SMOLUCHOWSKI LECTURE

PROBLEMS 3 – 5

3. **Problem**: Give the proof of the K-theorem (K > 0, dK / dt < 0) for several special stochastic processes:
   (i) the Smoluchowski equation,
   (ii) the Fokker-Planck equation,
   (iii) the Pauli equation,
   (iv) the Boltzmann equation in relaxation approximation.
Discuss the meaning of the K-functional.

4. **Problem**: Demonstrate by analytical considerations and/or by computer simulations for simple examples that the trajectories fill – under certain conditions - densely the phase space. Study (following Lecture Notes 3-4) the tent map and the Hamilton dynamics H = -a q + b p or look for other simple cases. Discuss the conditions for ergodic behaviour and master equations.

5. **Problem**: Study stochastic properties - similar as in problem 4 - of the two-dimensional Hamiltonian map, the Chirikov-Taylor standard map
   \[ x(t+1) = x(t) - (K/2 \pi) \sin (2 \pi y(t)) \]
   \[ y(t+1) = y(t) + x(t+1) \pmod{1}. \]
Try first to find a Perron-Frobenius equation for the phase-space density f(x,y,t) (or better for its Fourier series) and then a diffusion-like equation for the density n(x, t) neglecting memory effects.