The impact of the sigma meson on the freeze-out, pion condensation and proton puzzle at the LHC

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The phase diagram of strongly interacting matter

Thermal model gives the freeze-out curve
The fit of the LHC data gives the parameters that fall out to the "wrong" side

The prediction of thermal models gave too high ratios to pions, especially proton to pion ratio.

The best fit of the LHC data gives three standard deviations for protons.
Problems of hydrodynamic models with the pion spectra at the LHC

IP - Glasma + MUSIC:

pion enhancement

AdS + hydro + cascade:

pions well described, protons?

Hydro with dynamical freeze-out:

pion enhancement

pions well described, protons?
The phase-space distribution of the primordial particles has the form:

\[ f_i = g_i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\gamma_i^{-1} \exp(\sqrt{m^2 + p^2 / T}) \pm 1}, \text{ where } \gamma_i = \gamma_q^{N_{q_i}^I + N_{q_i}^\bar{I}} \gamma_s^{N_{s_i}^I + N_{s_i}^\bar{I}} \exp\left(\frac{\mu_B B_i + \mu_S S_i}{T}\right), \]

and \( N_{q_i}^I, N_{s_i}^I \) are the numbers of light \((u, d)\) and strange \((s)\) quarks in the \(i\)th hadron. It includes all well established resonances from PDG. Resonance decay according to their branching ratios.

**Single-freeze out model** (Broniowski, Florkowski, Phys. Rev. Lett. (2001))


The spectra are calculated from the Cooper-Frye formula at the freeze-out hyper surface

\[ \frac{dN}{dy d^2 \mathbf{p}_T} = \int d\Sigma \mu p^\mu f(p \cdot u), \quad t^2 = \tau_f^2 + x^2 + y^2 + z^2, \quad x^2 + y^2 \leq r_{\text{max}}^2, \]

assuming the Hubble-like flow: \( u^\mu = x^\mu / \tau_f. \)

There is **only one additional parameter** in the model, because the product \( \pi \tau_f r_{\text{max}}^2 \) is equal to the volume (per unit rapidity), while the ratio \( r_{\text{max}} / \tau_f \) determines the slope of the spectra.
Spectra of pions in Cracow model at the LHC. Linear scale

The fits to the ratios of hadron abundances (Petran, Letessier, Petracek, Rafelski, Phys. Rev. C (2013)) yield $\gamma_q$ which is very close to the critical pion chemical potential

$$\mu_\pi = 2T \ln \gamma_q \approx 134 \text{ MeV}$$

$$\approx m_{\pi^0} \approx 134.98 \text{ MeV}$$

It may suggest that a substantial part of $\pi^0$ mesons form the condensate.

The calculations of the pion spectra support the formation of the condensate at the LHC

Can the LHC data be explained by the updated sigma?

- The recent PDG reviews report much **lower mass** and width of the $f_0(500)$ or the **sigma** meson.

- The lower mass of the $\sigma$ would result in its **higher multiplicity**. It decays into pions, therefore it **could add** some of the **missing pions**.

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In thermal models the calculations are performed using the sum of contributions of all (stable and resonance) hadrons to the partition function

\[ \ln Z = \sum_k \ln Z_k^{\text{stable}} + \sum_k \ln Z_k^{\text{res}} \]

In practice, one uses the list of existing particles from the PDG.

In the limit where the decay widths of resonances and chemical potentials are neglected, one has

\[ \ln Z_k^{\text{stable, res}} = g_k V \int \frac{d^3p}{(2\pi)^3} \ln \left[ 1 \pm e^{-E_p/T} \right] \]

where \( g_k \) is the spin-isospin degeneracy, \( V \) – volume, \( p \) – momentum, \( M_k \) – the mass of the resonance, \( E_p = \sqrt{p^2 + M_k^2} \) – the energy, and the \( \pm \) corresponds to fermions or bosons.

As a better approximation for the partition function, one can take into account the finite widths of resonances:

\[ \ln Z_k^{\text{res}} = g_k V \int_0^\infty d_k(M) dM \int \frac{d^3p}{(2\pi)^3} \ln \left[ 1 - e^{-E_p/T} \right]^{-1} \]

For narrow resonances one can approximate \( d_k(M) \) with a (non-relativistic or relativistic) normalized Breit-Wigner function peaked at \( M_k \).
The 2 → 2 reactions are incorporated according to the formalism of Dashen, Ma, Bernstein, and Rajaraman. The mass distribution is given by the physical phase shifts $\delta$:

$$d_k(M) = \frac{d\delta(M)}{\pi dM}$$

One can get it for the relative radial wave function of a pair of scattered particles with angular momentum $l$, interacting with a central potential, which has the asymptotic

$$\psi_l(r) \propto \sin[kr - l\pi/2 + \delta]$$

where $k = |\vec{k}|$ is the length of the three-momentum, and $\delta$ is the phase shift. If we confine our system into a sphere of radius $R$, the condition

$$kR - l\pi/2 + \delta = n\pi \quad \text{with} \quad n = 0, 1, 2, \ldots$$

must be met, since $\psi_l(r)$ has to vanish at the boundary. Analogously, in a free system

$$kR - l\pi/2 = n_{\text{free}}\pi$$

In the limit $R \to \infty$, upon subtraction,

$$\delta = (n - n_{\text{free}})\pi$$

Differentiation with respect to $M$ yields the distribution $d\delta/(\pi dM)$.
The experimental $\pi\pi$ phase shifts and their derivatives

The isospin-spin channel $(0,0)$ that is responsible for the emergence of the $f_0(500)$ pole is attractive, while the channel $(2,0)$ is repulsive.

![Graph of phase shifts](image)

The derivative of the phase shift for the resonance $(0,0)$ is almost cancelled by the derivative of the $(2,0)$ channel until $f_0(980)$ takes over above $M \sim 0.85$ GeV.

This is achieved with the multiplication of the isotensor channel by the isospin degeneracy factor $(2I + 1) = 5$, which occurs for isospin-averaged quantities.

The case of $K_0^*(800)$

The attractive $\pi K$ channel with $I = 1/2$ and $J = 0$ is capable of the generation of a pole corresponding to the $K_0^*(800)$ or the $\kappa$ resonance. It is not included in the PDG, but is naturally expected to exist.

One can see that there is a **partial cancellation** and one can neglect the $K_0^*(800)$.

The error one would make by including the sigma

The **cancellation occurs** at the level of the distribution functions $d_k(M)$, therefore it persists in all isospin-averaged observables.

Left: the pions coming from the $\sigma$ decay. Right: the contribution to pressure, entropy, energy density and the "interaction measure" $\Delta = (\varepsilon - 3p)/T^4$.

The $\sigma$ is implemented as a Breit-Winger pole with $M_\sigma = 484$ and $\Gamma_\sigma/2 = 255$ MeV.

The famous $K/\pi$ horn is **affected**, as well as all ratios to pions.

V.B., Broniowski, Giacosa, arXiv:1506.01260
Conclusions

- The **contribution of** the resonance $f_0(500)$ or **sigma** meson to isospin-averaged observables like thermodynamic functions, pion yields, etc. **is cancelled** by the **repulsion** from the isotensor-scalar channel.

- The cancellation occurs from a “conspiracy” of the isospin degeneracy factor and the derivative of the phase shifts.

- There is **no cancellation** mechanism in correlation studies of pion pair production.

- We thus clearly see the potential importance of the $\sigma$ in studies of pion correlations.

- The **ratios** of particle multiplicities involving pions are **affected** up to 6%.

- The **cancellation enhances** the proton-pion puzzle at the LHC and **opens** even more **space** for possible **novel** interpretations.