QCD 2025/26

problem set 11

Tuesday, January 20, 2026

1. Generating functional for electrodynamics in a covariant gauge is defined as:

$$\begin{aligned} Z_0[j^\mu] &= \int [D\omega] \int [DA^\mu] \exp\left(-i\frac{\xi}{2}\int d^4x\,\omega^2\right) \\ &\times \delta(\partial_\mu A^\mu - \omega)\exp\left(i\int d^4x\left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + j_\mu A^\mu\right)\right), \end{aligned}$$

   where ω, A^μ and $F^{\mu\nu}$ are functions of x. Show that it is equal to

$$Z_0[j^\mu] = \int [DA^\mu]\,\exp\left(i\int d^4x\left(\frac{1}{2}A^\mu(g_{\mu\nu}\Box - (1-\xi)\partial_\mu\partial_\nu\right)A^\nu + j_\mu A^\mu\right).$$

   Transform this expression to momentum space.

2. In QCD infinitesimal change of the gauge field under the gauge transformation

$$\Omega(x) = \exp(i\theta_a(x)T^a)$$

   can be calculated from

$$\boldsymbol{A}_\mu^\Omega = \Omega^\dagger(x)\boldsymbol{A}_\mu\Omega(x) + \frac{i}{g}\Omega^\dagger(x)\partial_\mu\Omega(x)$$

   and reads (show it):

$$g\,\delta A_\mu^a = g f^{abc}\theta_b(x)A_\mu^c - \partial_\mu\theta_a(x).$$

   Calculate the change of the gauge condition

$$G^a(A_\mu) = n^\mu A_\mu^a$$

   $g\,\delta G^a$ under this transformation.

3. Next calculate matrix

$$\mathcal{M}_{ab} = g\frac{\delta G^a}{\delta\theta_b}$$

   and then rewrite the free part of the ghost action

$$\int d^4x\mathcal{L}_{FPG}^0 = \int d^4x\bar{\chi}_a\mathcal{M}_{ab}(g=0)\chi_b$$

   in momentum space. The inverse of this term is the ghost propagator.

4. Applying gauge condition from problem 2 to the free part of the gluon action calculate the gluon propagator (inverse of gluonic operator derived from the free part of $F_{\mu\nu}F^{\mu\nu}$) in the axial gauge.

5. For a system of SU(3) scalar fields $\hat{\phi}_i(x)$ with $i = 1, 2, 3$ that satisfy the following commutation rules

$$\left[\hat{\phi}_i(t, \vec{x}), \hat{\pi}_j(t, \vec{x}\,')\right] = i\delta^{(3)}(\vec{x} - \vec{x}\,')\delta_{ij}$$

one defines charge operators

$$\hat{Q}^a(t) = -i \int d^3\vec{x}\, \hat{\pi}_i(t, \vec{x}) T^a_{ij} \hat{\phi}_j(t, \vec{x})$$

where matrices T^a satisfy SU(3) commutation relations:

$$\left[T^a, T^b\right] = if^{abc}T^c.$$

Prove that

$$\left[\hat{Q}^a(t), \hat{Q}^b(t)\right] = if^{abc}\hat{Q}^c(t).$$

6. In the case of fermion fields, commutation relations of scalar fields are replaced by anticommutation relations:

$$\left\{q_{\alpha,k}(t, \vec{x}), q^\dagger_{\beta,l}(t, \vec{x}\,')\right\} = \delta^{(3)}(\vec{x} - \vec{x}\,')\delta_{\alpha\beta}\delta_{kl}$$

where α, β stand for Dirac indices and k, l denote SU(3) indices. Relevant charges are defined as

$$\begin{aligned}
\hat{Q}^a_{L,R}(t) &= \int d^3\vec{x}\, q^\dagger_{L,R}(t, \vec{x}) T^a q_{L,R}(t, \vec{x}), \\
\hat{Q}_V(t) &= \int d^3\vec{x}\, \left[q^\dagger_L(t, \vec{x}) q_L(t, \vec{x}) + q^\dagger_R(t, \vec{x}) q_R(t, \vec{x})\right]
\end{aligned}$$

where $T^a = \lambda^a/a$ are SU(3) generators (Gell-Mann matrices). Making use of the identity (prove it!)

$$\{ab, cd\} = a\{b, c\}d - ac\{b, d\} + \{a, c\}db - c\{a, d\}b$$

show that

$$\begin{aligned}
\left[\hat{Q}^a_L, \hat{Q}^b_L\right] &= if^{abc}\hat{Q}^c_L, \\
\left[\hat{Q}^a_R, \hat{Q}^b_R\right] &= if^{abc}\hat{Q}^c_R, \\
\left[\hat{Q}^a_L, \hat{Q}^b_R\right] &= 0, \\
\left[\hat{Q}^a_{L,R}, \hat{Q}_V\right] &= 0.
\end{aligned}$$