QCD

Anomaly November 26, 2025

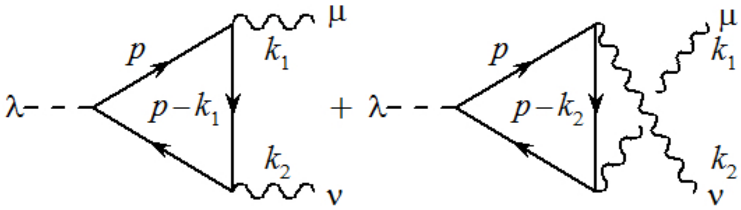

Figure 1: Loop diagrams contributing to the decay of axial-vector current (dashed line) to two photons.

Consider the following loop contribution to the decay of axial-vector current to two photons (Fig. 1):

$$\begin{aligned} T_{\mu\nu\lambda} &= -i\int\frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left[\frac{i}{\not p - m}\gamma_\lambda\gamma_5\frac{i}{(\not p-\not q)-m}\gamma_\nu\frac{i}{(\not p-\not k_1)-m}\gamma_\mu\right] \\ &\quad -i\int\frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left[\frac{i}{\not p - m}\gamma_\lambda\gamma_5\frac{i}{(\not p-\not q)-m}\gamma_\mu\frac{i}{(\not p-\not k_2)-m}\gamma_\nu\right] \end{aligned} \tag{1}$$

where

$$q = k_1 + k_2. \tag{2}$$

Note that the second line in (1) and the first line are related by a replacement $\mu \longleftrightarrow \nu$ and $k_1 \longleftrightarrow k_2$. We expect that vector currents are conserved

$$k_1^\mu T_{\mu\nu\lambda} = k_2^\nu T_{\mu\nu\lambda} = 0 \tag{3}$$

and that the axial current is conserved in a massless limit

$$q^\lambda T_{\mu\nu\lambda} = 2mT_{\mu\nu}. \tag{4}$$

In fact on general grounds we expect $T_{\mu\nu}$ to be obtained from $T_{\mu\nu\lambda}$ by replacing $\gamma_\lambda\gamma_5 \to \gamma_5$.

Let's first check vector current conservation $k_1^\mu T_{\mu\nu\lambda}$ with the help of

$$\not k_1 = (\not p - m) - ((\not p - \not k_1) - m), \tag{5}$$

which gives

$$\begin{aligned} &\mathrm{Tr}\left[\gamma_\lambda\gamma_5\frac{i}{(\not p-\not q)-m}\gamma_\nu\frac{i}{(\not p-\not k_1)-m}\not k_1\frac{i}{\not p-m}\right] \\ &= i\,\mathrm{Tr}\left[\gamma_\lambda\gamma_5\frac{i}{(\not p-\not q)-m}\gamma_\nu\frac{i}{(\not p-\not k_1)-m}\right] - i\,\mathrm{Tr}\left[\gamma_\lambda\gamma_5\frac{i}{(\not p-\not q)-m}\gamma_\nu\frac{i}{\not p-m}\right]. \end{aligned} \tag{6}$$

For the second trace we need

$$\not{k}_1 = (\not{p} - \not{k}_2 - m) - ((\not{p} - \not{k}_1 - \not{k}_2) - m) = (\not{p} - \not{k}_2 - m) - ((\not{p} - \not{q}) - m) \tag{7}$$

and get

$$\begin{aligned} & \mathrm{Tr}\left[\frac{i}{\not{p}-m}\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{q})-m}\not{k}_1\frac{i}{(\not{p}-\not{k}_2)-m}\gamma_\nu\right] \\ = \ & i\,\mathrm{Tr}\left[\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{q})-m}\gamma_\nu\frac{i}{\not{p}-m}\right] - i\,\mathrm{Tr}\left[\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{k}_2)-m}\gamma_\nu\frac{i}{\not{p}-m}\right] \end{aligned} \tag{8}$$

so the full result is proportional to

$$\begin{aligned} k_1^\mu T_{\mu\nu\lambda} \ \sim \ & \int \frac{d^4p}{(2\pi)^4} \\ & \left\{\mathrm{Tr}\left[\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{q})-m}\gamma_\nu\frac{i}{(\not{p}-\not{k}_1)-m}\right] - \mathrm{Tr}\left[\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{q})-m}\gamma_\nu\frac{i}{\not{p}-m}\right]\right. \\ & \left. + \mathrm{Tr}\left[\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{q})-m}\gamma_\nu\frac{i}{\not{p}-m}\right] - \mathrm{Tr}\left[\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{k}_2)-m}\gamma_\nu\frac{i}{\not{p}-m}\right]\right\} \\ = \ & \int \frac{d^4p}{(2\pi)^4} \\ & \left\{\mathrm{Tr}\left[\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{q})-m}\gamma_\nu\frac{i}{(\not{p}-\not{k}_1)-m}\right] - \mathrm{Tr}\left[\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{k}_2)-m}\gamma_\nu\frac{i}{\not{p}-m}\right]\right\}. \end{aligned} \tag{9}$$

Note that second and third term cancelled. When we change variable in the first integral $p \to p + k_1$ we get that $p - q \to p - k_2$ and it seems that also the two remaining integrals cancel.

To check axial current conservation let's use

$$\begin{aligned} \not{q}\gamma_5 &= -\gamma_5\not{q} \\ &= \gamma_5\left[(\not{p}-\not{q})-m\right] - \gamma_5\left[\not{p}-m\right] \\ &= \gamma_5\left[(\not{p}-\not{q})-m\right] + \left[\not{p}-m\right]\gamma_5 + 2m\gamma_5. \end{aligned} \tag{10}$$

This replacement results in

$$\begin{aligned} \left[\frac{i}{\not{p}-m}\not{q}\gamma_5\frac{i}{(\not{p}-\not{q})-m}\right] \ = \ & 2m\frac{i}{\not{p}-m}\gamma_5\frac{i}{(\not{p}-\not{q})-m} \\ & + i\frac{i}{\not{p}-m}\gamma_5 + i\gamma_5\frac{i}{(\not{p}-\not{q})-m}. \end{aligned} \tag{11}$$

Therefore from the loop diagram (1) we obtain that

$$q^\lambda T_{\mu\nu\lambda} = 2mT_{\mu\nu} + \Delta^{(1)}_{\mu\nu} + \Delta^{(2)}_{\mu\nu} \tag{12}$$

where

$$\Delta^{(1)}_{\mu\nu} + \Delta^{(2)}_{\mu\nu} \tag{13}$$

$$= \int \frac{d^4p}{(2\pi)^4} \,\mathrm{Tr}\left[\frac{i}{\not{p}-m}\gamma_5\gamma_\nu \frac{i}{(\not{p}-\not{k}_1)-m}\gamma_\mu + \gamma_5 \frac{i}{(\not{p}-\not{q})-m}\gamma_\nu \frac{i}{(\not{p}-\not{k}_1)-m}\gamma_\mu\right],$$

$$+ \int \frac{d^4k}{(2\pi)^4} \,\mathrm{Tr}\left[\frac{i}{\not{p}-m}\gamma_5\gamma_\mu \frac{i}{(\not{p}-\not{k}_2)-m}\gamma_\nu + \gamma_5 \frac{i}{(\not{p}-\not{q})-m}\gamma_\mu \frac{i}{(\not{p}-\not{k}_2)-m}\gamma_\nu\right].$$

In order to define $\Delta^{(1,2)}_{\mu\nu}$ separately let's combine the first term in the first line and the second term in the second line and the two remaining ones, use periodicity of trace and anticommutation of $\gamma_5$with γ_μ:

$$\begin{aligned}
\Delta^{(1)}_{\mu\nu} &= \int \frac{d^4p}{(2\pi)^4} \,\mathrm{Tr}\left[\frac{i}{\not{p}-m}\gamma_5\gamma_\nu \frac{i}{(\not{p}-\not{k}_1)-m}\gamma_\mu - \frac{i}{(\not{p}-\not{k}_2)-m}\gamma_5\gamma_\nu \frac{i}{(\not{p}-\not{q})-m}\gamma_\mu\right], \\
\Delta^{(2)}_{\mu\nu} &= \int \frac{d^4p}{(2\pi)^4} \,\mathrm{Tr}\left[\frac{i}{\not{p}-m}\gamma_5\gamma_\mu \frac{i}{(\not{p}-\not{k}_2)-m}\gamma_\nu - \frac{i}{(\not{p}-\not{k}_1)-m}\gamma_5\gamma_\mu \frac{i}{(\not{p}-\not{q})-m}\gamma_\nu\right].
\end{aligned} \tag{14}$$

The question is: are $\Delta^{(1,2)}_{\mu\nu}$ equal zero? At first sight it does seem so. Changing variables in the second part of $\Delta^{(1)}_{\mu\nu}$

$$p \to p + k_2 \tag{15}$$

and of $\Delta^{(2)}_{\mu\nu}$

$$p \to p + k_1 \tag{16}$$

seems to nullify $\Delta^{(1,2)}_{\mu\nu}$. However, the integrals (14) are UV divergent. Indeed

$$\Delta^{(1,2)}_{\mu\nu} \sim \int^{\infty} dp p^3 \frac{1}{p^2} \sim \int^{\infty} dp p. \tag{17}$$

Due to the angular integration in (14) the divergence is only linear. Nevertheless the change of variables in a linearly divergent integral is not well defined. To illustrate this consider an integral that naively is equal to zero

$$\int_{-\infty}^{\infty} dx\,[f(x+a) - f(x)] \tag{18}$$

where f is a function that does not vanish at infinity:

$$f(\pm\infty) \neq 0. \tag{19}$$

Expanding in a

$$\int_{-\infty}^{\infty} dx\,[f(x+a) - f(x)] = a\,[f(\infty) - f(-\infty)] + \frac{a^2}{2}\,[f'(\infty) - f'(-\infty)] + \dots \tag{20}$$

We see that there is a contribution from the integration limits even if $f'(\pm\infty) = 0$. Consider the n-dimensional Euclidean integral

$$\begin{aligned}
\Delta(\vec{a}) &= \int d^n\vec{r}\,[f(\vec{r}+\vec{a}) - f(\vec{r})] \\
&= \int d^n\vec{r}\,\vec{a}\cdot\vec{\nabla}f(\vec{r}) + \ldots \\
&= \vec{a}\cdot\vec{n}\,S_n(R)\ f(\vec{R})
\end{aligned} \tag{21}$$

where the last line has been obtained by applying the Gauss theorem and

$$\vec{n} = \frac{\vec{R}}{R} \tag{22}$$

with $S_n(R)$ being the surface of n sphere. To calculate the integral in Minkowski space we have to make Wick rotation by replacing $r_0 \to ir_0$, hence in 4 dimensions $d^4r = id^4\vec{r}$ and

$$\Delta(a) = 2i\pi^2 a^\mu \lim_{R\to\infty} R^2 R_\mu\, f(R). \tag{23}$$

We have used the formula for n sphere (for even n):

$$S_n(R) = \frac{2\pi^{n/2}}{(n/2-1)!}R^{n-1} = \begin{cases} 2\pi R & \text{for} \quad n=2 \\ 2\pi^2 R^3 & \text{for} \quad n=4 \end{cases} . \tag{24}$$

Now we shall calculate what is the change of (1) if the integration momentum p is shifted by a four-vector

$$a = \alpha k_1 + (\alpha-\beta)k_2. \tag{25}$$

Let's define the difference

$$\Delta_{\mu\nu\lambda}(a) = T_{\mu\nu\lambda}(p\to p+a) - T_{\mu\nu\lambda} \tag{26}$$

where $T_{\mu\nu\lambda}$ is defined by (1). We have

$$\begin{aligned}
\Delta_{\mu\nu\lambda}(a) = & -\int\frac{d^4p}{(2\pi)^4}\left\{\mathrm{Tr}\left[\frac{1}{\not p+\not a-m}\gamma_\lambda\gamma_5\frac{1}{(\not p+\not a-\not q)-m}\gamma_\nu\frac{1}{(\not p+\not a-\not k_1)-m}\gamma_\mu\right]\right. \\
& \left. -\,\mathrm{Tr}\left[\frac{1}{\not p-m}\gamma_\lambda\gamma_5\frac{1}{(\not p-\not q)-m}\gamma_\nu\frac{1}{(\not p-\not k_1)-m}\gamma_\mu\right]\right\} \\
& + (\mu\longleftrightarrow\nu, k_1\leftrightarrow k_2)\,.
\end{aligned} \tag{27}$$

Expanding (27) according to (23) we arrive at

$$\begin{aligned}
\Delta_{\mu\nu\lambda}(a) = & -\int\frac{d^4p}{(2\pi)^4}a^\sigma\frac{\partial}{\partial p^\sigma}\,\mathrm{Tr}\left[\frac{1}{\not p-m}\gamma_\lambda\gamma_5\frac{1}{(\not p-\not q)-m}\gamma_\nu\frac{1}{(\not p-\not k_1)-m}\gamma_\mu\right] \\
& + (\mu\longleftrightarrow\nu, k_1\leftrightarrow k_2)\,.
\end{aligned} \tag{28}$$

Since we are interested in $p \to \infty$ we can neglect finite pieces in the denominator:

$$\Delta_{\mu\nu\lambda}(a) = -\frac{1}{(2\pi)^4} 2i\pi^2 a^\sigma \lim_{P\to\infty} P^2 P_\sigma \, \mathrm{Tr}\left[\not{P}\gamma_\lambda\gamma_5\not{P}\gamma_\nu\not{P}\gamma_\mu\right] \frac{1}{P^6} + (\mu \longleftrightarrow \nu, k_1 \leftrightarrow k_2)\,. \quad (29)$$

With the help of[1]

$$\mathrm{Tr}\left[\not{P}\gamma_\lambda\gamma_5\not{P}\gamma_\nu\not{P}\gamma_\mu\right] = 4iP^2\varepsilon_{\alpha\mu\nu\lambda}P^\alpha \quad (30)$$

we arrive at

$$\Delta_{\mu\nu\lambda}(a) = \frac{1}{(2\pi)^4} 8\pi^2 \varepsilon_{\mu\nu\lambda\alpha}\, a_\sigma \lim_{P\to\infty} \frac{P^\sigma P^\alpha}{P^2} + (\mu \longleftrightarrow \nu, k_1 \leftrightarrow k_2)\,. \quad (31)$$

Taking symmetric limit

$$\lim_{P\to\infty} \frac{P^\sigma P^\alpha}{P^2} = \frac{1}{4} g^{\sigma\alpha} \quad (32)$$

we obtain

$$\begin{aligned} \Delta_{\mu\nu\lambda}(a) &= \frac{1}{8\pi^2}\varepsilon_{\alpha\mu\nu\lambda} a^\alpha + (\mu \longleftrightarrow \nu, k_1 \leftrightarrow k_2) \\ &= \frac{1}{8\pi^2}\varepsilon_{\alpha\mu\nu\lambda}\left(\alpha k_1^\alpha + (\alpha-\beta)k_2^\alpha - \alpha k_2^\alpha - (\alpha-\beta)k_1^\alpha\right) \\ &= \frac{\beta}{8\pi^2}\varepsilon_{\alpha\mu\nu\lambda}\left(k_1 - k_2\right)^\alpha . \end{aligned} \quad (33)$$

We see that there is an ambiguity in $\Delta_{\mu\nu\lambda}$. At this moment β is a free parameter. We can fix it by imposing current conservation, however – as we will see – no β exists so that both vector and axial-vector currents ar conserved simultaneously.

Lets calculate $\Delta^{(1,2)}_{\mu\nu}$using the same trick with shifting the integration variable. Indeed

$$\begin{aligned} \Delta^{(1)}_{\mu\nu} &= \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\frac{i}{\not{p}-m}\gamma_5\gamma_\nu\frac{i}{(\not{p}-\not{k}_1)-m}\gamma_\mu - \frac{i}{(\not{p}-\not{k}_2)-m}\gamma_5\gamma_\nu\frac{i}{(\not{p}-\not{q})-m}\gamma_\mu\right] \quad (34) \\ &= \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{(\not{p}-\not{k}_2)-m}\gamma_5\gamma_\nu\frac{1}{(\not{p}-\not{k}_2-\not{k}_1)-m}\gamma_\mu - \frac{1}{\not{p}-m}\gamma_5\gamma_\nu\frac{1}{(\not{p}-\not{k}_1)-m}\gamma_\mu\right] \end{aligned}$$

where the first part in the second line corresponds to the second part with variable p shifted by $p \to p - k_2$ and therefore can be evaluated wit the help of (23) where $a = -k_2$:

$$\Delta^{(1)}_{\mu\nu} = -\frac{1}{(2\pi)^4} 2i\pi^2 k_2^\rho \lim_{P\to\infty} \frac{P_\rho}{P^2} \mathrm{Tr}\left[\not{P}\gamma_5\gamma_\nu(\not{P}-\not{k}_1)\gamma_\mu\right] . \quad (35)$$

Note that we have included k_1 term because the trace with $\not{P}\ldots\not{P}$ vanishes, and also terms proportional to m vanish. We have therefore

$$\begin{aligned} \Delta^{(1)}_{\mu\nu} &= \frac{1}{(2\pi)^4} 2i\pi^2 k_2^\rho k_1^\sigma \lim_{P\to\infty} \frac{P_\rho P^\alpha}{P^2} \mathrm{Tr}\left[\gamma_\alpha\gamma_5\gamma_\nu\gamma_\sigma\gamma_\mu\right] \\ &= \frac{1}{(2\pi)^4} 2i\pi^2 k_2^\rho k_1^\sigma \frac{1}{4}(-)\underbrace{\mathrm{Tr}\left[\gamma_5\gamma_\rho\gamma_\nu\gamma_\sigma\gamma_\mu\right]}_{4i\varepsilon_{\rho\nu\sigma\mu}} \\ &= -\frac{1}{8\pi^2}\varepsilon_{\mu\nu\sigma\rho}\, k_1^\sigma k_2^\rho . \end{aligned} \quad (36)$$

[1]Remember that $\varepsilon_{\alpha\mu\nu\lambda} = -\varepsilon^{\alpha\mu\nu\lambda}$

We obtain $\Delta^{(2)}_{\mu\nu}$ by $\mu \longleftrightarrow \nu, k_1 \leftrightarrow k_2$, hence

$$\Delta^{(1)}_{\mu\nu} = \Delta^{(2)}_{\mu\nu}. \tag{37}$$

Suppose that we would have initially labelled the momenta in a different way in the e.g. *second* diagram in Fig.1. Since $\Delta^{(1,2)}_{\mu\nu}$correspond to the difference of two pieces each one coming from a different diagram (see Eqs. (13) and (14)), we would get a different result for $\Delta^{(1,2)}_{\mu\nu}$.

To fix this ambiguity we proceed in the following way. We calculate the divergemces of acial and vecor currents for an arbitrary routing obtained from the original routing of Fig. 1 by a shift $p \to p + a$. To this end we get

$$\begin{aligned} q^\lambda T_{\mu\nu l}(a) &= q^\lambda \left(T_{\mu\nu l}(a) - T_{\mu\nu l}(0)\right) + q^\lambda T_{\mu\nu l}(0) \\ &= q^\lambda \Delta_{\mu\nu\lambda}(a) + 2mT_{\mu\nu} + \Delta^{(1)}_{\mu\nu} + \Delta^{(2)}_{\mu\nu} \\ &= 2mT_{\mu\nu} - \frac{1}{4\pi^2}\varepsilon_{\mu\nu\sigma\rho}\, k_1^\sigma k_2^\rho + (k_1+k_2)^\lambda \frac{\beta}{8\pi^2}\varepsilon_{\alpha\mu\nu\lambda}\,(k_1-k_2)^\alpha \\ &= 2mT_{\mu\nu} - \frac{1-\beta}{4\pi^2}\varepsilon_{\mu\nu\sigma\rho}\, k_1^\sigma k_2^\rho. \end{aligned} \tag{38}$$

We shall now apply the same procedure to calculate

$$\begin{aligned} k_1^\mu T_{\mu\nu\lambda}(a) &= k_1^\mu \left(T_{\mu\nu\lambda}(a) - T_{\mu\nu\lambda}(0)\right) + k_1^\mu T_{\mu\nu\lambda}(0) \\ &= k_1^\mu T_{\mu\nu\lambda}(0) + k_1^\mu \frac{\beta}{8\pi^2}\varepsilon_{\alpha\mu\nu\lambda}\,(k_1-k_2)^\alpha \\ &= k_1^\mu T_{\mu\nu\lambda}(0) + \frac{\beta}{8\pi^2}\varepsilon_{\nu\lambda\sigma\rho} k_1^\sigma k_2^\rho. \end{aligned} \tag{39}$$

Now we have to calculate $k_1^\mu T_{\mu\nu\lambda}(0)$ directly

$$\begin{aligned} k_1^\mu T_{\mu\nu\lambda} &= -\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left[\frac{1}{\not p - m}\gamma_\lambda\gamma_5 \frac{1}{(\not p - \not q) - m}\gamma_\nu \frac{1}{(\not p - \not k_1) - m}\not k_1\right] \\ &\quad -\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left[\frac{1}{\not p - m}\gamma_\lambda\gamma_5 \frac{1}{(\not p - \not q) - m}\not k_1 \frac{1}{(\not p - \not k_2) - m}\gamma_\nu\right] \end{aligned} \tag{40}$$

Now we shall use

$$\begin{aligned} \not k_1 &= (\not p - m) - \big((\not p - \not k_1) - m\big) \\ &= \big((\not p - \not k_2) - m\big) - \big((\not p - \not q) - m\big), \end{aligned} \tag{41}$$

which gives (see the beginning of this note)

$$\begin{aligned}
k_1^\mu T_{\mu\nu\lambda} &= -\int \frac{d^4p}{(2\pi)^4} \\
&\left\{ \mathrm{Tr}\left[\gamma_\lambda\gamma_5 \frac{1}{(\not p - \not q) - m}\gamma_\nu \frac{1}{(\not p - \not k_1) - m}\right] - \mathrm{Tr}\left[\frac{1}{\not p - m}\gamma_\lambda\gamma_5 \frac{1}{(\not p - \not q) - m}\gamma_\nu\right]\right. \\
&\left. + \mathrm{Tr}\left[\frac{1}{\not p - m}\gamma_\lambda\gamma_5 \frac{1}{(\not p - \not q) - m}\gamma_\nu\right] - \mathrm{Tr}\left[\frac{1}{\not p - m}\gamma_\lambda\gamma_5 \frac{1}{(\not p - \not k_2) - m}\gamma_\nu\right]\right\} \\
&= -\int \frac{d^4p}{(2\pi)^4} \qquad (42) \\
&\left\{ \mathrm{Tr}\left[\gamma_\lambda\gamma_5 \frac{1}{(\not p - \not q) - m}\gamma_\nu \frac{1}{(\not p - \not k_1) - m}\right] - \mathrm{Tr}\left[\gamma_\lambda\gamma_5 \frac{1}{(\not p - \not k_2) - m}\gamma_\nu \frac{1}{\not p - m}\right]\right\}.
\end{aligned}$$

We see that the first piece can be obtained from the second one by the shift $p \to p - k_1$ and can be evaluated by (23):

$$\begin{aligned}
k_1^\mu T_{\mu\nu\lambda} &= -\frac{1}{(2\pi)^4} 2i\pi^2 (-) k_1^\sigma \lim_{P\to\infty} \frac{P_\sigma}{P^2} \mathrm{Tr}\left[\gamma_\lambda\gamma_5 (\not P - \not k_2)\gamma_\nu \not P\right] \\
&= -\frac{1}{8\pi^2} i \frac{1}{4} \mathrm{Tr}\left[\gamma_\lambda\gamma_5\gamma_\rho\gamma_\nu\gamma_\sigma\right] k_1^\sigma k_2^\rho \\
&= \frac{1}{8\pi^2}\varepsilon_{\nu\lambda\sigma\rho} k_1^\sigma k_2^\rho. \qquad (43)
\end{aligned}$$

Hence

$$k_1^\mu T_{\mu\nu l}(\beta) = \frac{1+\beta}{8\pi^2}\varepsilon_{\nu\lambda\sigma\rho} k_1^\sigma k_2^\rho. \qquad (44)$$

We see that it is impossible to maintain both Ward identities (38) and (44) by a suitable choice of β. Because we know that vector current (charge) is conserved, we are forced to choose $\beta = -1$.Then

$$q^\lambda T_{\mu\nu l} = 2m T_{\mu\nu} - \frac{1}{2\pi^2}\varepsilon_{\mu\nu\sigma\rho}\, k_1^\sigma k_2^\rho, \qquad (45)$$

which means that axial current is *anomalous*. This can be translated to the configuration space

$$\partial^\lambda A_\lambda(x) = \frac{1}{(4\pi)^2}\varepsilon_{\mu\nu\sigma\rho} F^{\mu\nu}(x) F^{\sigma\rho}(x) + \mathcal{O}(m). \qquad (46)$$