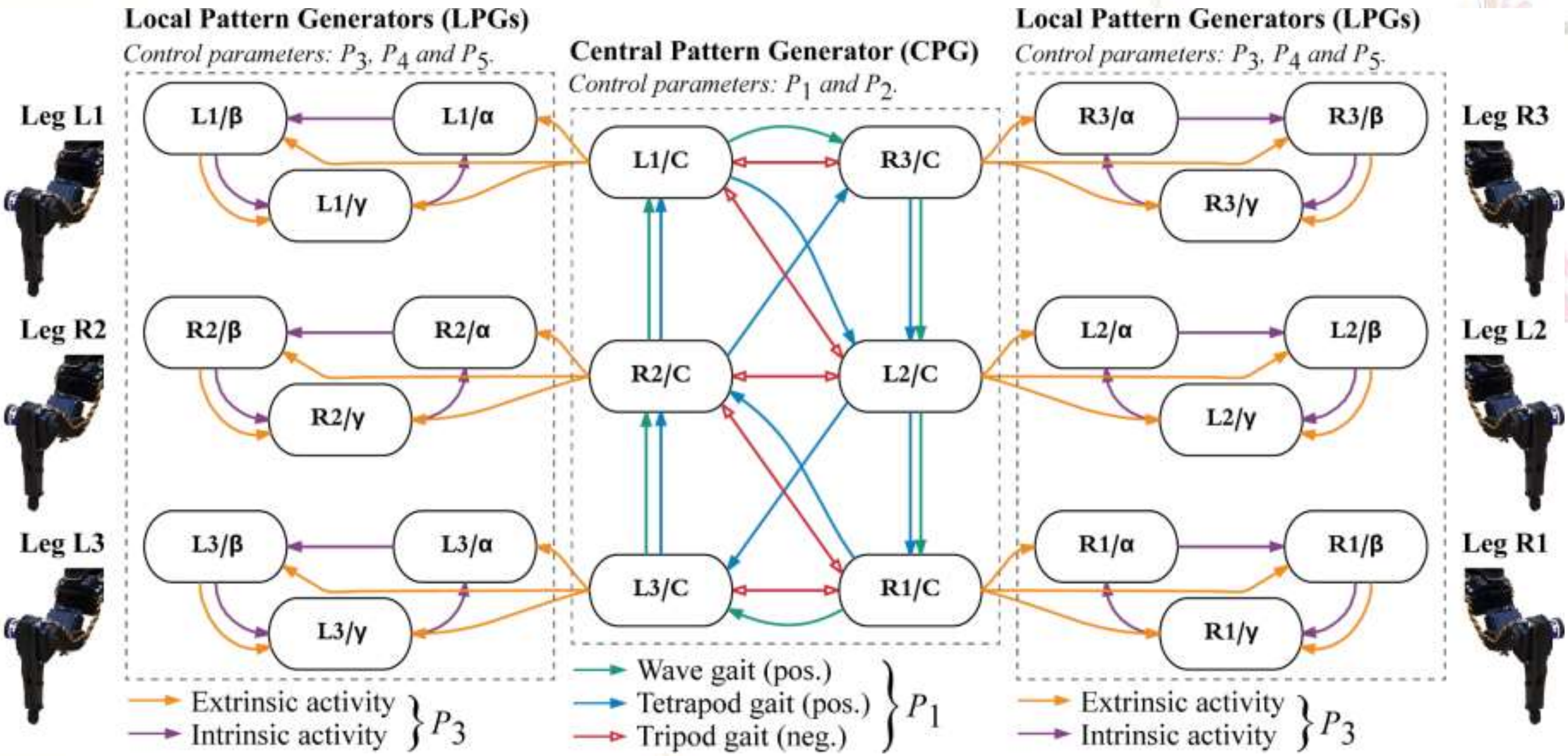


Image source: Minati et al., IEEE Access 2018. © IEEE

Phase vs. amplitude synchronization

Phase coherence $r_{ij} = |\langle e^{i[\varphi_i(t) - \varphi_j(t)]} \rangle|$

Instantaneous amplitude (envelope)

$$v_i(t) + i\hat{v}_i(t) = A_i(t)e^{i\varphi_i(t)}$$

where $\hat{v}_i$ is the Hilbert transform of $v_i(t)$

$$\hat{v}_i(t) = \frac{1}{\pi}\text{p.v.}\left[\int_{-\infty}^{\infty} \frac{v_i(\tau)}{t-\tau} d\tau\right]$$

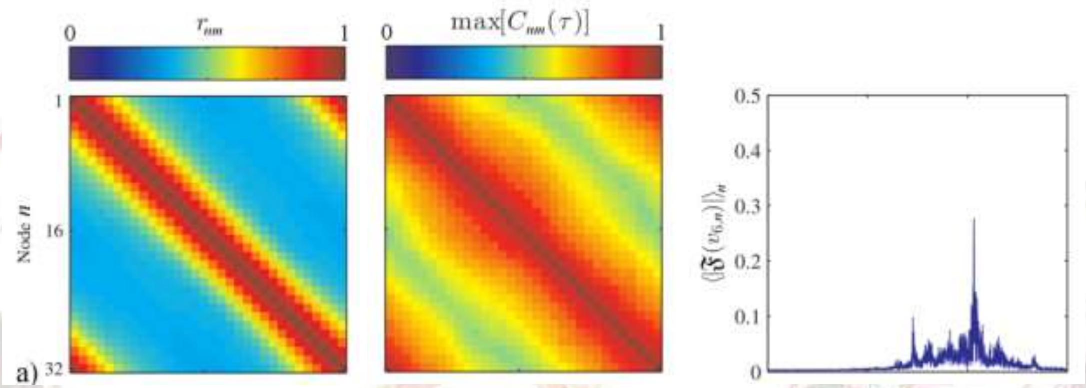
and where p.v. denotes the Cauchy principal value of the integral[18].

Maximum cross-correlation or mutual information

$$C_{XY}(\tau) = \frac{k_{XY}(\tau)}{\sqrt{\sigma_X^2 \sigma_Y^2}} \qquad N_{XY}(d) = \frac{I_{XY}(d)}{\sqrt{H_X H_Y}}$$

Numerical simulations reveal three regimes

a: G_6=0.196, G_7=-1.365

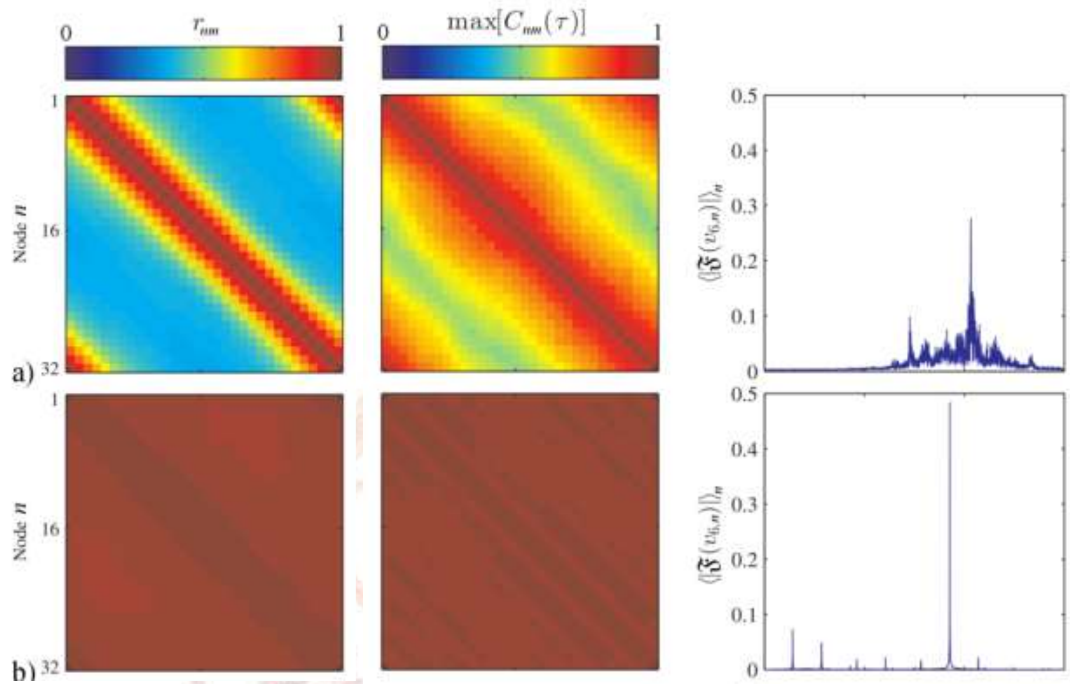


Broadband chaos

Numerical simulations reveal three regimes

a: G_6=0.196, G_7=-1.365

b: G_6=0.096, G_7=-1.53



Broadband chaos

Quasi-periodicity

Image source: Minati et al., CHAOS 2015. © AIP

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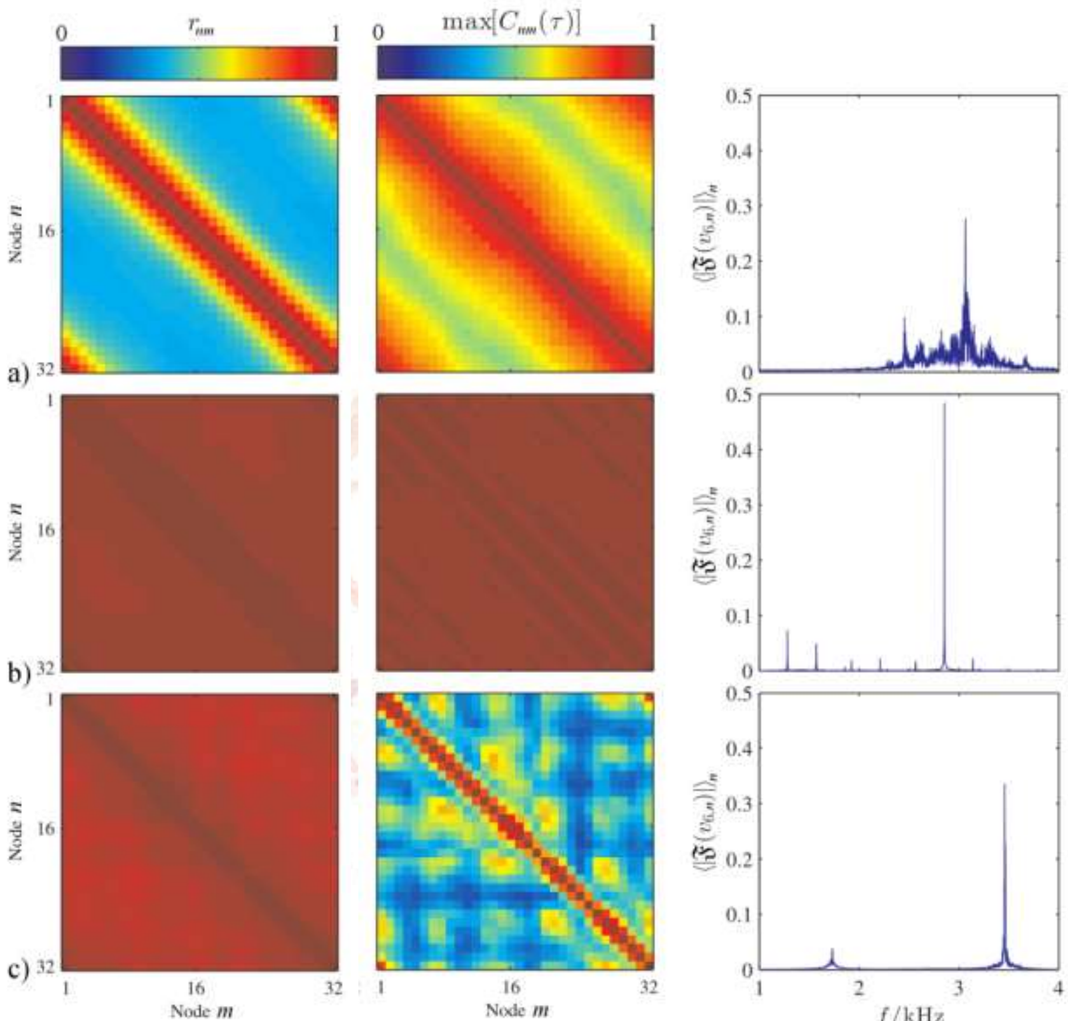
Numerical simulations reveal three regimes

a: G_6=0.196, G_7=-1.365

b: G_6=0.096, G_7=-1.53

c: G_6=0.188, G_7=-1.14



Broadband chaos

Quasi-periodicity

Narrowband chaos

Image source: Minati et al., CHAOS 2015. © AIP

Numerical simulations reveal three regimes

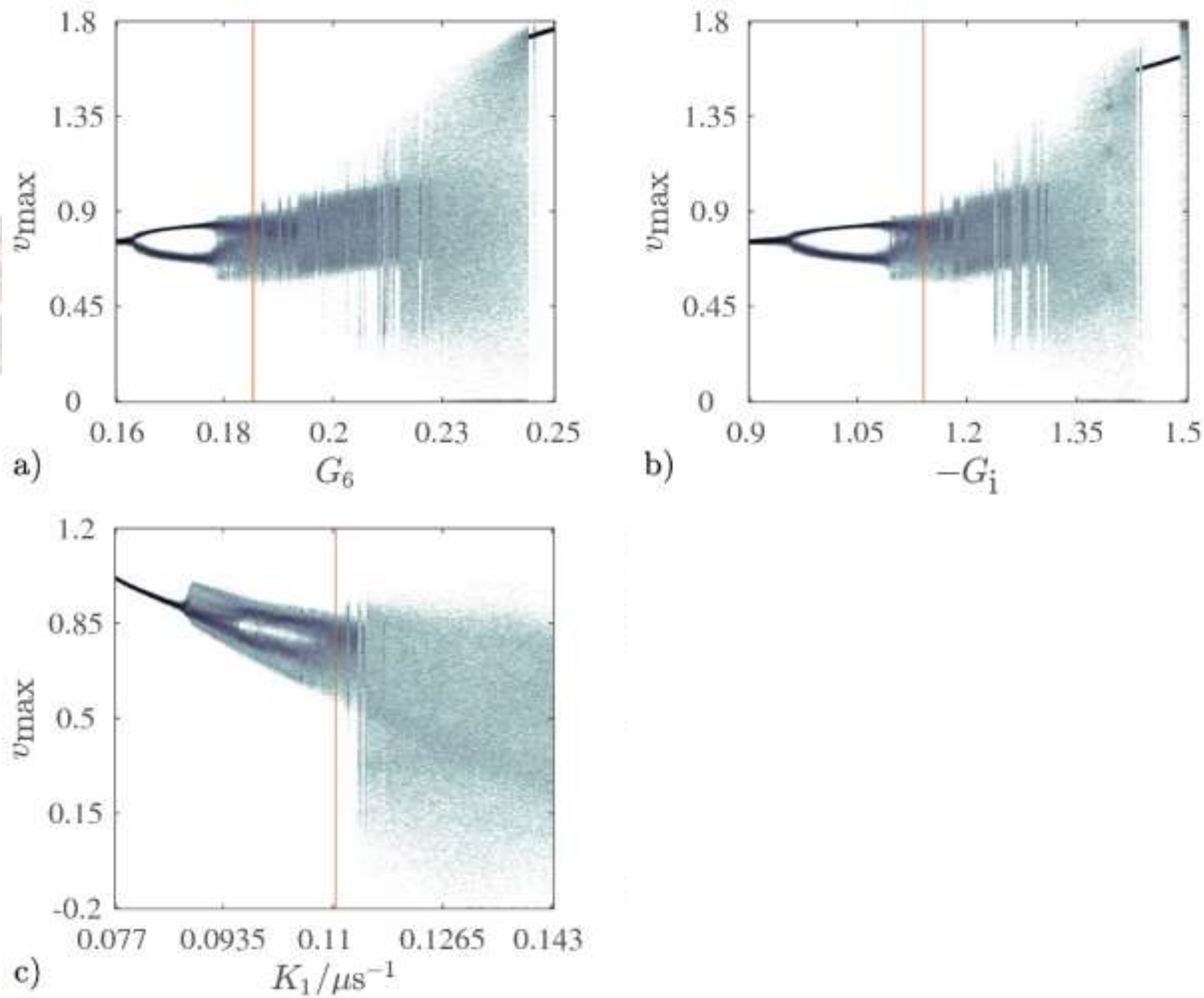
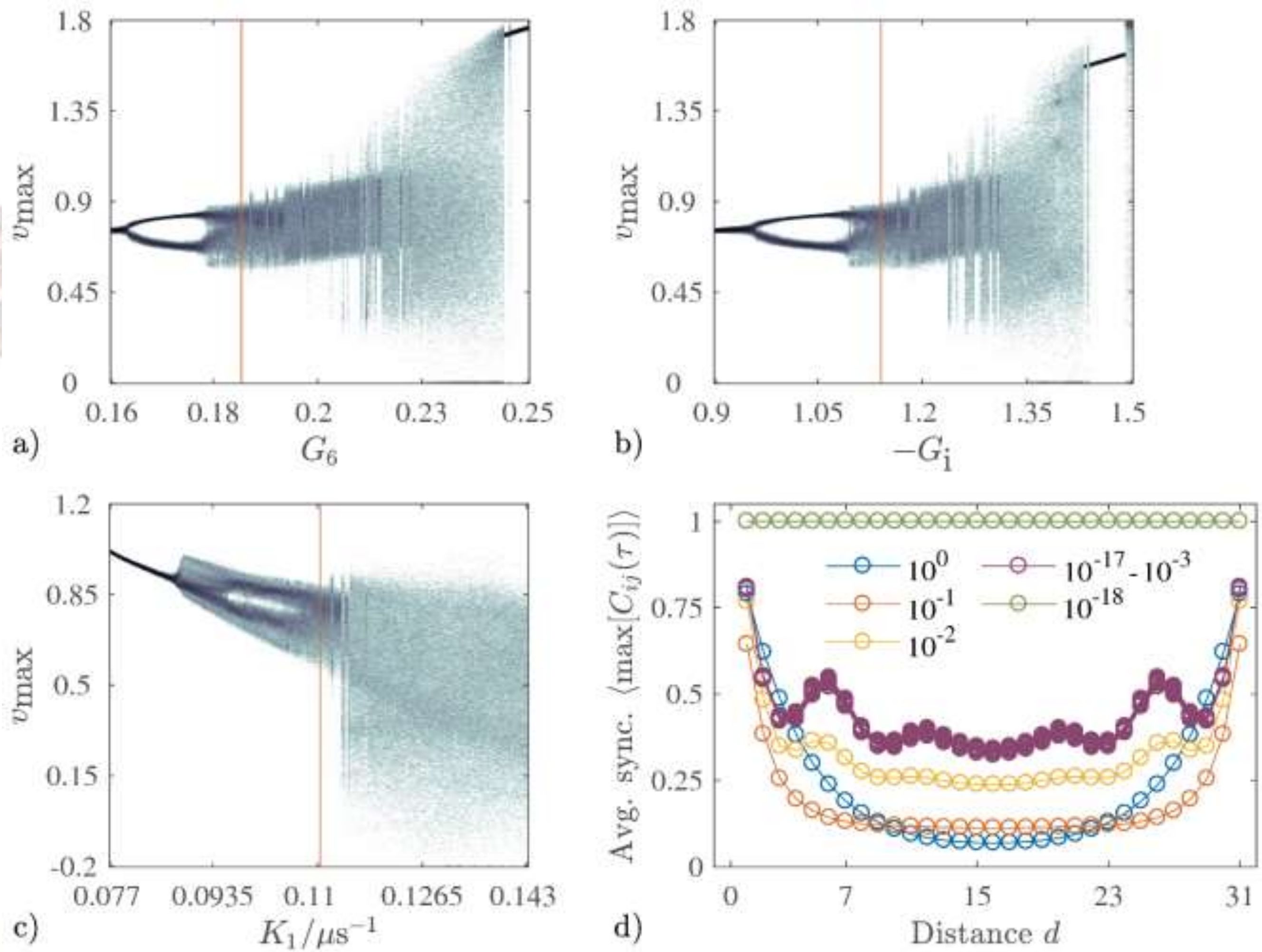


Image source: Minati et al., CHAOS 2018. © AIP

Numerical simulations reveal three regimes



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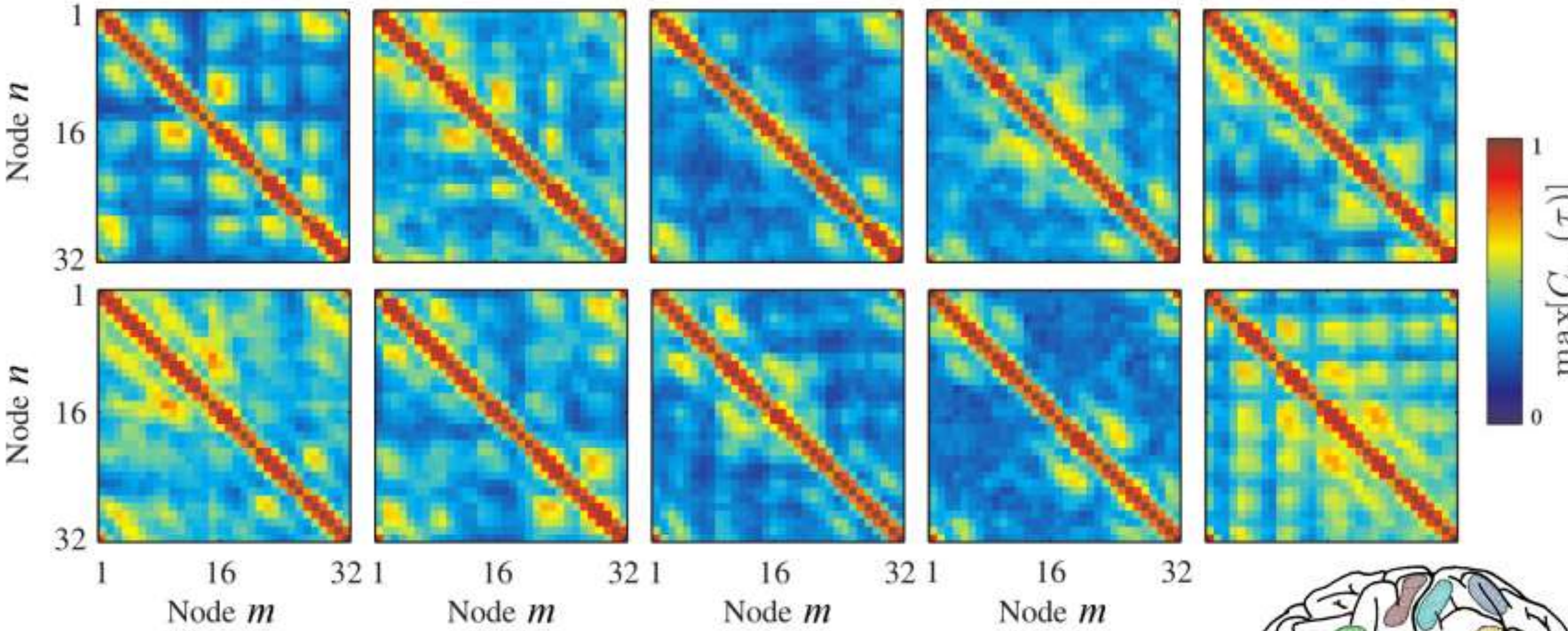
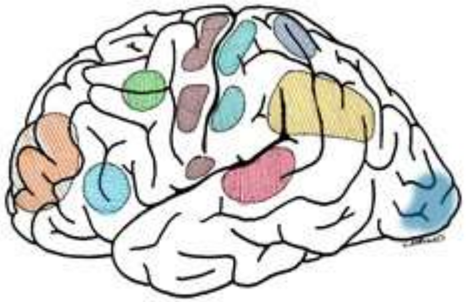
Effect of parametric mismatches





Experimental implementation

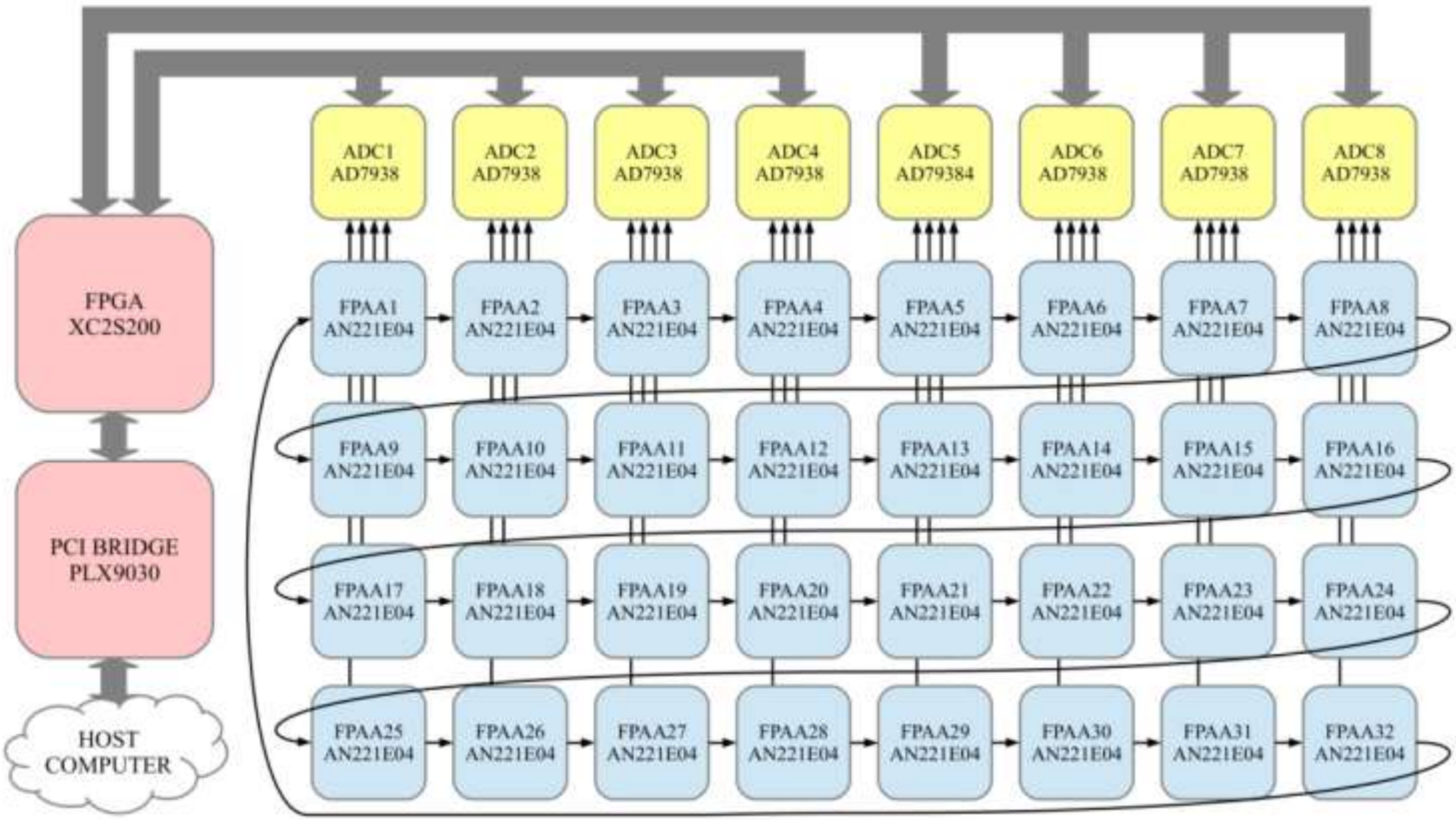
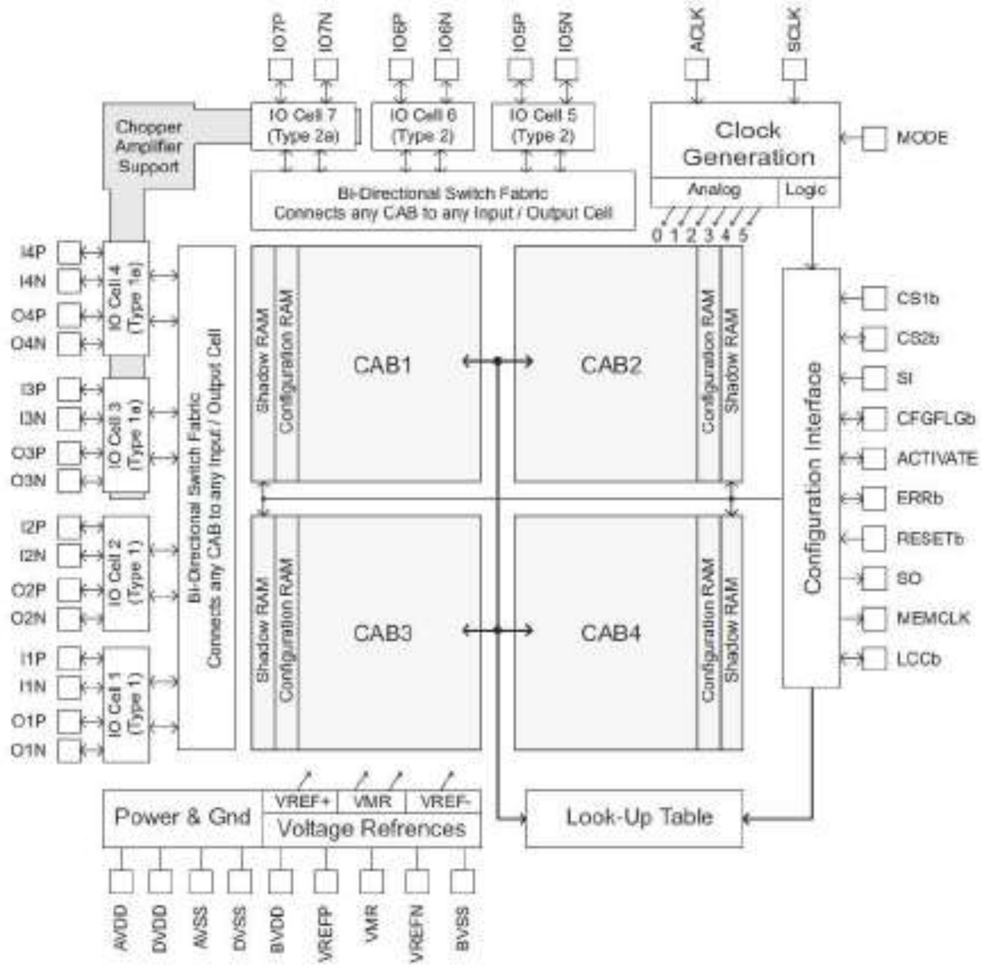


Image source: Minati et al., CHAOS 2015. © AIP

Experimental implementation

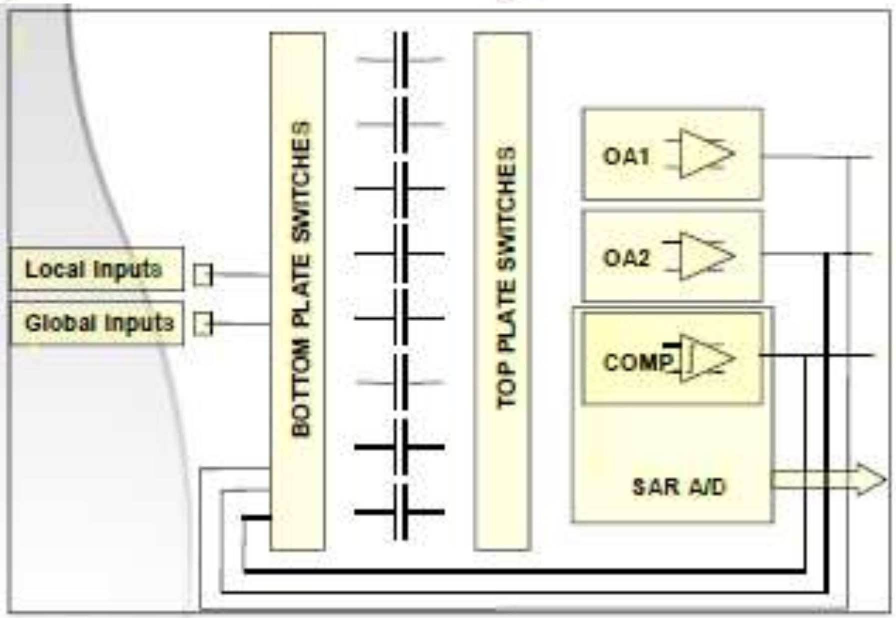
Top-level FPAA architecture



Upon permission and courtesy of Anadigm, Inc.

Experimental implementation

The Configurable Analog Module (CAM)



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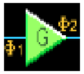
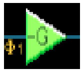
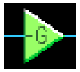
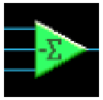
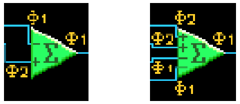
Experimental implementation

- **GainHalf** 
  - Half-cycle
- **GainHold** 
  - Inverting only
- **GainInv** 
  - Continuous Time
- **SumInv** 
  - Up to three inputs
- **SumDiff (SumHalf)** 
  - Up to four inputs
  - Add or subtract since input branches can be inverting or non-inverting
- **RectifierFilter** 
  - Full Wave/Half Wave
  - Inverting/non-inverting
- **RectifierHalf** 
  - Full Wave/Half Wave
  - Inverting/non-inverting
- **RectifierHold** 
  - Half Wave Inverting only

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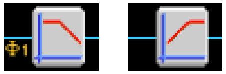
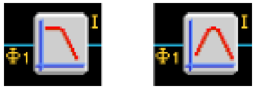
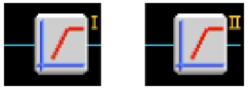
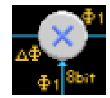
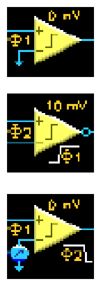
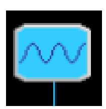
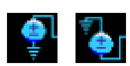
Experimental implementation

- **FilterBilinear – One pole** 
  - Low Pass/High Pass/All Pass
- **FilterBiquad – Two poles** 
  - Low Pass/High Pass/Band Pass/Band Stop
  - Automatically chooses from multiple circuit topologies 
- **Differentiator** 
  - Output voltage slews – see documentation
- **Integrator** 
  - Optional reset 

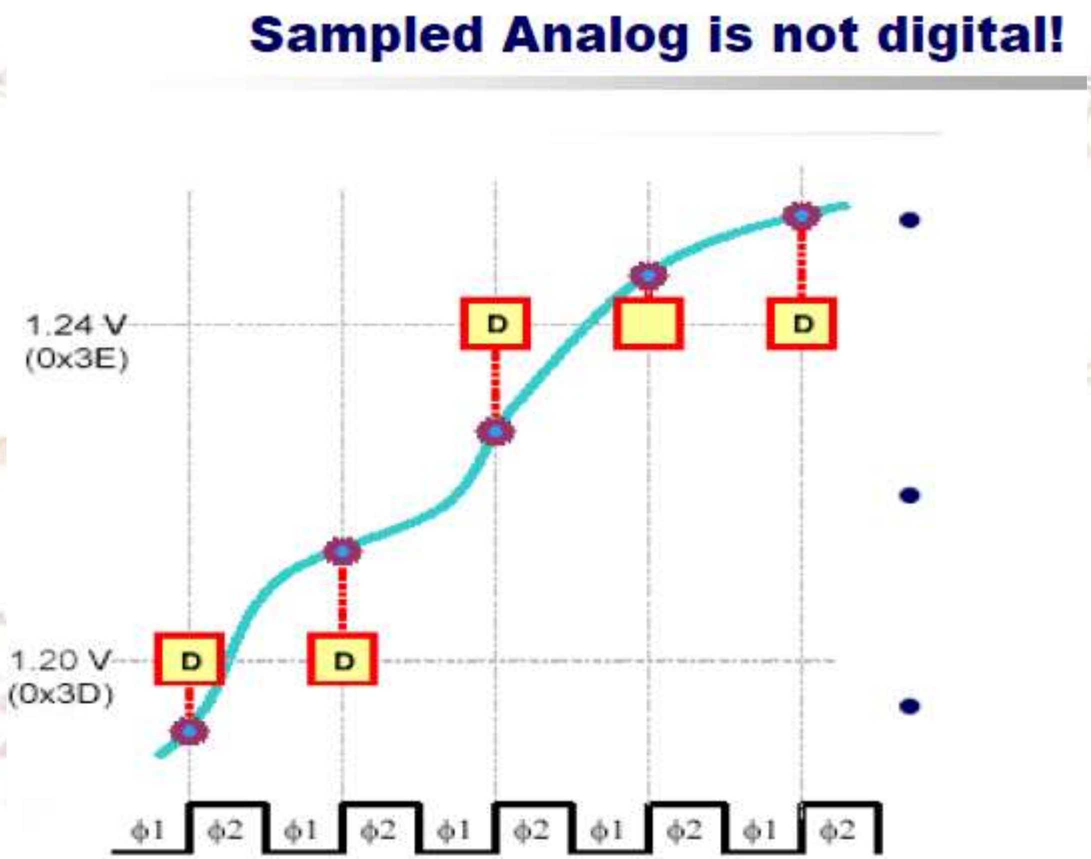
Experimental implementation

- **Multiplier** 
  - Uses SAR (Input Y is quantized)
  - Subject to internal reference voltage error
  - Optional sample and hold on input X to equalize sampling time of two inputs (uses chip resources)

- **Comparator** 
  - Single/Dual Input
  - Variable Reference
- **Hold – Sample and hold** 
- **OscillatorSine** 
  - Subject to internal reference voltage error
- **Voltage (+/- 3 VDC)** 
  - Subject to internal reference voltage error

Experimental implementation

Continuous-value,
discrete-time



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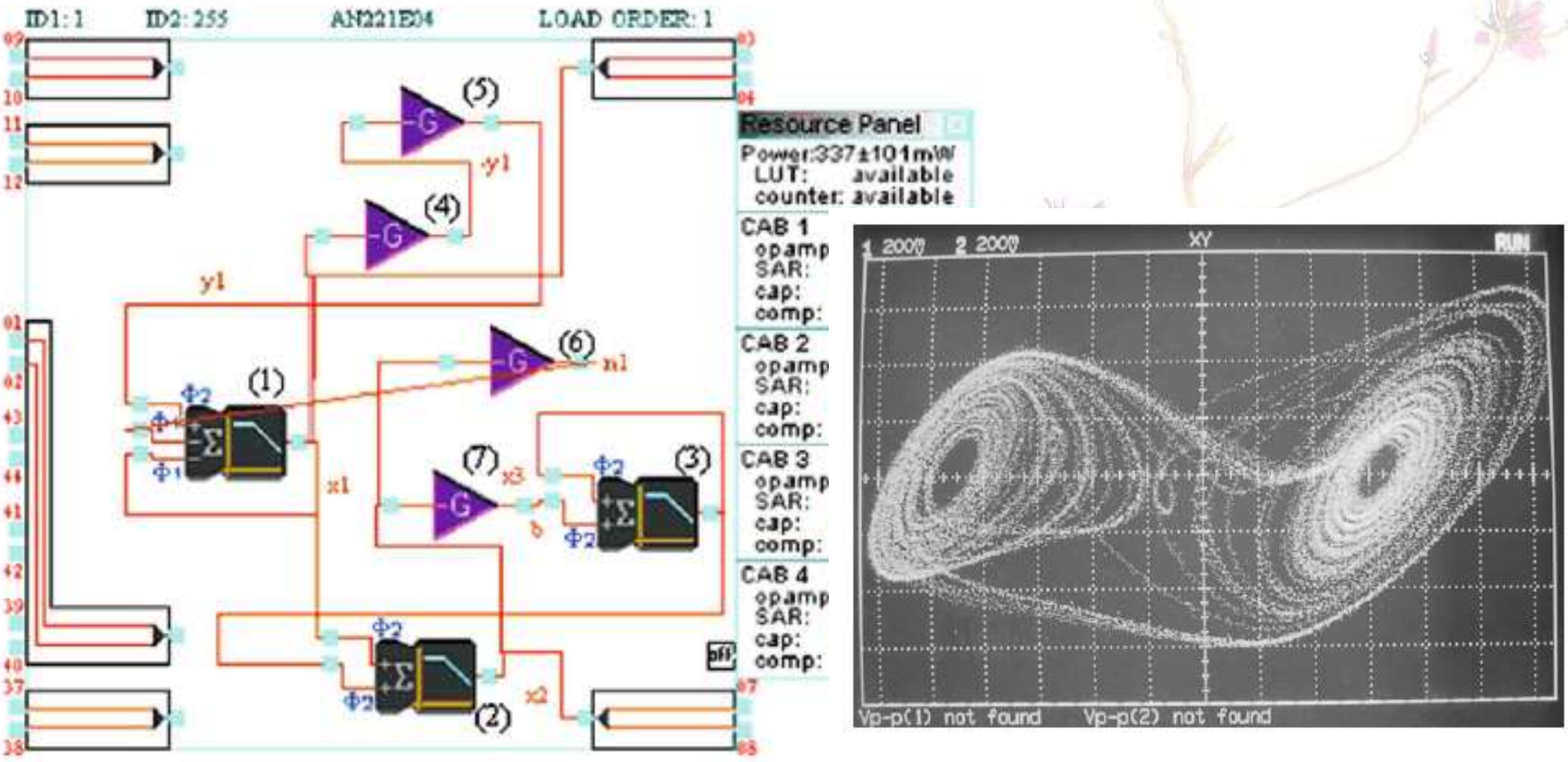
Experimental implementation



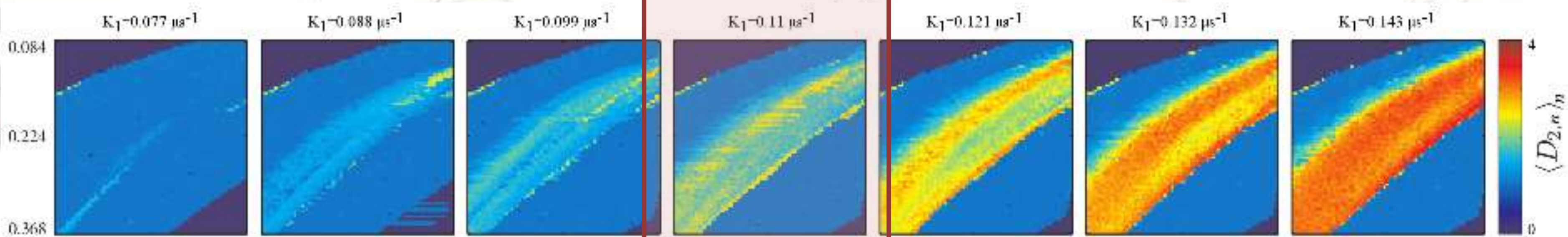
From: A. Buscarino et al., A Concise Guide to Chaotic Electronic Circuits, 73 , SpringerBriefs in Applied Sciences and Technology

Experimental implementation

A soft of "Chimera": an analog plug-in system for digital computer

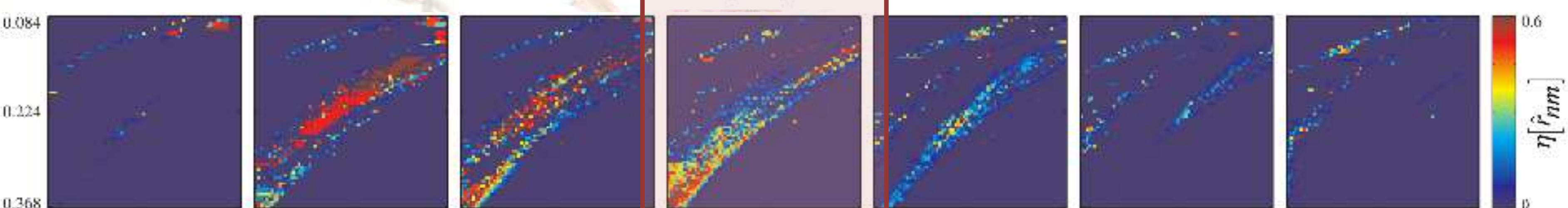
Remote sync. close to quasi-periodicity

Correlation dimension (D_2)



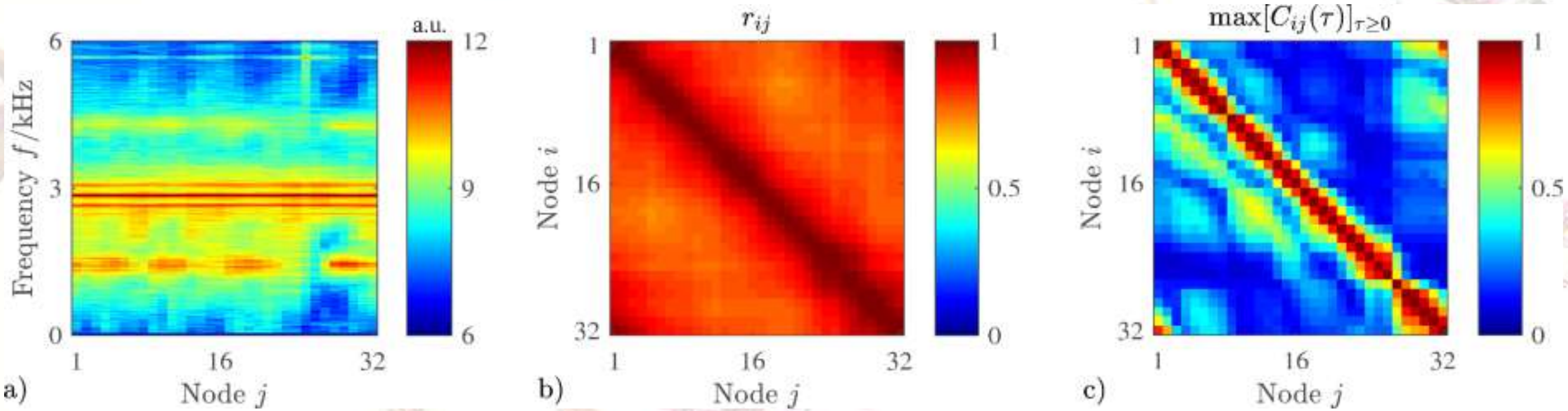
Remote synchronization (η)

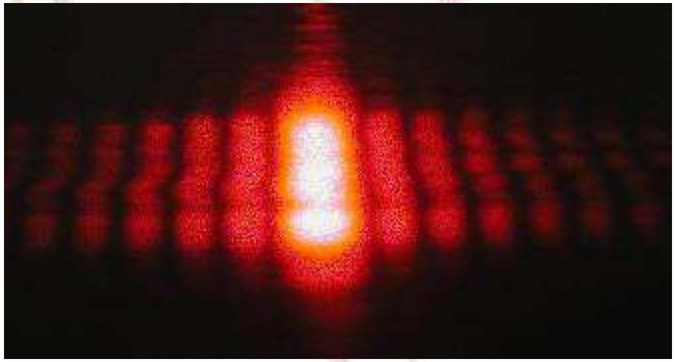
$$\eta\left[r_{nm}\right] = \sum_{n=1}^{32} \sum_{m=1}^{32} \Theta[H(r-r')]_{nm} \Big/ \sum_{n=1}^{32} \sum_{m=1}^{32} H(r_{nm}-r'),$$



Experimental data – basics

Spectrogram | Phase sync. | Amplitude sync.





Reminiscent of a diffraction pattern?

Image source: Minati et al., CHAOS 2018. © AIP, Wikipedia

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Non-stationarity

Adjacent time-windows…

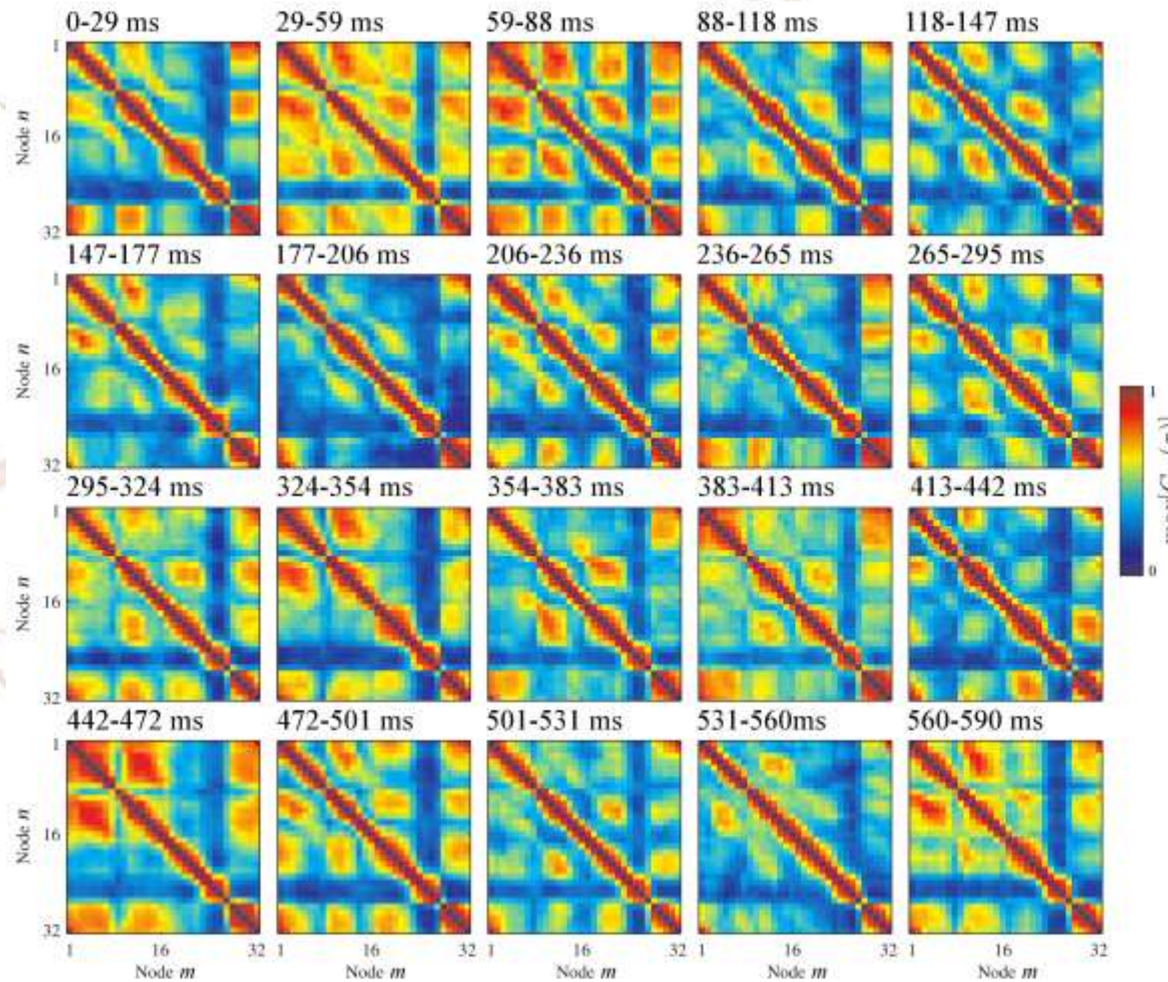
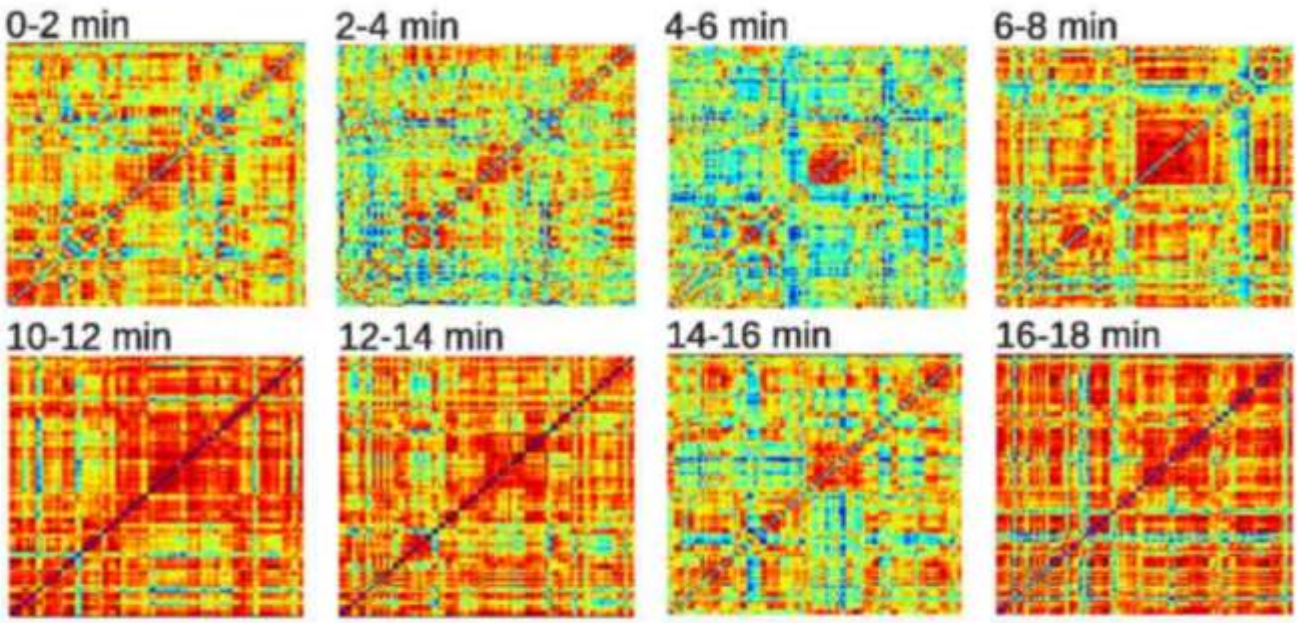


Image source: Minati et al., CHAOS 2015. © AIP

Non-stationarity

…reminiscent of observations in resting-state functional MRI



Source: Tagliazucchi et al., Front. Hum. Neurosci 2012

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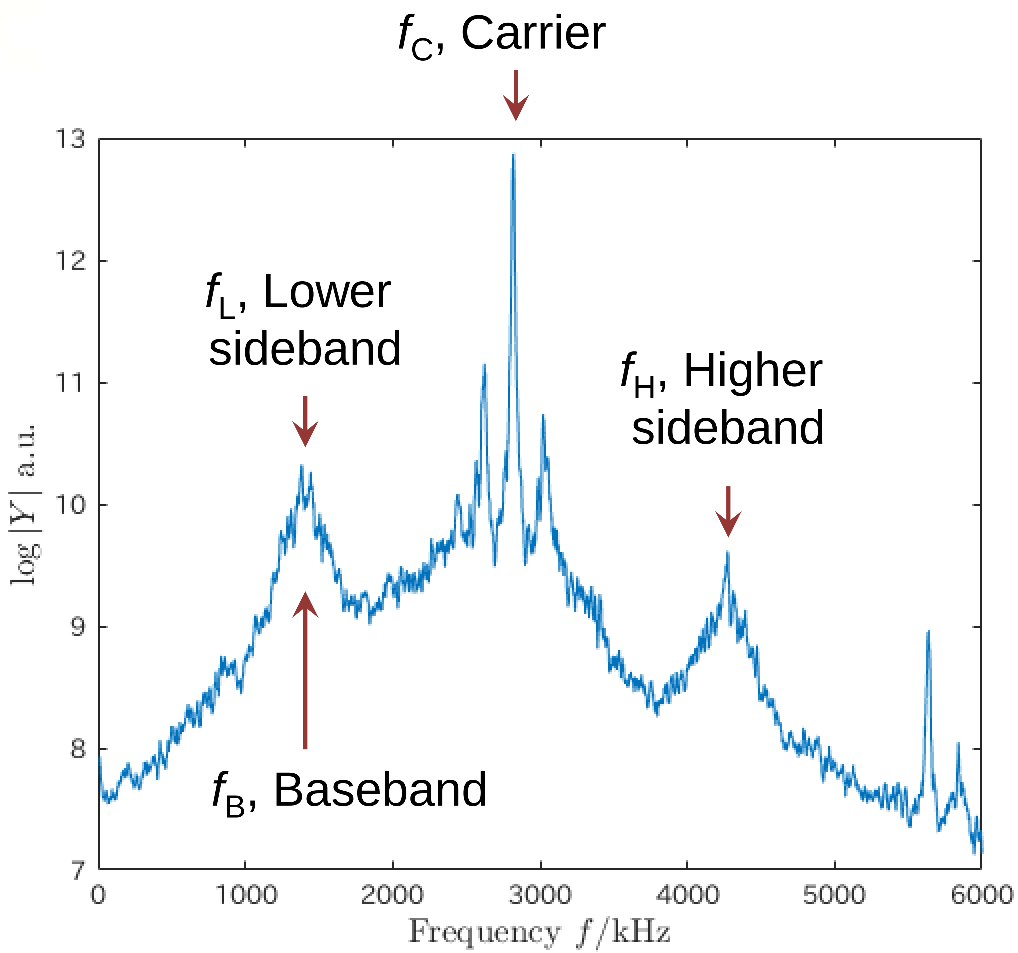
Experimental data – focus on spectrum



Lower sideband and baseband overlap!

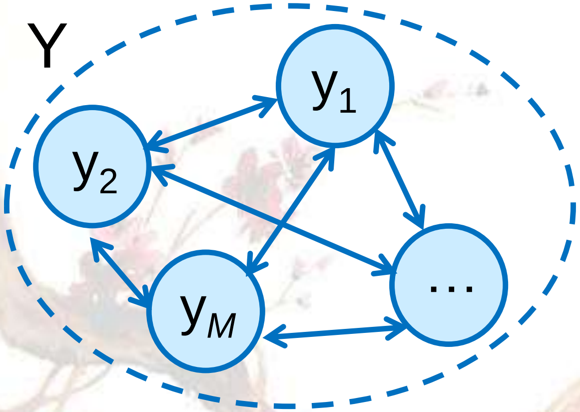
Two concomitant spectral relations:

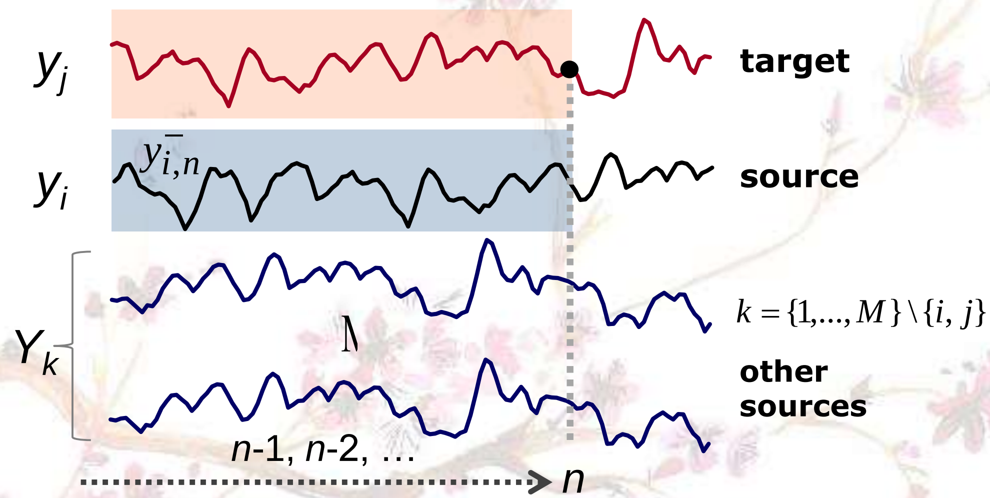
1) $f_B = f_H - f_C = f_C - f_L$

2) $f_H = f_L + f_C$ $\longrightarrow$ $\mathbf{f_L = f_B}$

Reminiscent of classic AM modulation!

From synchronization to causality





- Regression of the present of the target on its own past:

$$e_{j|j,n} = y_{j,n} - \mathrm{E}[y_{j,n} \mid y^-_{j,n}] \quad \Rightarrow \quad \lambda_{j|j} = \mathrm{E}[e^2_{j|j,n}]$$

- Regression of the present of the target on its past and the past of the source:

$$e_{j|ji,n} = y_{j,n} - \mathrm{E}[y_{j,n} \mid y^-_{j,n}, y^-_{i,n}] \quad \Rightarrow \quad \lambda_{j|ji} = \mathrm{E}[e^2_{j|ji,n}]$$

Granger causality (GC)

$$F_{i \to j} = \ln \frac{\lambda_{j|j}}{\lambda_{j|ji}}$$

[J.F. Geweke, J. AM. Stat. Assoc. 77, 1982]

Gaussian

Transfer Entropy (TE)

$$T_{i \to j} = \frac{1}{2} \ln \frac{\lambda_{j|j}}{\lambda_{j|ji}}$$

[L. Barnett et al., Phys. Rev. Lett. 103, 2009]

Courtesy: Luca Faes

Mutual information and causality

Mutual info.

Linear Granger

Granger with quadratic+ cross-terms

Transfer Entropy

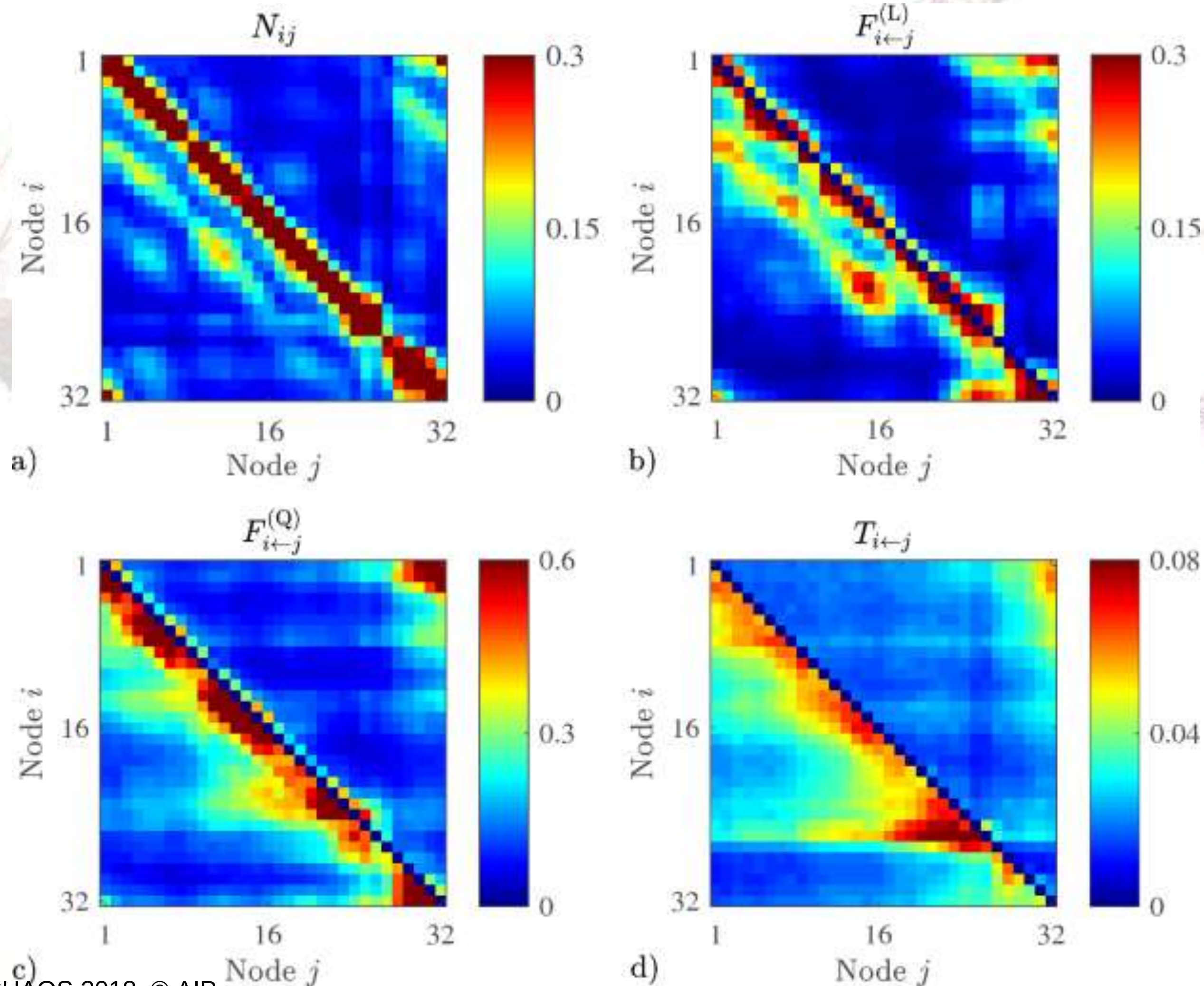
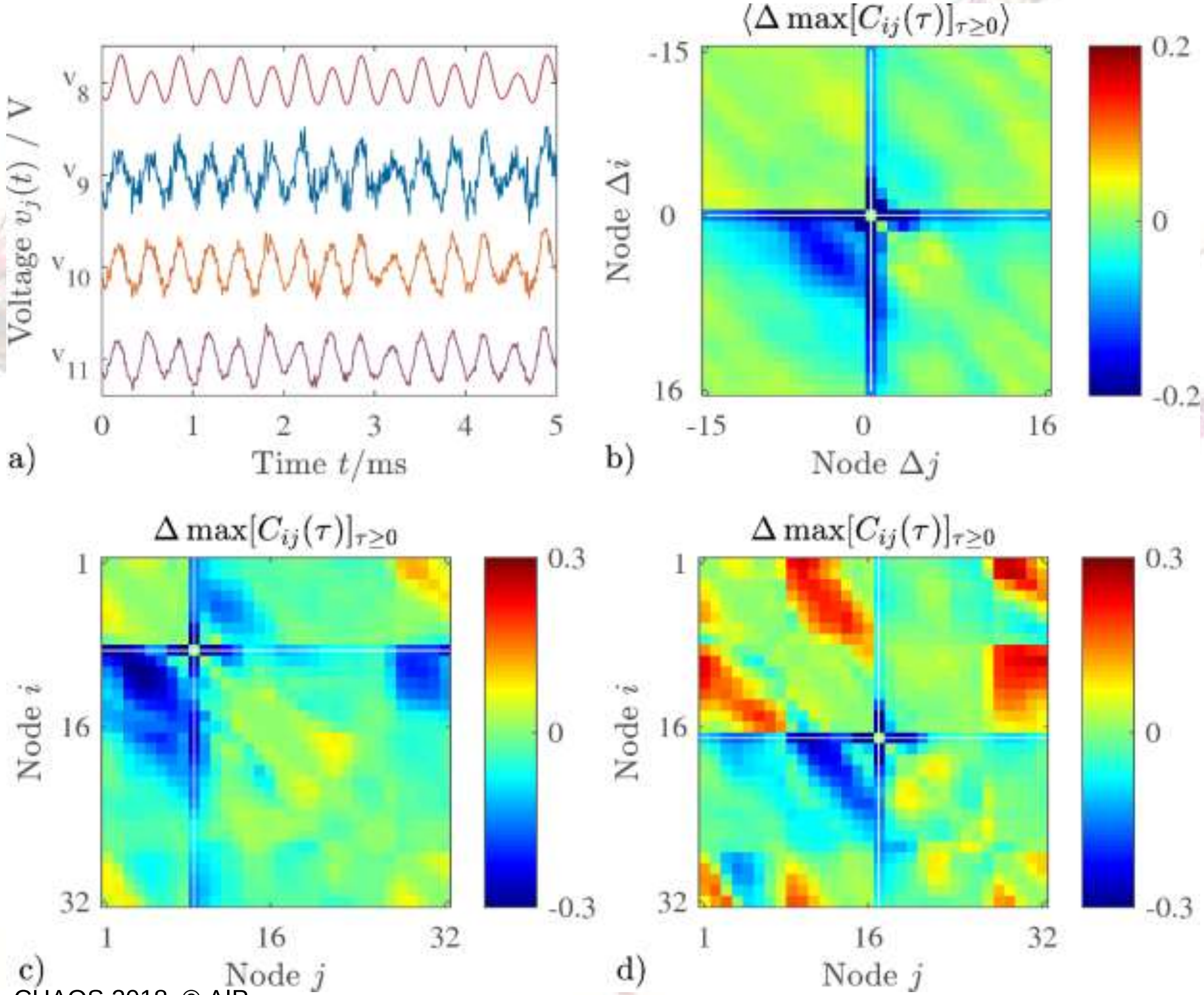


Image source: Minati et al., CHAOS 2018. © AIP

Effect of "lesioning" by noise injection



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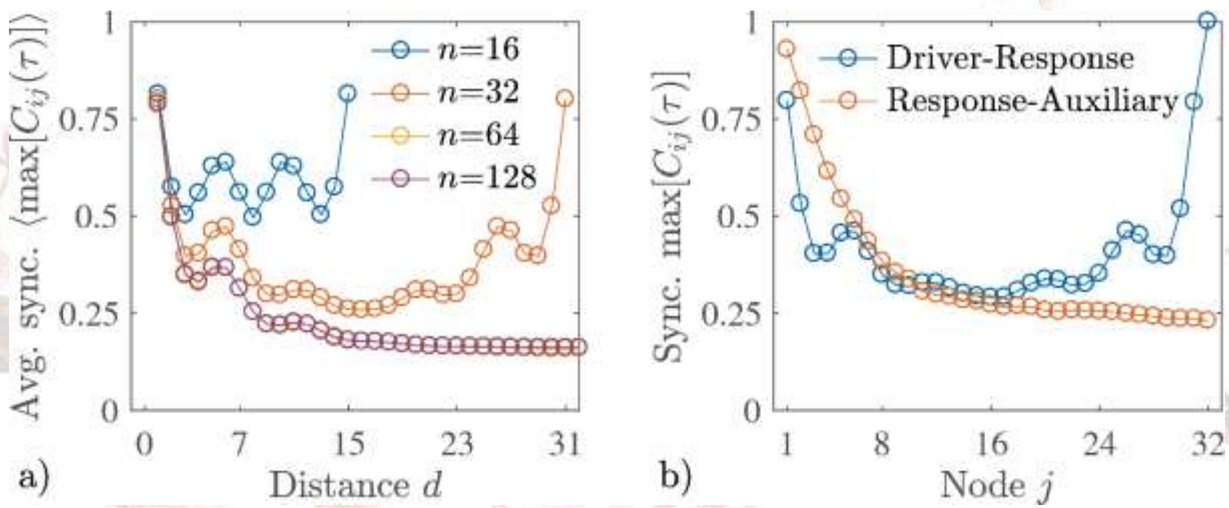
Ring size and auxiliary system simulations



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Propagation of external perturbations

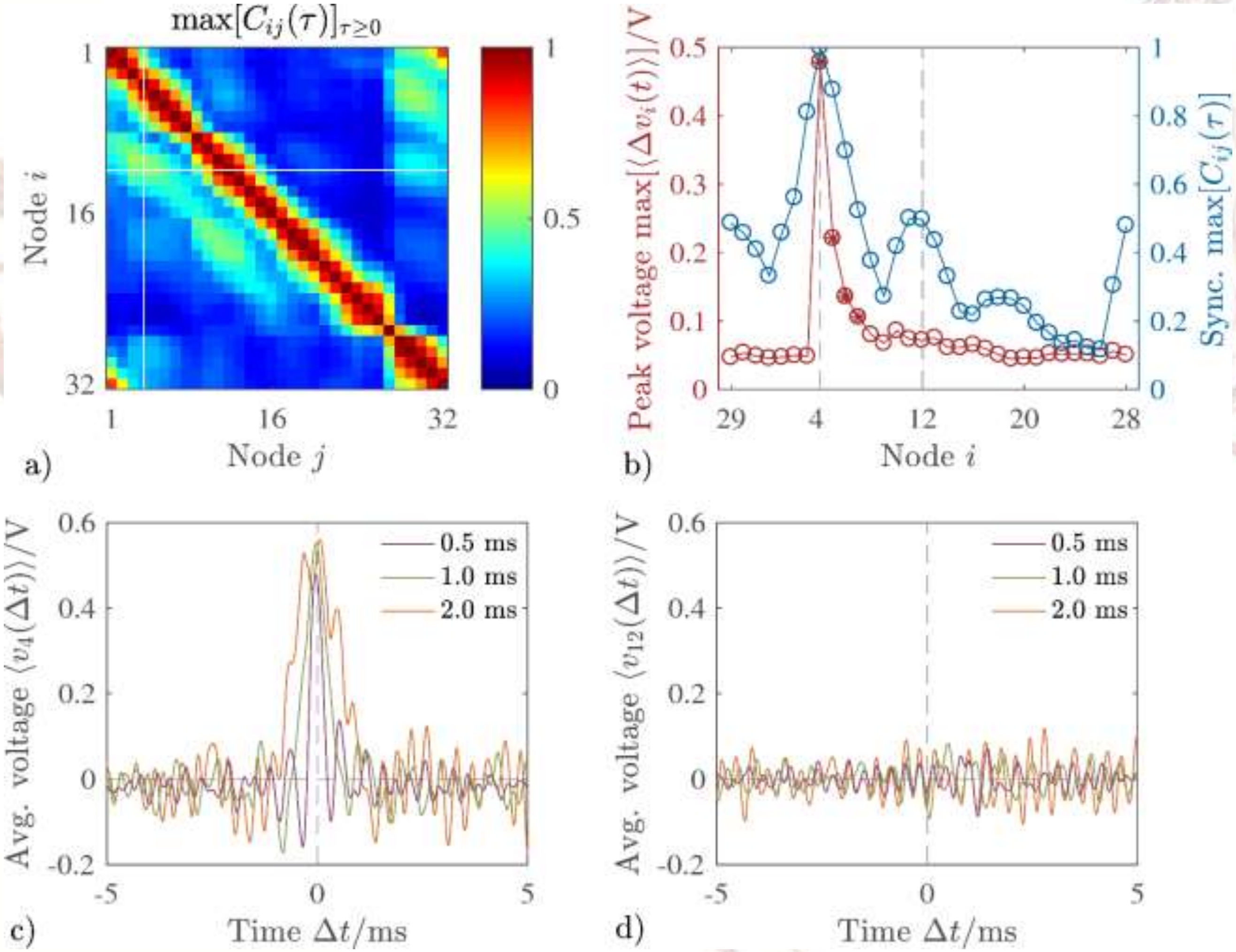


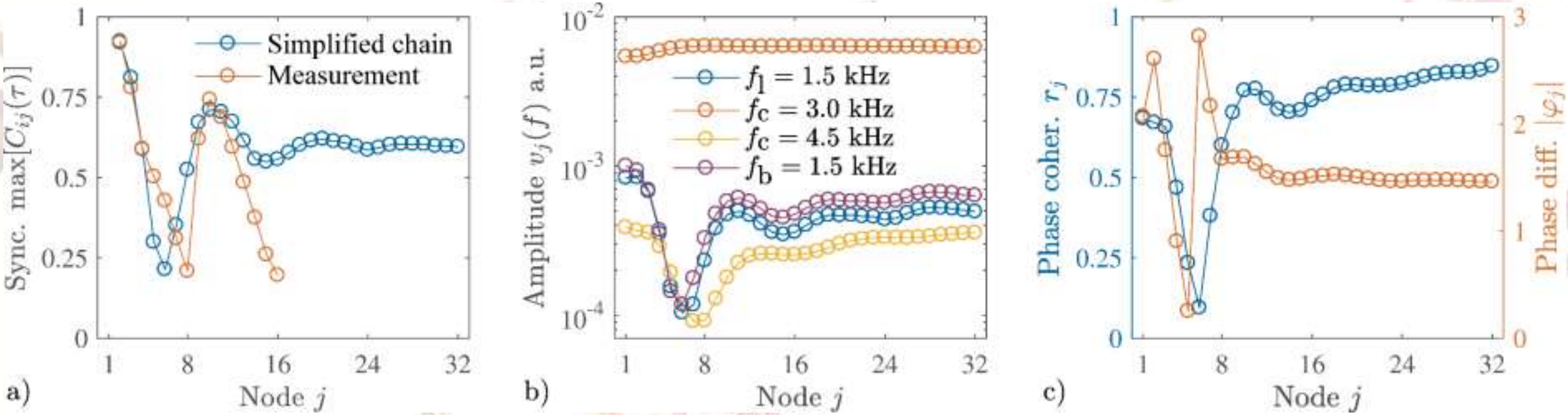
Image source: Minati et al., CHAOS 2018. © AIP, ANT Neuro

Simplified chain model

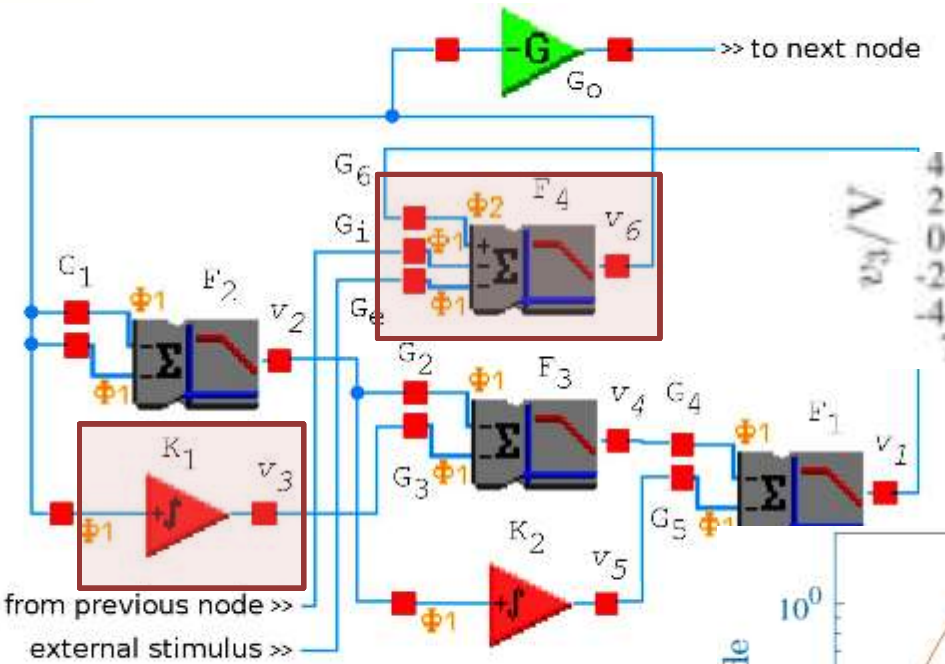
1) An open network is considered in the form of a chain.
2) Two dynamical equations are removed.
3) $\Gamma(x,y)$ is removed for all voltages except v_3.
4) The parameters are set identically across all nodes.

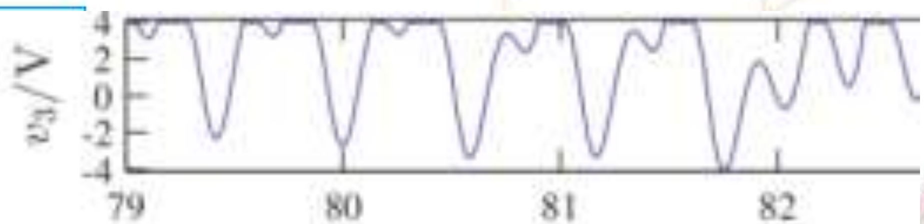
$$\begin{cases} \dfrac{\mathrm{d}v_1}{\mathrm{d}t} = 2\pi F(G_4 v_4 - v_1) \\ \dfrac{\mathrm{d}v_2}{\mathrm{d}t} = 2\pi F(G_1 v_\mathrm{o} - v_2) \\ \dfrac{\mathrm{d}v_3}{\mathrm{d}t} = \Gamma(K v_\mathrm{o}, v_3) \\ \dfrac{\mathrm{d}v_4}{\mathrm{d}t} = 2\pi F(G_2 v_2 + G_3 v_3 - v_4) \end{cases}$$

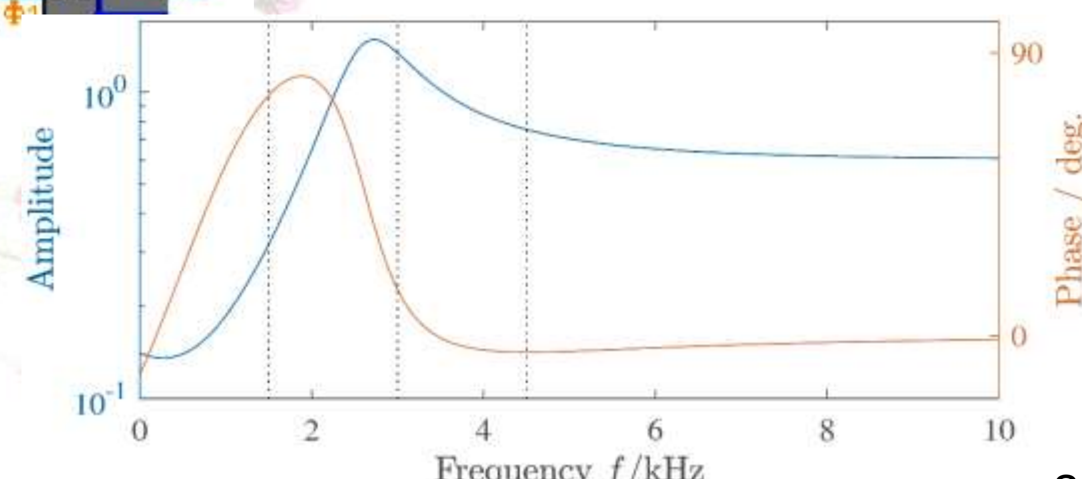
Simplified chain model



Demodulation and interference







Demodulation and interference

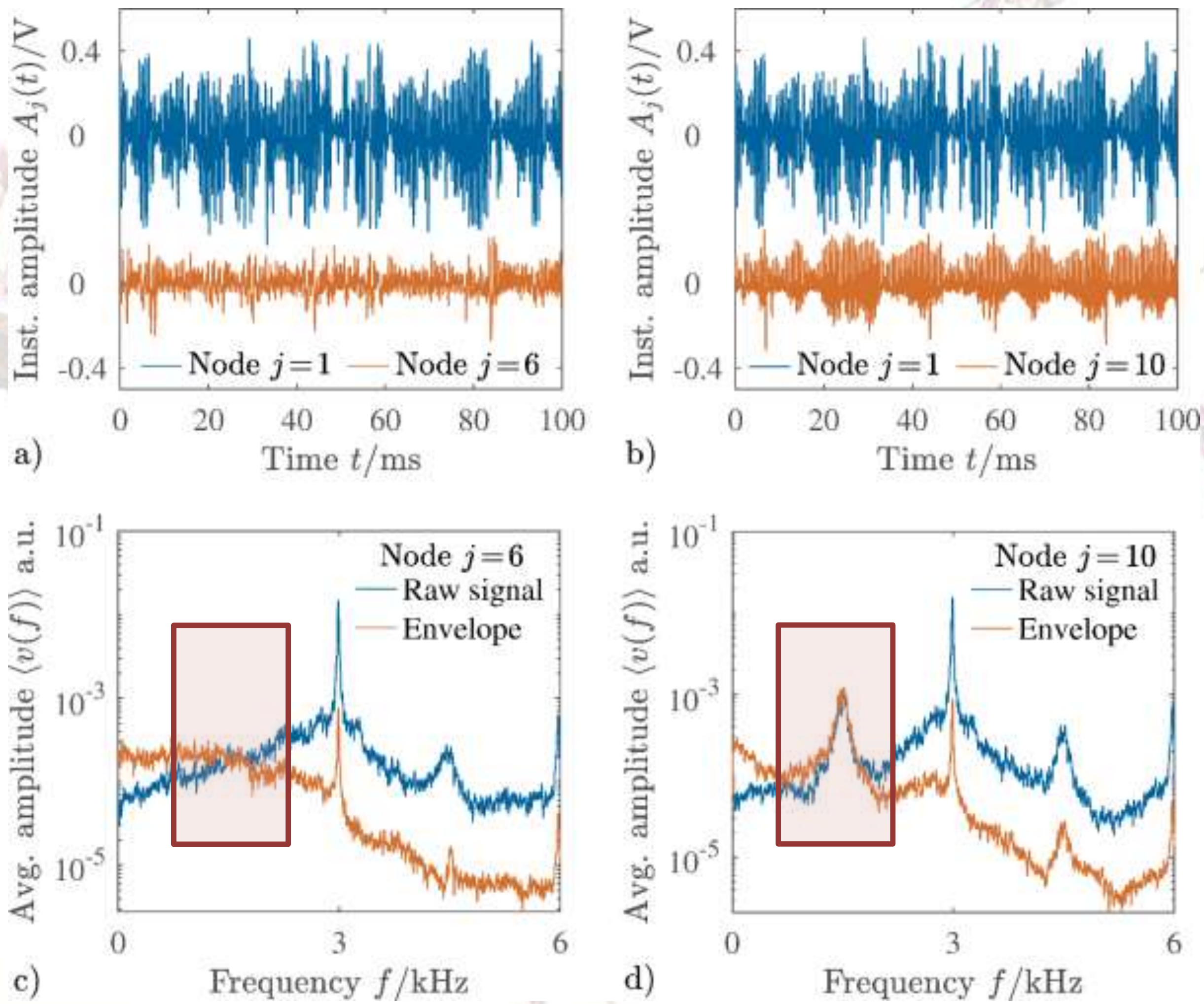
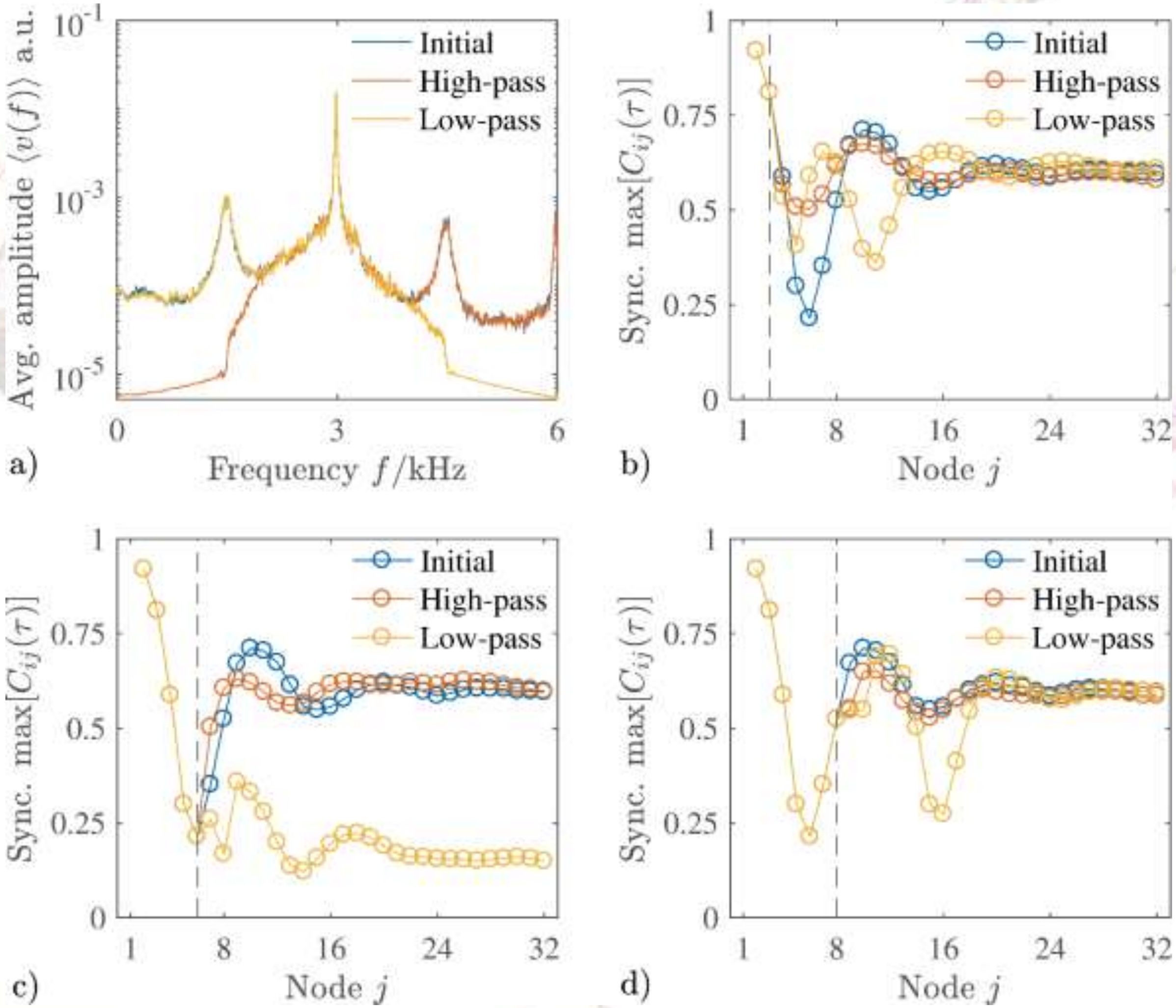


Image source: Minati et al., CHAOS 2018. © AIP

Instancing filters at specific points of chain



Revised Granger model: baseband + sideband

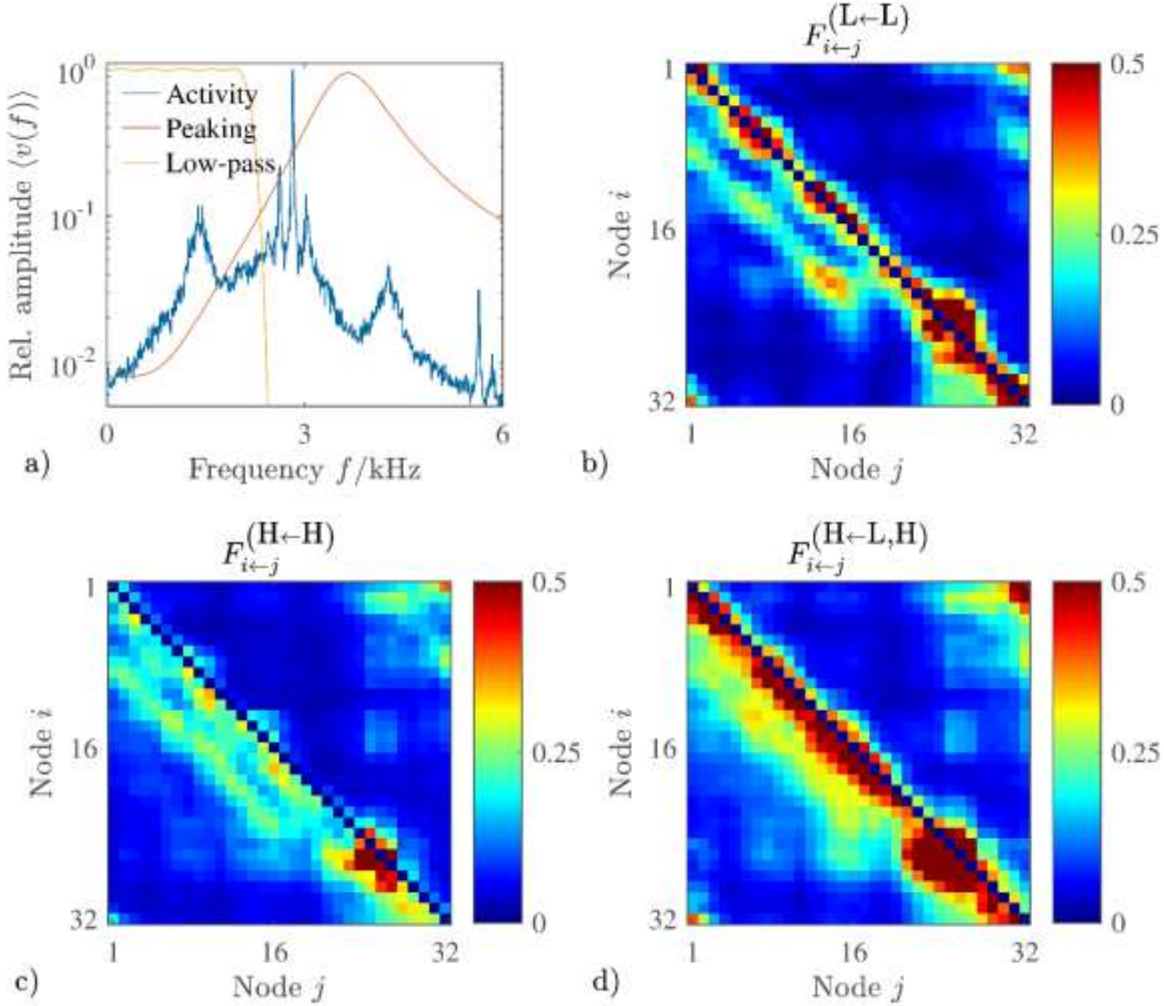
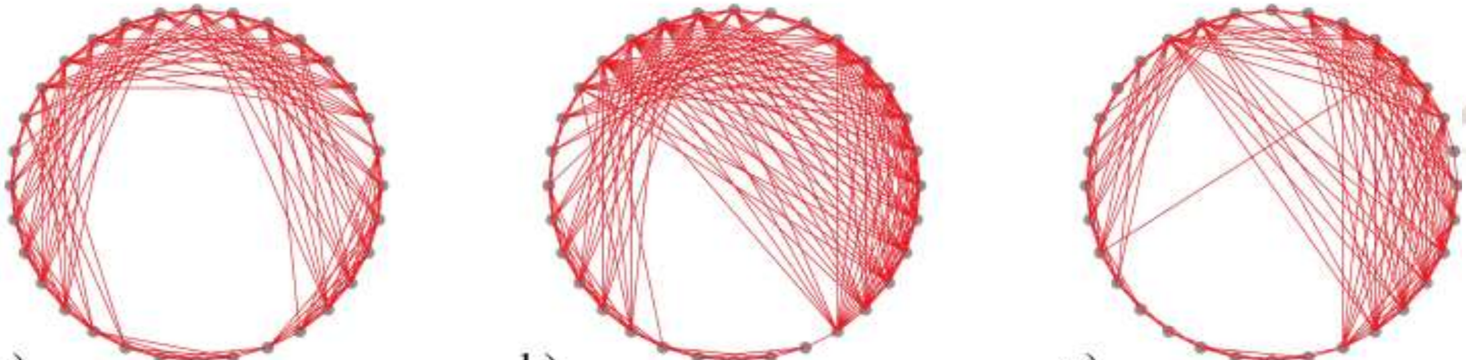


Image source: Minati et al., CHAOS 2018. © AIP

Small-world features, nonetheless...

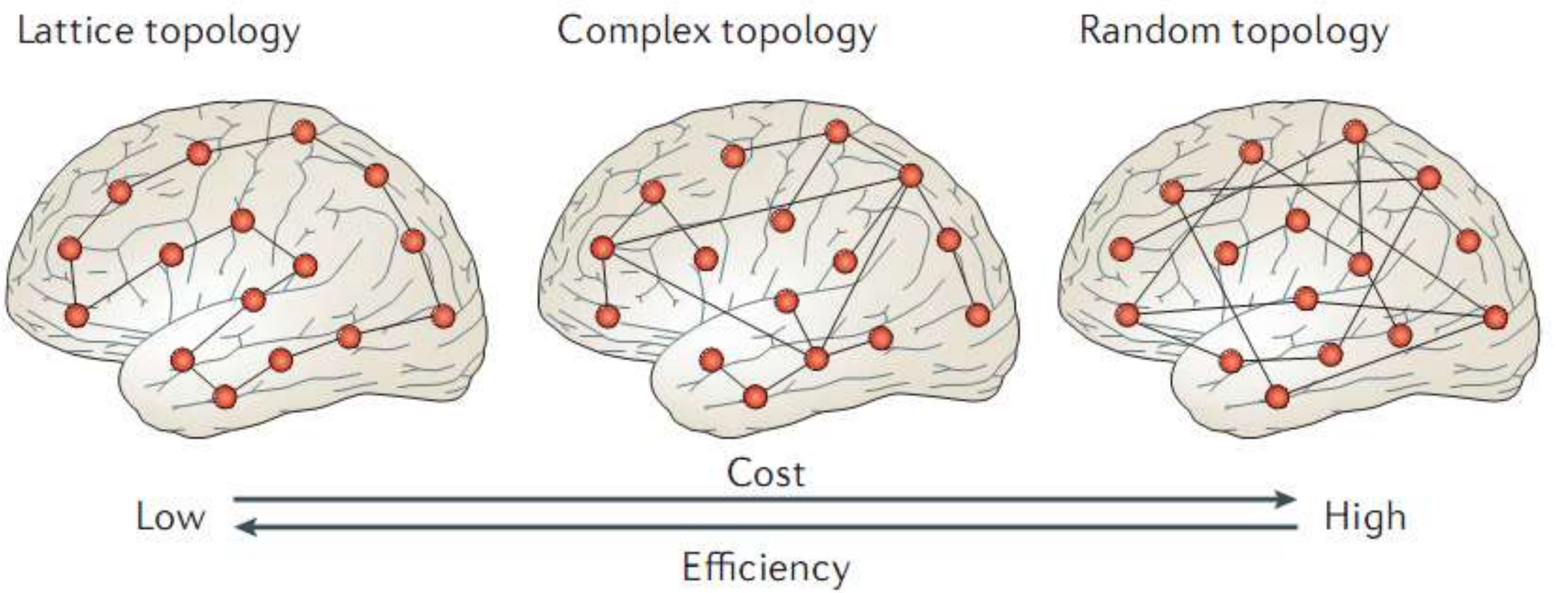
$$S^{\mathrm{WS}} = \gamma_g^{\mathrm{WS}} / \lambda_g = \left(C_g^{\mathrm{WS}} L_{\mathrm{rand}}\right) / \left(C_{\mathrm{rand}}^{\mathrm{WS}} L_g\right)$$



a) b) c)

Small-world features, nonetheless...

Small-worldness in the brain (and not only) is an <u>efficient trade-off</u>!



Source: Bullmore & Sporns, 2012; Nat Rev Neurosci, © 2012 Macmillan Publishers Limited

Conclusions

1) A complex mechanism of pattern generation was demonstrated.

2) Is this just “apparent” remoteness?
Central importance of measure choice…

3) To what systems may such mechanism apply?
Broadband vs. narrowband chaos, spectral relationships

Thank you for your attention

References:

1. Minati L. Remote synchronization of amplitudes across an experimental ring of non-linear oscillators. CHAOS. 2015 Dec; 25(12):123107.
2. Minati L, Faes L, Frasca M, Oświęcimka P, Drożdż S. Apparent remote synchronization of amplitudes: a demodulation and interference effect. CHAOS. 2018, in press

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