QCD Lecture 2a

October 15, 2025

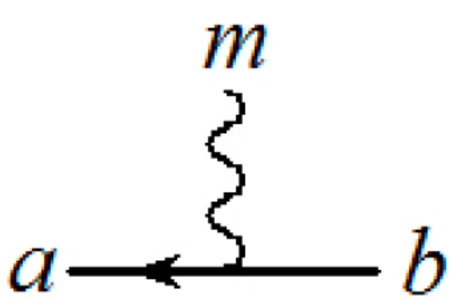
Color factors

each Feynman diagram is a product of a momentum-Dirac structure (like in QED) and a color factor

to calculate color factors it is very practical to use the graphical notation

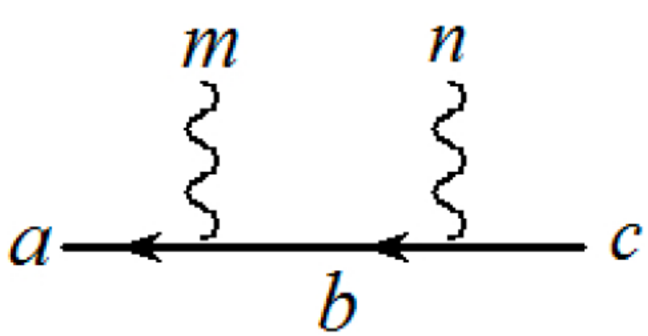
fundamental geneator:

$m,n = 1,2,\ldots N^2\text{-}1, \quad a,b = 1,2,\ldots N$



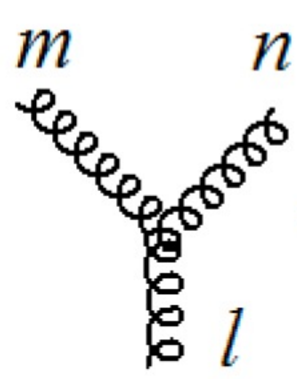
$= T^m_{ab}$

multiplication:



$= T^m_{ab}\, T^n_{bc}$

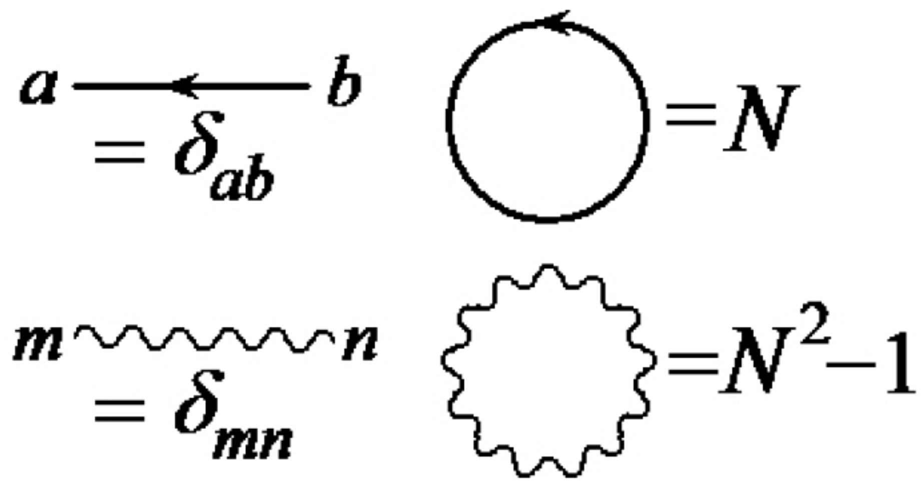
adjoint generator:



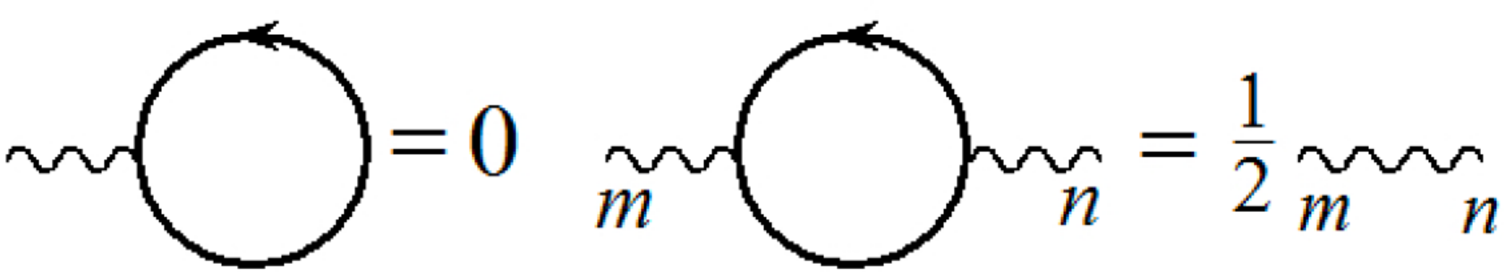
$= if_{mnl} = -if_{nml}$

Color factors

Kroneker deltas and traces:



generators are tracless and normalized to 1/2

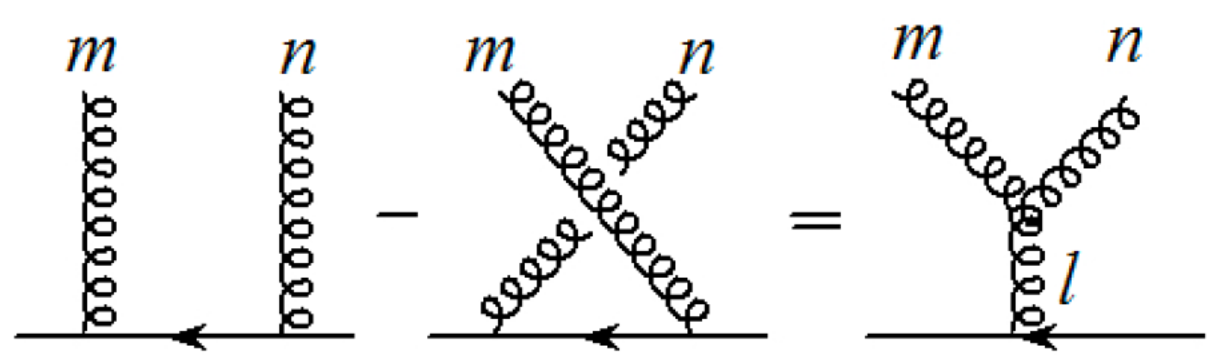


$$\mathrm{Tr}(T_m T_n) = \frac{1}{2}\delta_{mn}$$

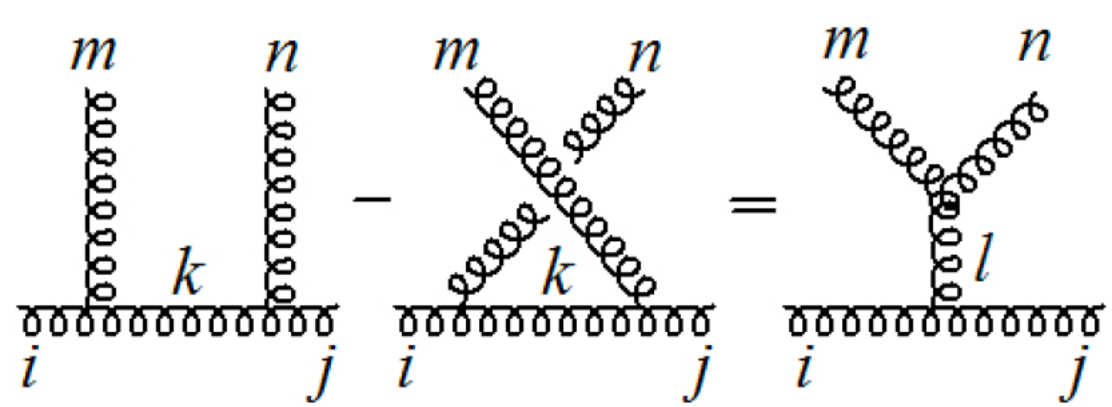
Color factors

commutation relations: $[T_m, T_n] = i f_{mnl} T_l$

fundamental:



adjoint:



Color factors

Example:

Casimir operator for the fundamental representation

quadratic Casimir operator is the sum over all generators squared
and it is proportional to unity multiplied by a number,
which is simply called “Casimir”

$$\sum_n (T^n)^2 = C_F \, \mathbf{1}$$

In SU(2) for any representation of spin *s* it is equal to

$$\sum_n \hat{S}_n^2 = s(s+1) \, \mathbf{1}$$

Color factors

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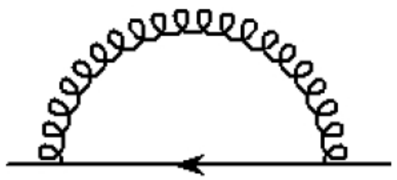
Casimir operator for the fundamental representation

$$\sum_n (T^n)^2 = C_F\,\mathbf{1}$$

$\Sigma_n (T^n)^2 =$  $= C_F$ 

Color factors

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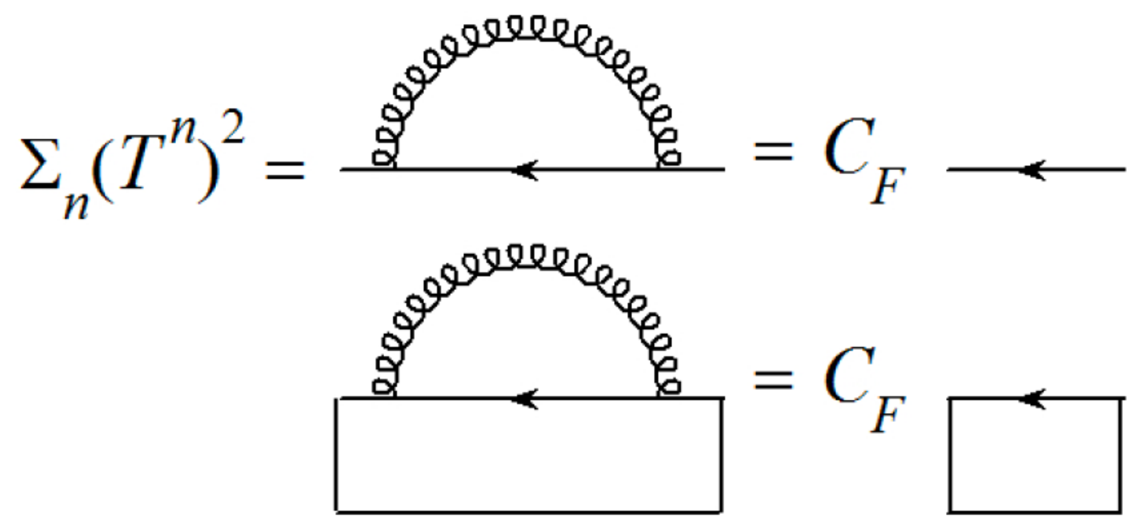
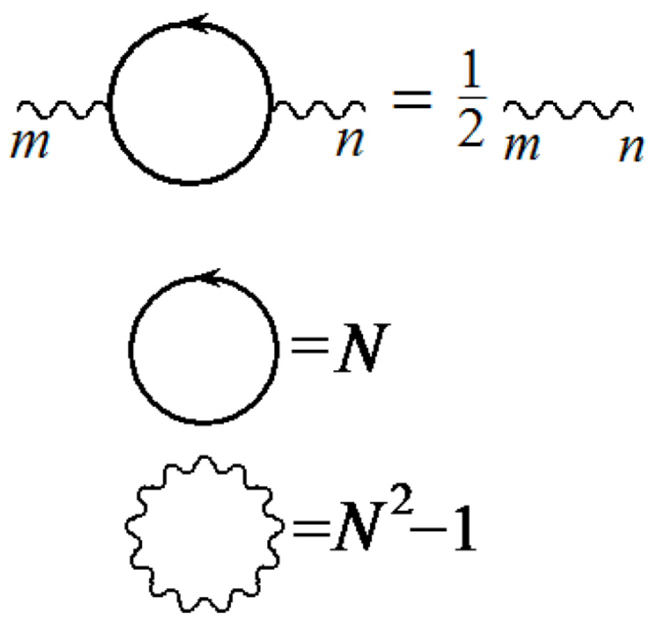


contract fermion line:

use:



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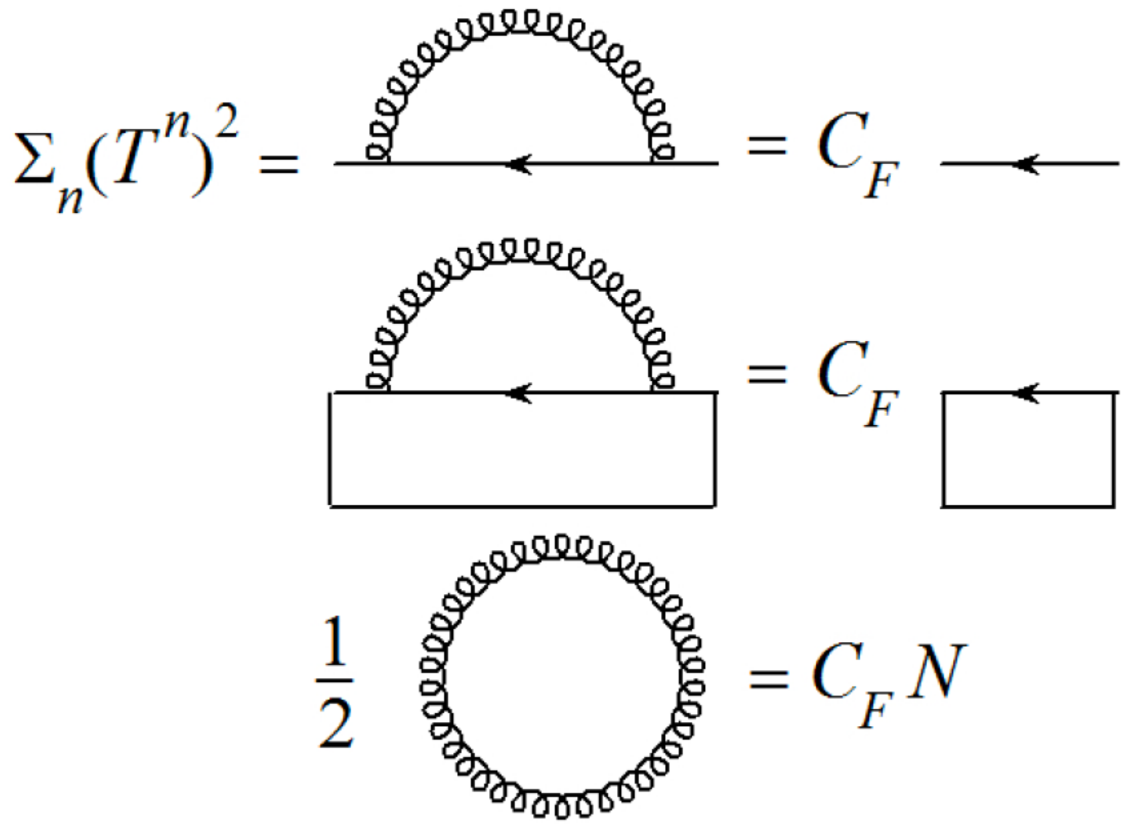

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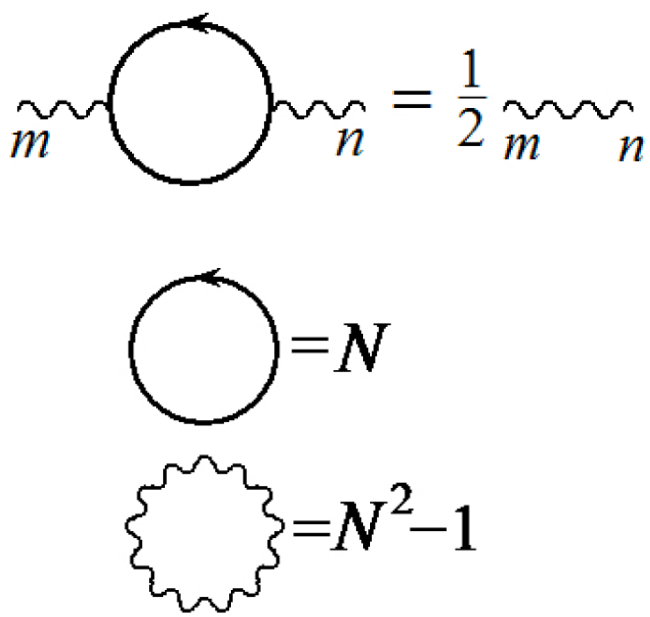

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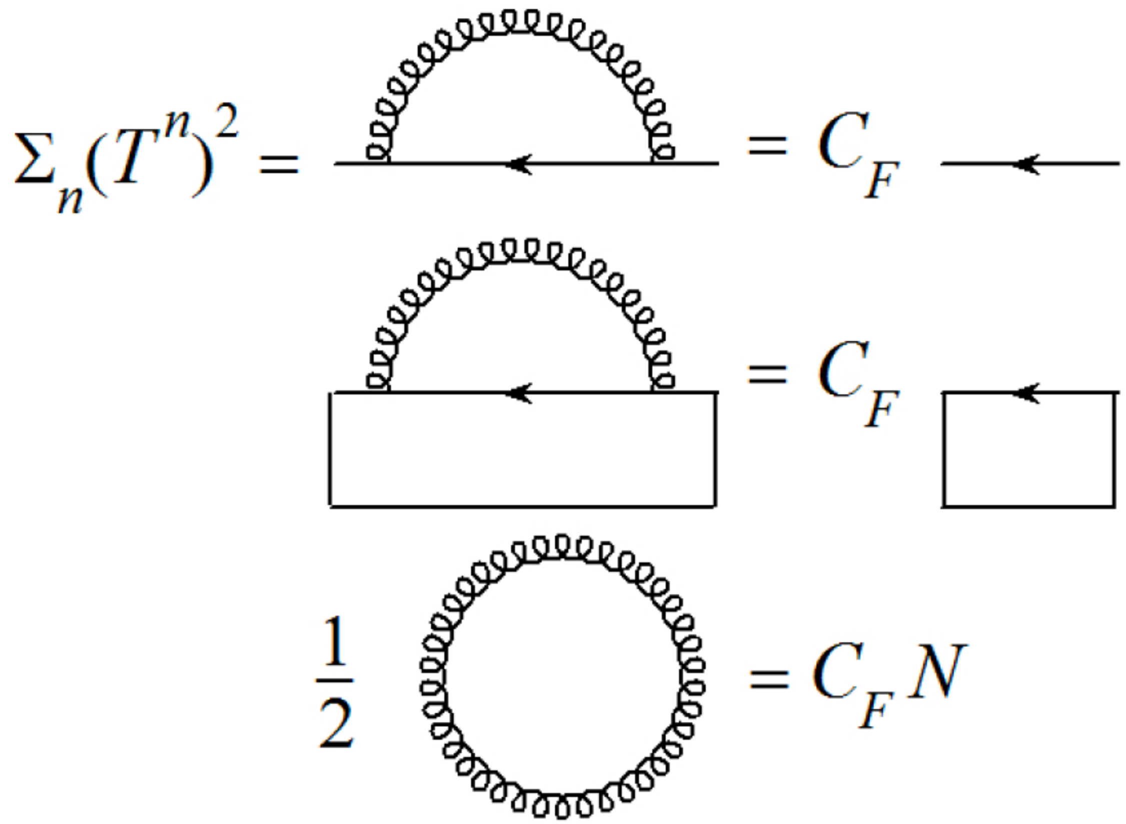

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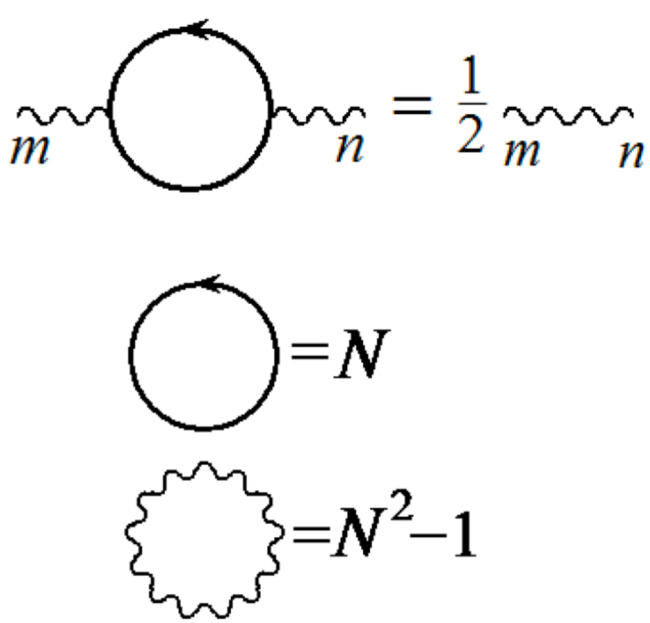

$$C_F = \frac{N^2-1}{2N} = \begin{cases} \frac{3}{4} & \text{SU(2)} \\ \frac{4}{3} & \text{SU(3)} \end{cases}$$