

# QCD lecture 11

January 14

# Scalar propagator

The free functional integral can be easily performed, because it is Gaussian in  $\phi$

$$\mathcal{L}_0(\phi) = \frac{1}{2}(1 + i0^+)\dot{\phi}^2 - \frac{1}{2}(1 - i0^+)((\nabla\phi) \cdot (\nabla\phi) + m^2\phi^2)$$

$$Z_0[j] \equiv \int [D\phi(x)] \exp \left\{ i \int d^4x (\mathcal{L}_0(\phi) + j(x)\phi(x)) \right\}$$

Recall

$$\int_{-\infty}^{+\infty} dx e^{ax^2+bx} = \sqrt{\frac{\pi}{-a}} e^{-\frac{b^2}{4a}}, \quad \text{Re } a \leq 0$$

and we get

$$Z_0[j] = \exp \left\{ -\frac{1}{2} \int d^4x d^4y j(x)j(y) G_F^0(x, y) \right\}$$

where  $G_F^0(x, y)$  is an inverse of  $i \left[ (1 + i0^+)\partial_0^2 - (1 - i0^+)(\nabla^2 - m^2) \right]$  which is obtained by integration by parts:

$$(\dot{\phi})^2 \rightarrow -\phi \partial_0^2 \phi, \quad (\nabla\phi) \cdot (\nabla\phi) \rightarrow -\phi \nabla^2 \phi$$

# Scalar propagator

Inverse of  $i\left[(1+i0^+)\partial_0^2 - (1-i0^+)(\nabla^2 - m^2)\right]$  can be evaluated in momentum space

$$i\partial_0 \rightarrow k_0, \quad -i\nabla \rightarrow \mathbf{k}$$

yielding 
$$\frac{i}{(1+i0^+)k_0^2 - (1-i0^+)(\mathbf{k}^2 + m^2)}$$

This is of course the same result as the one obtained in the canonical approach

$$\tilde{G}_F^0(\mathbf{k}) = \frac{i}{k^2 - m^2 + i0^+}$$

Exercise: show that the pole structure of the two expressions is the same

# Functional integral for photons

Here we would naively think that we will have a scalar integral for each component  $A_\mu$  (Lecture 7).

However gauge invariance complicates things. Even more so for QCD. Let's first write a naive functional integral

$$Z_0[j^\mu] \equiv \int [DA_\mu(x)] \exp \left\{ i \int d^4x \left( -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + j^\mu A_\mu \right) \right\}$$

This is a Gaussian integral, because  $F^{\mu\nu} F_{\mu\nu}$  is quadratic in  $A_\mu$ . (exercise)

$$\begin{aligned} -\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} &= -\frac{1}{4} \int d^4x (\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ &= +\frac{1}{2} \int d^4x A^\mu (g_{\mu\nu} \square - \partial_\mu \partial_\nu) A^\nu \\ &= -\frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \tilde{A}^\mu(k) (g_{\mu\nu} k^2 - k_\mu k_\nu) \tilde{A}^\nu(-k) \end{aligned}$$

We need to invert  $(g_{\mu\nu} k^2 - k_\mu k_\nu)$  to perform the integral over  $A_\mu$ .

# Functional integral for photons

Inverting photonic operator: find  $\alpha$  and  $\beta$

$$\underbrace{(g_{\mu\nu}k^2 - k_\mu k_\nu)}_{\alpha k^2 \delta_\mu^\rho - \alpha k_\mu k^\rho} (\alpha g^{\nu\rho} + \beta \frac{k^\nu k^\rho}{k^2}) = \delta_\mu^\rho$$

This operator is not invertible: some eigenvalues are zero. These flat directions

correspond to the projection of  $\tilde{A}^\mu(k)$  along  $k^\mu$

## Landau gauge

Decompose  $A^\mu = A_\perp^\mu + A_\parallel^\mu$

in the following way:  $\tilde{A}_\perp^\mu(k) \equiv \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) \tilde{A}_\nu(k)$

$$\tilde{A}_\parallel^\mu(k) \equiv \left(\frac{k^\mu k^\nu}{k^2}\right) \tilde{A}_\nu(k).$$

The functional measure can be therefore factorized

$$[DA^\mu] = [DA_\perp^\mu] [DA_\parallel^\mu]$$

# Functional integral for photons

We have 
$$-\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} = -\frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \tilde{A}^\mu(k) (g_{\mu\nu} k^2 - k_\mu k_\nu) \tilde{A}^\nu(-k)$$

So the  $F^{\mu\nu} F_{\mu\nu}$  part is purely transverse, and

$$Z_0[j^\mu] \equiv \int [DA_{\parallel}^\mu(x)] \exp \left\{ i \int d^4x j_\mu A_{\parallel}^\mu \right\} \\ \times \int [DA_{\perp}^\mu(x)] \exp \left\{ i \int d^4x \left( -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + j_\mu A_{\perp}^\mu \right) \right\}$$

Recall  $\tilde{A}_{\parallel}^\mu(k) \equiv \left( \frac{k^\mu k^\nu}{k^2} \right) \tilde{A}_\nu(k)$  but vector current is conserved  $k^\mu j_\mu = 0$

and  $\int [DA_{\parallel}^\mu(x)]$  is an infinite constant that has to be divided out.

When restricted to the transverse directions  $g_{\mu\nu} k^2 - k^\mu k^\nu$  is invertible and we get

$$Z_0[j^\mu] = \exp \left\{ -\frac{1}{2} \int d^4x d^4y j_\mu(x) G_F^{0\mu\nu}(x, y) j_\nu(y) \right\} \quad G_F^{0\mu\nu}(p) \equiv \frac{-i}{p^2 + i0^+} \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right)$$

again  $i0^+$  prescription selects the ground state for large times.

# General covariant gauges

Note that to get  $G_F^{0\mu\nu}(p) \equiv \frac{-i}{p^2 + i0^+} \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right)$  we demanded  $\partial_\mu A^\mu = 0$ .

This is called Landau or Lorentz gauge.

In general we may require:

$$\partial_\mu A^\mu(x) = \omega(x)$$

This can be done by introducing a delta function into the functional integral

$$Z_0[j^\mu] \equiv \int [D\omega(x)] \exp \left\{ -i \frac{\xi}{2} \int d^4x \omega^2(x) \right\} \\ \times \int [DA_\mu(x)] \delta[\partial_\mu A^\mu - \omega] \exp \left\{ i \int d^4x \left( -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + j^\mu A_\mu \right) \right\}$$

where  $\xi$  is an arbitrary constant. Note that for fixed  $\omega$  we break Lorentz invariance. To mitigate this problem we integrate over all  $\omega$ 's with the Gaussian weight. We can do this Gaussian integral and integrating by parts (exercise) we arrive at

$$Z_0[j^\mu] = \int [DA_\mu(x)] \exp \left\{ i \int d^4x \left( \frac{1}{2} A^\mu (g_{\mu\nu} \square - (1-\xi) \partial_\mu \partial_\nu) A^\nu + j^\mu A_\mu \right) \right\}$$

We need to find inverse of  $i(g_{\mu\nu} p^2 - (1-\xi) p_\mu p_\nu)$

# General covariant gauges

To invert  $i(g_{\mu\nu}p^2 - (1 - \xi)p_\mu p_\nu)$

we look for the inverse operator in a form:  $\alpha g^{\nu\rho} + \beta \frac{p^\nu p^\rho}{p^2}$

The result reads (exercise)

$$G_F^{0\mu\nu}(p) = \frac{-i g^{\mu\nu}}{p^2 + i0^+} + \frac{i}{p^2 + i0^+} \left(1 - \frac{1}{\xi}\right) \frac{p^\mu p^\nu}{p^2}$$

Landau gauge:  $\xi \rightarrow \infty$

Feynman gauge:  $\xi = 1$

As we will see in QCD the gauge condition will be more like  $[F(\partial_\mu A^\mu) - \omega] = 0$  and then we will need a Jacobian (to be discussed later)

$$\int [D\omega(x)] \exp \left\{ -i \frac{\xi}{2} \int d^4x \omega^2(x) \right\} \int [DA_\mu(x)] \underbrace{F'(\partial_\mu A^\mu)}_{\text{Jacobian}} \delta[F(\partial_\mu A^\mu) - \omega]$$

# Quantization of QCD

Consider expectation value of some gauge invariant operator

$$\langle \mathcal{O} \rangle \equiv \int [DA_{\mu}^a(x)] \mathcal{O}(A_{\mu}) \exp \left\{ i \underbrace{\int d^4x \left( -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} \right)}_{\mathcal{S}_{\text{YM}}[A_{\mu}]} \right\}$$

When we perform gauge transformation

$$A_{\mu}(x) \rightarrow A_{\mu}^{\Omega}(x) \equiv \Omega^{\dagger}(x) A_{\mu}(x) \Omega(x) + \frac{i}{g} \Omega^{\dagger}(x) \partial_{\mu} \Omega(x)$$

the integration measure changes

$$[DA_{\mu}^a(x)] = [DA_{\mu}(x)] \det \left[ \left( \frac{\delta A_{\mu}^a(x)}{\delta A_{\nu}^b(y)} \right) \right]$$

We need to calculate the Jacobian. For this we need a small reminder from group theory.

# Digression

Consider some representation  $r$

$$\mathbf{X} = X_a T^a(r), \quad \mathbf{Y} = Y_a T^a(r)$$

Let's calculate an object analogous to gauge transformation:

$$\begin{aligned} e^{-i\mathbf{X}} \mathbf{Y} e^{i\mathbf{X}} &= \mathbf{Y} - i [\mathbf{X}, \mathbf{Y}] + \dots \\ &= \mathbf{Y} - i X_a Y_b [T^a, T^b] + \dots \\ &= [Y_c - i(-if_{acb}) X_a Y_b] T^c + \dots \\ &= \left[ Y_c - i X_a (T_{\text{adj}}^a)_{cb} Y_b + \dots \right] T^c \end{aligned}$$

This can be written in short  $[e^{-i\mathbf{X}} \mathbf{Y} e^{i\mathbf{X}}]_c = [\exp(-i X_a T_{\text{adj}}^a)]_{cb} Y_b$

Gauge transformation

$$\Omega^\dagger \mathbf{A} \Omega = e^{-i \text{ad}_\Omega} \mathbf{A}$$



Lecture 1

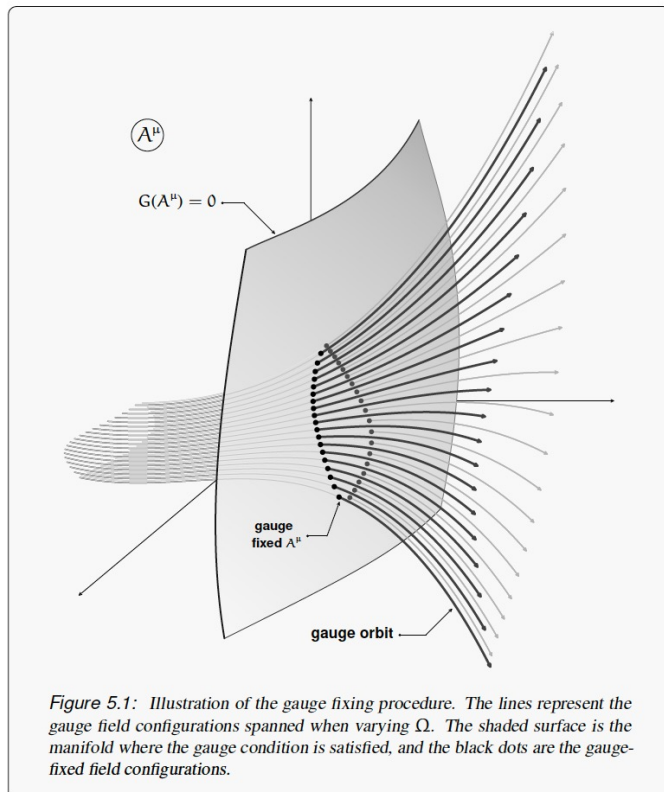
$$\frac{\delta A_{a\mu}^\Omega(x)}{\delta A_{b\nu}(y)} = \delta_\mu^\nu \delta(x-y) (e^{-i \text{ad}_\Omega})_{ab} \quad \longrightarrow \quad \det \left( \frac{\delta A_{a\mu}^\Omega(x)}{\delta A_{b\nu}(y)} \right) = 1$$

# Quantization of QCD

Changing gauge does not change the integration measure

$$[DA_{\alpha\mu}^{\Omega}(x)] = [DA_{\alpha\mu}(x)]$$

So the path integral is infinite. To eliminate gauge redundancy we have to fix the gauge.



$$G^a(A_\mu(x)) = 0$$

[ This condition may have many solutions (Gribov copies) but only one of them is perturbative, others are  $\sim 1/g$  ]

We want to split the functional integration into a physical component in the gauge fixing manifold and a component along the gauge orbit (analogue of the longitudinal QED field). This can be done by inserting

$$\delta[G^a(A_\mu)]$$

into the functional integral.

How this behaves under the gauge transformation?

# Quantization of QCD

Toy model example  $f(x_0) = 0$

$$\int dx \delta(f(x)) = \int dx \frac{1}{|f'(x)|} \delta(x - x_0) = \frac{1}{|f'(x)|} \Big|_{x=x_0}$$

Define  $\Delta^{-1}[A_\mu] \equiv \int [D\Omega(x)] \delta[G^a(A_\mu^\Omega)]$

then

$$\Delta(A_\mu) = \det \left( \frac{\delta G^a}{\delta \Omega} \right)_{G^a(A_\mu^\Omega)=0} \quad [\text{Faddeev - Popov determinant}]$$

In QED  $\Delta$  does not depend on  $A_\mu$  but in QCD it does, because gauge transformation is non-linear:

$$A_\mu^\Omega(x) \equiv \underline{\Omega^\dagger(x) A_\mu(x) \Omega(x)} + \frac{i}{g} \Omega^\dagger(x) \partial_\mu \Omega(x)$$

# Quantization of QCD

First we prove that  $\Delta[A_\mu]$  is gauge invariant

$$\begin{aligned}\Delta^{-1}[A_\mu^\Theta] &= \int [D\Omega(x)] \delta[G^a(A_\mu^{\overbrace{\Theta\Omega}^{\Omega'}})] \\ &= \int [D(\Theta^\dagger(x)\Omega'(x))] \delta[G^a(A_\mu^{\Omega'})] \\ &= \int [D\Omega'(x)] \delta[G^a(A_\mu^{\Omega'})] = \Delta^{-1}[A_\mu]\end{aligned}$$

Last step follows from the unitarity of gauge transformations (there exists a group invariant measure on a Lie group).

Hence

$$1 = \Delta[A_\mu] \int [D\Omega(x)] \delta[G^a(A_\mu^\Omega)]$$

and we will insert this unity under the functional integral.

# Quantization of QCD

Expectation value of a gauge invariant operator:

$$\langle \mathcal{O} \rangle = \int [\underline{D\Omega(x)}] \int [DA_\mu^a(x)] \Delta[A_\mu] \delta[\underline{G^a(A_\mu^a)}] \mathcal{O}(A_\mu) e^{iS_{YM}[A_\mu]}$$

Change variables:  $A_\mu \rightarrow A_\mu^{\Omega^\dagger}$

Invariants:

$$\begin{aligned} [DA_\mu^{\Omega^\dagger}] &= [DA_\mu], \\ S_{YM}[A_\mu^{\Omega^\dagger}] &= S_{YM}[A_\mu], \\ \mathcal{O}[A_\mu^{\Omega^\dagger}] &= \mathcal{O}[A_\mu], \\ \Delta[A_\mu^{\Omega^\dagger}] &= \Delta[A_\mu], \end{aligned}$$

At this point the functional integral does not contain the gauge transformation

$$\langle \mathcal{O} \rangle = \int [\underline{D\Omega(x)}] \int [DA_\mu^a(x)] \Delta[A_\mu] \delta[G^a(A_\mu)] \mathcal{O}(A_\mu) e^{iS_{YM}[A_\mu]}$$

We can now drop  $[D\bar{\Omega}]$ . So functional integral has been factored out into a gauge orbit part at the expense of  $\Delta[A_\mu]$  that modifies QCD Feynman rules.

# Quantization of QCD

We need to find a functional representation for the Faddeev-Popov determinant.  
Recall:

$$\det(\mathbf{M}) \equiv \int d^N \xi d^N \psi \exp(\psi_i M_{ij} \xi_j)$$

Let's introduce new fermion fields (Faddeev-Popov ghosts)

$$\begin{aligned} \det(i\mathcal{M}) &= \int [D\chi_a(x) D\bar{\chi}_a(x)] \\ &\quad \times \exp \left\{ i \int d^4x d^4y \bar{\chi}_a(x) \mathcal{M}_{ab}(x, y) \chi_b(y) \right\} \end{aligned}$$

and use a trick for covariant gauges in QED

$$\int [D\omega(x)] \exp \left\{ -i \frac{\xi}{2} \int d^4x \omega^2(x) \right\} \delta[G^a(A_\mu(x)) - \omega]$$

# Quantization of QCD

After integration over  $D[\omega]$

$$\langle \mathcal{O} \rangle = \int [DA_\mu^a(x)] [D\chi_a(x)D\bar{\chi}_a(x)] \mathcal{O}(A_\mu) \\ \times \exp i \int d^4x \left( \underbrace{-\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}}_{\mathcal{L}_{YM}} - \underbrace{\frac{\xi}{2} (G^a(A_\mu))^2}_{\mathcal{L}_{GF}} + \underbrace{\bar{\chi}_a \mathcal{M}_{ab} \chi_b}_{\mathcal{L}_{FPG}} \right)$$

Note that  $\mathcal{M}_{ab} \sim g \left( \frac{\delta G^a}{\delta \Omega_b} \right)$  at  $\Omega = 1$  (det is gauge inv.)

and therefore is a function of  $A_\mu$ . Ghost fields couple to the gauge fields and appear only inside loops. In practice they remove contributions from the "longitudinal" gauge fields. They ensure that the theory is unitary.

Both GF and FPG depend on the gauge choice (choice of function  $G$ ). Typically we choose  $G$  linear in  $A_\mu$ , so the gluon propagator will depend on  $\xi$  and will be the same as in QED, up to the color factor.

[ Matrix  $\mathcal{M}_{ab}$  can be scaled by any factor  $\mathcal{M} \rightarrow \kappa \mathcal{M}$ , this changes the propagator  $S \rightarrow \kappa^{-1} S$  and vertices  $V \rightarrow \kappa V$  leaving the final result invariant.]

# Covariant gauge

## EXAMPLE

Covariant gauge  $G^a(\mathcal{A}) \equiv \partial^\mu \mathcal{A}_\mu^a - \omega^a(x)$

gluon propagator (as in QED)

$$G_{F\ ab}^{0\ \mu\nu}(p) = \frac{p}{\text{wavy line}} = \frac{-i g^{\mu\nu} \delta_{ab}}{p^2 + i0^+} + \frac{i \delta_{ab}}{p^2 + i0^+} \left(1 - \frac{1}{\xi}\right) \frac{p^\mu p^\nu}{p^2}$$

We need to calculate matrix  $\mathcal{M}_{ab}$

Gauge transformation  $\mathcal{A}_\mu^\Omega(x) \equiv \Omega^\dagger(x) \mathcal{A}_\mu(x) \Omega(x) + \frac{i}{g} \Omega^\dagger(x) \partial_\mu \Omega(x)$   $\Omega(x) = \exp(i\theta_a(x)T^a)$

infinitesimal  $g \delta \mathcal{A}_{a\ \mu}(x) = g f^{abc} \theta_b(x) \mathcal{A}_{c\ \mu}(x) - \partial_\mu \theta_a(x)$

which yields:  $g \delta G^a = g f^{abc} (\partial^\mu \theta_b(x)) \mathcal{A}_{c\ \mu}(x) + g f^{abc} \theta_b(x) (\partial^\mu \mathcal{A}_{c\ \mu}(x)) - \square \theta_a(x)$

and:  $\mathcal{M}_{ab} = g \frac{\delta G^a(\mathcal{A})}{\delta \theta^b} = g f^{abc} (\partial^\mu \mathcal{A}_{c\ \mu}(x)) + g f^{abc} \mathcal{A}_{c\ \mu}(x) \partial^\mu - \delta_{ab} \square$

So this matrix contains gluon-ghost interactions and ghost propagator (inverse)

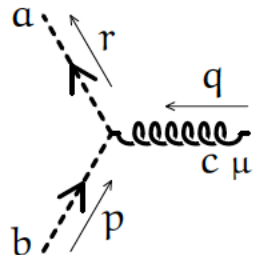
# Covariant gauge

This results in the following FPG langrangian:

$$\mathcal{L}_{\text{FPG}} = \bar{\chi}_a \left( -\delta_{ab} \square + g f^{abc} (\partial^\mu A_{c\mu}(x)) + g f^{abc} A_{c\mu}(x) \partial^\mu \right) \chi_b$$

and the following Feynman rules:

$$G_{\text{F}}^0(p) = \begin{array}{c} \xrightarrow{p} \\ \text{---} \rightarrow \text{---} \end{array} = \frac{i \delta_{ab}}{p^2 + i0^+}$$



$$= g f^{abc} (p_\mu + q_\mu) = g f^{abc} r_\mu$$

$$\begin{aligned}
 \begin{array}{c} \overrightarrow{p} \\ \text{wavy line} \end{array} &= \frac{-i g^{\mu\nu} \delta_{ab}}{p^2 + i0^+} + \frac{i \delta_{ab}}{p^2 + i0^+} \left(1 - \frac{1}{\xi}\right) \frac{p^\mu p^\nu}{p^2} \\
 \begin{array}{c} \overrightarrow{p} \\ \text{solid line} \end{array} &= \frac{i \delta_{ij}}{p' - m + i0^+} \\
 \begin{array}{c} \overrightarrow{p} \\ \text{dashed line} \end{array} &= \frac{i \delta_{ab}}{p^2 + i0^+} \\
 \begin{array}{c} \begin{array}{c} a \mu \\ \uparrow \\ \text{wavy line} \\ \downarrow \\ k \end{array} \\ \begin{array}{c} \overrightarrow{p} \\ \text{wavy line} \\ \downarrow \\ b \nu \end{array} \\ \begin{array}{c} \text{wavy line} \\ \downarrow \\ q \\ \text{wavy line} \\ \downarrow \\ c \rho \end{array} \end{array} &= g f^{abc} \{ g^{\mu\nu} (k - p)^\rho \\ &\quad + g^{\nu\rho} (p - q)^\mu + g^{\rho\mu} (q - k)^\nu \} \\
 \begin{array}{c} a \mu \\ \text{wavy line} \\ \downarrow \\ c \rho \end{array} \\ \begin{array}{c} b \nu \\ \text{wavy line} \\ \downarrow \\ d \sigma \end{array} \end{array} &= -i g^2 \{ f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ &\quad + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ &\quad + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}) \} \\
 \begin{array}{c} i \\ \uparrow \\ \text{solid line} \\ \downarrow \\ j \end{array} \\ \begin{array}{c} \text{wavy line} \\ \downarrow \\ a \mu \end{array} \end{array} &= -i g \gamma^\mu (t_r^a)_{ij} \\
 \begin{array}{c} a \\ \uparrow \\ \text{dashed line} \\ \downarrow \\ r \end{array} \\ \begin{array}{c} \text{dashed line} \\ \downarrow \\ q \\ \text{wavy line} \\ \downarrow \\ c \mu \end{array} \\ \begin{array}{c} \text{dashed line} \\ \downarrow \\ p \\ \text{dashed line} \\ \downarrow \\ b \end{array} \end{array} &= g f^{abc} (p_\mu + q_\mu) = g f^{abc} r_\mu
 \end{aligned}$$

Figure 5.2: Feynman rules of non-Abelian gauge theories in covariant gauge. We also list the rules involving fermions for completeness. Latin characters  $a, b, c$  refer to the adjoint representation, while the letters  $i, j$  refer to the representation  $r$  in which the fermions live.

# Axial gauge

Usefull class of gauges

$$G^a(\mathcal{A}) \equiv n^\mu A_\mu^a - \omega^a(x)$$

where  $n^\mu$  is a fixed four-vector. If it is time-like – temporal gauge  
light-like – light-cone gauge

Then (exercise)

$$\frac{1}{2} A_\mu^a (g^{\mu\nu} \square - \partial^\mu \partial^\nu - \xi n^\mu n^\nu) A_\nu^a$$

and we have to invert the following matrix:

$$g^{\mu\nu} p^2 - p^\mu p^\nu + \xi n^\mu n^\nu$$

which gives (exercise):

$$G_{F\ ab}^{0\ \mu\nu}(p) = \frac{-i\delta_{ab}}{p^2 + i0^+} \left[ g^{\mu\nu} - \frac{p^\mu n^\nu + p^\nu n^\mu}{p \cdot n} + \frac{p^\mu p^\nu}{(p \cdot n)^2} (n^2 + \xi^{-1} p^2) \right]$$

# Axial gauge

Final result (exercise)

$$\mathcal{L}_{\text{FPG}} = \bar{\chi}_a \left( -\delta_{ab} n^\mu \partial_\mu + g f^{abc} n^\mu A_{c\mu}(x) \right) \chi_b$$

and the ghost propagator and ghost vertex look like:

$$G_F^0(p) = \begin{array}{c} \xrightarrow{p} \\ \text{---} \rightarrow \text{---} \end{array} = -\frac{\delta_{ab}}{p \cdot n + i0^+}$$

$$\begin{array}{c} a \\ \nearrow \\ \text{---} \\ \nearrow \\ b \end{array} \begin{array}{c} r \\ \nearrow \\ \text{---} \\ \nearrow \\ p \end{array} \begin{array}{c} \leftarrow q \\ \text{---} \\ c \mu \end{array} = i g f^{abc} n_\mu .$$