where is the problem?
hadronic final states - jets, heavy quarks - even at Tevatron
approximations

doing it better!
CCFM equation, solution
implementation into new hadron level MC CASCADE and LDC
solve problems, also for heavy quarks and even for Tevatron

conclusion
Recent results from CCFM Evolution
dedicated to the memory of Jan Kwiecinski

H. Jung, University of Lund
ISMD03, Cracow, 8. Sept 2003

- where is the problem?
  hadronic final states - jets, heavy quarks - even at Tevatron
  approximations

- doing it better!
  CCFM equation, solution
  implementation into new hadron level MC CASCADE and LDC

- solve problems, also for heavy quarks and even for Tevatron

- conclusion

- many topics Jan had suggested and investigated:
  CCFM, unintegrated pdf, forward jets, $\phi$ decorrelation etc ...
The structure function $F_2(x, Q^2)$: DGLAP

$F_2(x, Q^2) = \sum_i e_i^2 x q_i(x, Q^2)$

$x = \frac{Q^2}{W^2 + Q^2}$

Scaling violations perfectly described with DGLAP:

0.63 $\cdot$ $10^{-5}$ < $x$ < 0.65

1 < $Q^2$ < 25000 GeV$^2$

- adjust input pdf to fit $F_2$ data
- BUT different sets: MRS, GRV etc

- use extracted pdf to predict $x$ - sections
- even at $p\bar{p}$

BUT for reliable predictions at HERA II, HERA III, Tevatron, LHC etc

- better understand pdf’s
Where is the problem?
Bottom at HERA and Tevatron

HERA

ZEUS (ZEUS Coll. *EPJC* (2001))

\[ Q^2 < 1 \text{ GeV}^2, \ 0.2 < y < 0.8, \]

\[ p_T^b > 5 \text{ GeV}, |\eta^b| < 2 \]

\[ \sigma = 1.6 \pm 0.4 (\text{stat.})^{+0.3}_{-0.5} (\text{syst.})^{+0.2}_{-0.4} (\text{ext.}) \text{ nb} \]

NLO: \[ \sigma = 0.64^{+0.15}_{-0.1} \text{ nb} \]

safe extrapolation from visible to total x-section ???

\[ b\bar{b} \text{ in photoproduction and hadroproduction: } \]

- standard DGLAP with NLO calculation
- \( \sim \) factor 2 - 4 too small!

D0 Data
Where is the problem: Di - jets in DIS

- $\phi$ - decorrelation (J. Kwiecinski et al)
- Measurement of $\frac{d\sigma}{d\Delta\phi}$ exp. difficult
- Measure (A. Szczurek):
  $$S(x, Q^2, \Delta\Phi) = \frac{\int_{\Delta\Phi^*}^{120^\circ} d\sigma d\Phi}{\int_{180^\circ}^{120^\circ} d\sigma d\Phi}$$

Data much higher than:
- $\text{NLO-}\mathcal{O}(\alpha_s)$
- $\text{NLO-}\mathcal{O}(\alpha_s^2)$

new dynamics ???
Where is the problem? Forward Jets

Mueller - Navelet jets in DIS: Jet in $p$ - direction with

$$p_t^2 \sim Q^2, \quad x_{jet} \text{ large, BUT small } x_{bj}$$

- suppress DGLAP evolution allow evolution in $x$
- standard DGLAP $\sim$ factor 2 too small!
Approximation in QCD cascade: Factorization

Gluon Bremsstrahlung:
\[ \sim \frac{1}{k_t^2} \left( \frac{1}{z} + \cdots \right) \]

\( x \)
\( z \)
\( k \)
\( p_t \)
\( x \)

\( 1/E \)

\( \text{collinear approximation} \)
\( \text{collinear factorization} \)
\( \text{`small x` approximation} \)
\( k_t \text{ factorization} \)

DGLAP:
- collinear singularities
  factorized in pdf
- evolution in \( Q^2 \sim k_t^2 \), \( k_t^2 \) or \( p_t^2 \)
- \( \sigma = \sigma_0 \int \frac{dz}{z} C^a \left( \frac{x}{z} \right) f_a(z, Q^2) \)

BFKL:
- \( k_t \) dependent pdf
  → unintegrated pdf
- evolution in \( x \)
- \( \sigma = \int \frac{dz}{z} d^2 k_t \hat{\sigma} \left( \frac{x}{z}, k_t \right) F(z, k_t) \)
The problem of Asymptotia...

DGLAP is great at large $Q^2 \to \infty$

But has problems:

- Small $x$ processes:
  - heavy quarks
  - particle spectra
  - jets

BFKL is great at small $x \to 0$

But has problems:

- at finite $x$:
  - NLO corrections
  - predictive power
  - how to simulate?

But asymptotia still far away even for LHC and ...
Attempts to survive in reality

- hack DGLAP and BFKL prediction?
- introduce new concepts: resolved (virtual) photons
  - evolve with DGLAP from proton and photon side
  - similar to $p\bar{p}$
  - works nicely ($\rightarrow$ RAPGAP MC generator)
- **BUT** theoretical questions: which scale etc ???
- **CCFM** - new investigation of color coherence
Attempts to survive in reality

- hack DGLAP and BFKL prediction?
- introduce new concepts: resolved (virtual) photons
  - evolve with DGLAP from proton and photon side
  - similar to $p \bar{p}$
  - works nicely ($\rightarrow$ RAPGAP MC generator)
- **BUT** theoretical questions: which scale etc ???
- **CCFM** - new investigation of color coherence

hacker: person paid to do hard and uninteresting work ...

Oxford Advanced Dictionary
including color coherence effects in multi-gluon emissions

angular ordering of emission angles:

ordering in $q$ (DGLAP) implies also angular ordering
unification of DGLAP and BFKL

**WOW**

for small $z$ no restriction in $q$: random walk in $q$
including color coherence effects in multi-gluon emissions

angular ordering of emission angles:

\[
\begin{align*}
E_i & \quad \theta_i \quad q_i \\
E_{i-1} & \quad \theta_{i-1} \quad q_{i-1}
\end{align*}
\]

\[
p_{ti} = |q_i^0| \sin \Theta_i, \quad z = \frac{E_i}{E_{i-1}}
\]

\[
E_{i-1} = E_i + q_i^0 = z E_{i-1} + q_i^0, \quad q_i^0 = (1 - z) E_{i-1}
\]

\[
p_{ti} = q_i^0 \sin \Theta_i \sim (1 - z) E_{i-1} \Theta_i
\]

\[
\frac{p_{ti}}{z} \sim E_{i-1} \Theta_i
\]

with: \( q_i = \frac{p_{ti}}{z_i} \) \( \Theta_i = \frac{q_i}{E_{i-1}} \) and \( \Theta_{i+1} = \frac{q_i + 1}{E_i} \)

ordering in \( q \) (DGLAP) implies also angular ordering

unification of DGLAP and BFKL


\[
\text{WOW}
\]

for small \( z \) no restriction in \( q \): \( \text{random walk in } q \)
including color coherence effects in multi-gluon emissions

angular ordering of emission angles:

\[ \Theta_{i+1} > \Theta_i \]
\[ q_{i+1} > z_i q_i \]

with \( q = \frac{p_t}{1-z} \)

ordering in \( q \) (DGLAP) implies also angular ordering

unification of DGLAP and BFKL

**WOW**

for small \( z \) no restriction in \( q \): random walk in \( q \)
CCFM solves the problems

Solve CCFM equation to fit $F_2$ data from HERA

- obtain CCFM un-integrated gluon
- CASCADE MC implements CCFM:
  - predict fwd jet x-section at HERA ✔
  - predict charm at HERA ✔
  - predict bottom at HERA ✔

- test universality of un-integrated gluon density from HERA
- predict bottom at Tevatron ✔
- w/o additional free parameters

WOW !!!
Basic idea - $k_t$ factorisation

\[ \sigma(e p \to e' q \bar{q}) = \int \frac{d y}{y} d^2 Q \frac{d x_g}{x_g} \int d^2 k_t \hat{\sigma}(\hat{s}, k_t, Q) x_g A(x_g, k_t, \bar{q}) \]

with

\[ \int d^2 k_t x_g A(x_g, k_t, \bar{q}) \approx x_g G(x_g, Q^2) \]

BGF matrix element

evolution of parton cascade

initial distribution:
Basic idea - $k_t$ factorisation

CCFM

- BGF matrix element off mass shell
- evolution of parton cascade with CCFM splitting fct.

\[ \tilde{P} = \bar{\alpha}_s \left( \frac{1}{1-z} + \frac{1}{z} \Delta_{\text{ns}} + \ldots \right) \]

- initial distribution: flat?

\[
\sigma(ep \rightarrow e' q\bar{q}) = \int \frac{du}{y} d^2 Q \frac{dx_g}{x_g} \int d^2 k_t \hat{\sigma}(\hat{s}, k_t, Q)x_g A(x_g, k_t, \bar{q}) \\
\text{with } \int d^2 k_t x_g A(x_g, k_t, \bar{q}) \simeq x_g G(x_g, Q^2)
\]
CCFM equation: small and large $x$

$$A(x, k_t, \bar{q}) = A_0(x, k_t) \Delta_s(\bar{q}, Q_0) + \int \frac{dz}{z} \int \frac{d^2q}{\pi q^2} \Theta(\bar{q} - zq) \cdot \Delta_s(\bar{q}, zq) \tilde{P}(z, q, k_t) A\left(\frac{x}{z}, k'_t, q\right)$$

**CCFM Splitting fct:**

$$\tilde{P}(z, q, k_t) = \frac{\tilde{\alpha}_s(q(1-z))}{1-z} + \frac{\tilde{\alpha}_s(k_t)}{z} \Delta_{ns}(z, q, k_t)$$

**Sudakov $\Delta_s(a, b):$**

probability for no radiation in $[a, b]$

**angular ordering:**

$$\bar{q} > z_n q_n, q_n > z_{n-1} q_{n-1}, \ldots, q_1 > Q_0$$

**small $x$:**

- BFKL limit ($z \to 0$)
- angular ordering
- no restriction on $q_i$

**large $x$:**

- DGLAP limit ($z \gg 0$)
- DGLAP splitting fct $\tilde{P}$ with $\Delta_{ns} = 1$
- angular ordering $\to q_i$ ordering
Non-Sudakov and all - loop resummation

Splitting Fct: \[ \tilde{P} = \frac{\tilde{\alpha}_s(q(1-z))}{1-z} + \frac{\tilde{\alpha}_s(k_t)}{z} \Delta_{ns}(z, q, k_t) \]

Non - Sudakov form factor ➤ all loop resummation:

\[ \Delta_{ns} = \exp \left[ -\tilde{\alpha}_s(k_t^2) \int_0^1 \frac{dz'}{z'} \int \frac{dq^2}{q^2} \Theta(k_t - q)\Theta(q - z'q_t) \right] \]
\[ \Delta_{ns} = 1 + \left( -\tilde{\alpha}_s(k_t^2) \int \frac{dz'}{z'} \int \frac{dq^2}{q^2} \right) + \frac{1}{2!} \left( -\tilde{\alpha}_s(k_t^2) \int \frac{dz'}{z'} \int \frac{dq^2}{q^2} \right)^2 + \ldots \]
Non-Sudakov and all-loop resummation

Splitting Fct: \[ \tilde{P} = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z} \Delta_{ns}(z, q, k_t) \]

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\[ \bar{\alpha}_s(k_t) \frac{1}{z} \left[ 1 \right] \]
Non-Sudakov and all-loop resummation

**Splitting Fct:**

\[ \tilde{P} = \bar{\alpha}_s(q(1-z)) \frac{1}{1-z} + \bar{\alpha}_s(k_t) \Delta_{ns}(z, q, k_t) \]

**Non-Sudakov form factor ➤ all loop resummation:**

\[ \Delta_{ns} = \exp \left[ -\bar{\alpha}_s(k_t^2) \int_0^1 \frac{dz'}{z'} \int \frac{dq^2}{q^2} \Theta(k_t - q) \Theta(q - z'q_t) \right] \]

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\[ \bar{\alpha}_s(k_t) \frac{1}{z} \left[ 1 + \bar{\alpha}_s \log \left( \frac{z}{z_0} \right) \log \left( \frac{k_t^2}{z_0 q^2} \right) \right] \]
Non-Sudakov and all-loop resummation

Splitting Fct: \[ \tilde{P} = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z} \Delta_{ns}(z, q, k_t) \]

Non-Sudakov form factor ➤ all loop resummation:

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\[ \bar{\alpha}_s(k_t^2) \frac{1}{z} \left[ 1 + \bar{\alpha}_s \log \left( \frac{z}{z_0} \right) \log \left( \frac{k_t^2}{z_0zq^2} \right) + \frac{1}{2!} \left( \bar{\alpha}_s \log \left( \frac{z}{z_0} \right) \log \left( \frac{k_t^2}{z_0zq^2} \right) \right)^2 + \ldots \right] \]
Advantage of CCFM: parton emissions

- DGLAP or BFKL
- Only inclusive predictions
- No info on emitted partons !!!

CCFM treats explicitly:
- Partons emitted during cascade
- Color coherence
- Energy momentum conservation

Best to implement in MC generator

Compare evolution and MC

Cascade MC generator

LDC MC generator

Evolution - MC parton shower comparison never shown for DGLAP type MC's !!!
CCFM backward evolution implemented in MC generator CASCADE (http://www.quark.lu.se/hannes/cascade)

initial state CCFM cascade with strict angular ordering

off-shell hard scattering processes:

\[ \gamma g^* \rightarrow q\bar{q}, \gamma^* g^* \rightarrow Q\bar{Q}, \gamma g^* \rightarrow J/\psi g, \gamma\gamma \rightarrow Q\bar{Q} \]

\[ g^* g^* \rightarrow q\bar{q}, g^* g^* \rightarrow Q\bar{Q}, g^* g^* \rightarrow h \]

\(P\)-remnant treatment like in PYTHIA (q-di-\(q\), primordial \(k_t\))

final state parton showers added to quarks hadronization via JETSET/PYTHIA

CASCADE is MC implementation of CCFM for \(ep, ee, \gamma\gamma\) and also for \(p\bar{p}\)
**L**inked **D**ipole **C**hain is reformulation of CCFM

- redefinition of initial and final emissions
- $q_{\perp i} > \min(k_{\perp i}, k_{\perp i-1})$
- to cancel non-Sudakov $\Delta_{n,s}$
- forward - backward symmetry
- evolve from proton side or from photon side
- essentially one scale unintegrated pdf

**MC generator** LDCMC (http://www.thep.lu.se/leif/ariadne/)

- also final state cascade included
- optionally full splitting functions and quark ladders
- hadronization via JETSET/PYTHIA

LDCMC is MC implementation of LDC
Structure Function $F_2(x, Q^2)$

Together with G.P. Salam, EPJC 19, 351 (2001)

With $\sigma = \int dk_t^2 dx_g A(x_g, k_t^2, \bar{q})\sigma(\gamma^* g^* \to q\bar{q})$ fit $F_2(x, Q^2)$

(data from H1 Coll, NPB 470 (1996) 3.)

Parameters in fit
(fitted for $Q^2 > 5$ GeV$^2$, $x < 10^{-2}$)

- collinear cut-off
  $Q_0 = 1.4$ GeV
- initial gluon $x A_0(x, k_t^2)$
- freezing of $\alpha_s(k_t)$ for $k_t \to 0$
  $k_t$ not constrained ...
- light quark masses:
  $m_q = 0.250$ GeV,
  $m_c = 1.5$ GeV

unintegrated gluon density $x A(x, k_t^2, \bar{q})$
obtained from fit to $F_2$
Precision fits to $F_2(x, Q^2)$

With $\sigma = \int dk_t^2 dx_g A(x_g, k_t^2, \bar{q}) \sigma(\gamma^* g^* \rightarrow q \bar{q})$ fit $F_2(x, Q^2)$

- more precise data:
- fit $Q^2 > 4.5$ GeV$^2$, $x < 0.005$
- small $k_t^2$ - region?
- full splitting function?

<table>
<thead>
<tr>
<th>Fits to $F_2(x, Q^2)$</th>
<th>$k_t^{cut}$</th>
<th>$\chi^2/ndf$</th>
</tr>
</thead>
<tbody>
<tr>
<td>set</td>
<td>(GeV)</td>
<td>ndf = 248</td>
</tr>
<tr>
<td>JS2001</td>
<td>0.25</td>
<td>4.8</td>
</tr>
<tr>
<td>$k_t^{cut} = Q_0$</td>
<td>1.33</td>
<td>1.29</td>
</tr>
<tr>
<td>full splitting</td>
<td>1.18</td>
<td>1.18</td>
</tr>
</tbody>
</table>
### Unintegrated gluon density

#### JS (CCFM) gluon
H. Jung, G.P. Salam, EPJC 19, 351 (2001)

- Constrained from $F_2$ fit only for $x_g < 0.03$
- For HERA $F_2 \ x < 0.01, Q^2 > 5$

#### LDC gluonic

- Only gluons
- Including full splitting fct
- For HERA $F_2 \ x < 0.013, Q^2 > 3.5$

#### J2003 sets
- For HERA $F_2 \ x < 0.005, Q^2 > 4.5$
- At $\bar{q} = 10$ GeV:
  - Small $k_t$ region
  - Full splitting
CCFM unintegrated gluon density in transverse coordinate representation

Jan Kwiecinski

\[ x\mathcal{A}(x, k_t, \bar{q}) = \int d b b J_0(k_t b) x\bar{A}(x, b, \bar{q}) \]

- use one loop approximation
- without non-sudakov
- understand interplay of \( k_t \) and \( \bar{q} \)
- \( x\bar{A}(x, 0, \bar{q}) = 0.5 x g(x, Q^2) \)
- with starting distribution \( x g_0(x) = 3(1 - x)^5 \)
- solve exactly
- compare to MC solution of CCFM
- connection to dipole models....
Solution to the problem: Forward Jets

Require jets with $p_t > 3.5$ GeV and $0.5 < E_t^2/Q^2 < 2$

- CASCADE (and LDCMC) well in shape and normalization !!!
- NLO off as expected from DGLAP type evolution

H1 Forward Jet Data

- $x_{bj}$ small
- Evolution from large to small $x$
- 'Forward' jet $x_{jet} = E_{jet}/E_{proton} = large$

$H1$ data, prel.
Where is the problem: Di - jets in DIS

\[ S(x, Q^2, \Delta \Phi) = \int_0^{120^\circ} d\sigma d\Phi \int_0^{180^\circ} d\sigma d\Phi \]

- **NLO-** \( \mathcal{O}(\alpha_s) \) and **NLO-** \( \mathcal{O}(\alpha_s^2) \) too small at small \( x \)
- **CCFM** unintegrated gluon needed for rise at small \( x \) !!!

ISMD03, Cracow, 8. Sept 2003 – p.22
**Production at HERA: H1 and ZEUS**

**H1** (H1 Coll. *PLB* 467 (1999) 156)

\[ Q^2 < 1 \text{ GeV}^2, \ 0.1 < y < 0.8, \]
\[ p_t^\mu > 2 \text{ GeV}, \ 35^\circ < \theta^\mu < 130^\circ \]

**Visible x-section** \( ep \to b\bar{b}X \to \mu X \):

\[ \sigma_{vis} = 176 \pm 16(\text{stat.})^{+26}_{-17}(\text{syst.}) \text{ pb} \]

**NLO:** \( \sigma = 54 \pm 9 \text{ pb} \)

CASCADE \( \sigma(ep \to e' b\bar{b}X \to \mu X) = 65 \text{ pb} \)

**H1**

\[ R_{MC}(H1) = \frac{\sigma_{data}}{\sigma_{MC}} = 2.7 \pm 0.25^{+0.4}_{-0.26} \]

**ZEUS** (ZEUS Coll. *EPJC* (2001))

\[ Q^2 < 1 \text{ GeV}^2, \ 0.2 < y < 0.8, \]
\[ p_t^b > 5 \text{ GeV}, \ |\eta^b| < 2 \]

\[ \sigma = 1.6 \pm 0.4(\text{stat.})^{+0.3}_{-0.5}(\text{syst.})^{+0.2}_{-0.4}(\text{ext.}) \text{ nb} \]

**NLO:** \( \sigma = 0.64^{+0.15}_{-0.1} \text{ nb} \)

CASCADE \( \sigma(ep \to e' b\bar{b}X) = 0.88 \pm 0.08 \text{ nb} \)

\[ R_{MC}(ZEUS) = \frac{\sigma_{data}}{\sigma_{MC}} = 1.9 \pm 0.45^{+0.34}_{-0.57} \]

Lightning bolt: Measurements rely on large extrapolation from visible to total x-section. Really safe ?????
Solution to the problem at HERA: $\bar{b}b$

- large extrapolation from visible to total x-section
- look at visible x-section

**CASCADE ~ ok for visible $\mu$’s (similar to NLO)**
Solution to the problem: $b\bar{b}$ production at Tevatron

Test universality of unintegrated gluon density from HERA

- use unintegrated gluon as before (from $F_2$ fit at HERA)
- use off-shell matrix element for $g^* g^* \rightarrow b\bar{b}$ with $m_b = 4.75$ GeV.

NOTE NLO off by factor 2

CASCADE w/o additional free parameters
Solution to the problem: $b\bar{b}$ production at Tevatron

- **CASCADE** describes $\mu$ spectrum over huge range well
- **NLO** fails by factor $\sim 2$ (central) and $\sim 4$ (forward)

Why does $k_t$-factorization help for $b\bar{b}$ production at Tevatron

estimate higher order corrections

Nr of gluons with $p_t > p_t^{b\bar{b}}$

LO: $\mathcal{O}(\alpha_s^2) \rightarrow N_g = 0$

NLO: $\mathcal{O}(\alpha_s^3) \rightarrow N_g = 1$

NNLO: $\mathcal{O}(\alpha_s^4) \rightarrow N_g = 2$

.....

CASCADE $\rightarrow \mathcal{O}(\alpha_s^6)$

CASCADE with $k_t$ factorization for estimation of higher order corrections
\( k_t \) - factorization very successful

- unintegrated gluon density from CCFM / LDC
- precision fits to \( F_2 \), including full splitting fct
- describes measurements at HERA
- even where collinear NLO fails
- works also for diffraction
- works also for bottom in \( pp \)
- attempts also for bottom in \( \gamma\gamma \)
- also for Higgs at LHC

\( k_t \) - factorization useful for estimate of higher order corrections

Increasing theoretical interest in \( k_t \) - factorization:

- Lund small \( x \) workshops and Small \( x \) collaboration
Even if there is still a bit to go for a Theory of Everything, we are facing the beginning of an interesting, bright and challenging future in small $\chi$ physics.
New fit: small $k_t$ - region

- use H1 + ZEUS $F_2$ data (from 94 and 96-97)
- fit for $x < 0.005$ $Q^2 > 4.5$ GeV$^2$
- fit $Q_0$ and normalization in initial pdf $xA_0 = N(1 - x)^4$

Treatment of soft region
- no $k_t$ ordering $\Rightarrow$ diffusion into soft
- what about $\alpha_s$ for $k_t < k^\text{cut}_t$ in

- saturation of x-section for $k_t < k^\text{cut}_t$

What is actual cut - what is soft?
- JS2001 had soft cut $k_t > 0.25$ GeV
- now $k_t > Q_0$
New fit: full splitting function

- Improve splitting function
  \[ P_{gg} \sim \bar{\alpha}_s \left( \frac{1}{z} \Delta_{ns} + \frac{1}{1-z} \right) \]

- To include non-singular terms
  \[ P = \bar{\alpha}_s \left( \frac{1-z}{z} + \frac{z}{2} \right) \Delta_{ns} \]
  \[ + \bar{\alpha}_s \left( \frac{z}{1-z} + \frac{z(1-z)}{2} \right) \]

- Need also new Sudakov
  - New non-Sudakov

- Gluon PDFs are different

- Effect of non-singular terms visible
test machinery with one-loop (DGLAP)

use gluon in photon from GRV as input
use normalization at input scale

apply CCFM evolution (sing. terms only)
with parameters obtained from proton ($Q_0 = 1.4$ GeV)

First un-integrated gluon density of real photon with full CCFM evolution
use matrix elements in $k_t$ -factorization

- $\gamma\gamma \rightarrow b\bar{b}$
- $\gamma g \rightarrow b\bar{b}$
- $gg \rightarrow b\bar{b}$
- universality...

**compare** $k_t$ -factorization & CCFM with NLO:

- using norm. from pdf
  - CCFM similar to NLO
- determine norm. for gluon from charm ($n = 1.7$ for res. $\gamma$)
  - CCFM larger than NLO
  - BUT still low for $\gamma\gamma \rightarrow b\bar{b}$
**Diffractive Di-Jets and CCFM**

**BJKLW calculation**

$\gamma p \rightarrow qq$

$\gamma p \rightarrow q\bar{q}g$ (2-gluon exchange)


$\gamma p \rightarrow q\bar{q}g$ (2-gluon exchange)


Apply only to $x_F < 0.01$ !!!

pert. QCD calculation ...

**H1 Diffractive Dijets - $x_{IP}<0.01$**

$4 < Q^2 < 80 \text{ GeV}^2$, $0.1 < y < 0.7$, $x_{IP} < 0.05$, $M_Y < 1.6 \text{ GeV}$,
$E_{T,\text{jet}} > 4$ GeV, $-1.0 < \eta_{jet} < 2.2$

**Gluon from CCFM** ✔

ISMD03, Cracow, 8. Sept 2003 – p.34
Diffractive Di - Jets and CCFM

BJKLW calculation

\( x_{bj} \)

\( \gamma p \rightarrow qq \)

\( \gamma p \rightarrow q\bar{q}g \)

un-integrated gluon density

\( x_P \)

\( p \)

\( p' \)

\( 4 < Q^2 < 80 \text{ GeV}^2, \ 0.1 < y < 0.7, x_{IP} < 0.05, \ M_Y < 1.6 \text{ GeV}, \ \eta_{jet} > 4 \text{ GeV}, \ -1.0 < n_{jet}^{lab} < 2.2 \)

H1 Diffractive Dijets - \( x_{IP} < 0.01 \)

- H1 Data
- CCFM-JS2001
- new-soft
- full-splitt.
- \( \alpha_s \)

Gluon from CCFM ✔

cutoff \( p_t > 2.5 \text{ GeV} \) needed

- non-ordered emissions:
  - \( p_t^{\text{gluon}} > p_t^{q\bar{q}}: \sim 30 \% \)
  - not possible in res. pom
  - nor SCI
Forward $\pi^0$: H1

**DIS**: forward $\pi^0$ (instead of jet)
- $5^\circ < \theta_{\pi} < 25.0^\circ$
- $x_{\pi} > 0.01$

DGLAP too small, need:
- resolved virtual photons ??!
- CCFM too small at small $x$ ??!
- WHY ??!

**H1 Forward $\pi^0$**

- H1 preliminary
- DIR + RES, $Q^2 + 4p_t^2$
- CCFM (CASCADE)
- DIR (LO DGLAP)

$2.0 < Q^2 < 8.0$ GeV$^2$

$8.0 < Q^2 < 20.0$ GeV$^2$

$p_{T,\pi} > 3.5$ GeV
- $5^\circ < \theta_{\pi} < 25^\circ$

ISMD03, Cracow, 8. Sept 2003 – p.35
**H1**(prel.)

\[ 2 < Q^2 < 100 \text{ GeV}^2, \ 0.1 < y < 0.8, \]
\[ p_t^\mu > 2 \text{ GeV}, \ 35^\circ < \theta^\mu < 130^\circ \]

**visible x-section** \( ep \rightarrow e'b\bar{b}X \rightarrow \mu X \):

\[ \sigma = 39 \pm 8 \text{(stat.)} \pm 10 \text{(syst.)} \text{ pb} \]

**NLO:** \( \sigma = 11 \pm 2 \text{ pb} \)

**CASCADE** \( \sigma(ep \rightarrow e'b\bar{b}X) = 15 \text{ pb} \)

\[ R_{MC}(H1) = \frac{\sigma_{measured}}{\sigma_{MC}} = 2.6 \]

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**ZEUS**(prel.) ICHEP 2002

\[ Q^2 > 2 \text{ GeV}^2, \ 0.05 < y < 0.7, \]
\[ E_{T,jet}^{Breit} > 6 \text{ GeV}, \ -2 < \eta_{jet}^{lab} < 2.5 \]
\[ p_t^\mu > 2 \text{ GeV}, \ 30^\circ < \theta^\mu < 160^\circ \]

**x-section** \( ep \rightarrow e'b\bar{b}X \rightarrow e'jet\mu X \):

\[ \sigma = 38.7 \pm 7.7 \text{(stat.)}^{6.1}_{5.0} \text{(syst.)} \text{ pb} \]

**NLO:** \( \sigma = 28.1 \pm 2 \text{ pb} \)

**CASCADE** \( \sigma(ep \rightarrow e'b\bar{b}X) = 35 \text{ pb} \)

\[ R_{MC}(ZEUS) = \frac{\sigma_{measured}}{\sigma_{MC}} = 1.1 \]
**ZEUS (prel.) 99-00**

$O(\alpha_s)$ QCD $\otimes$ CCFM (CASCADE)

$O(\alpha_s)$ QCD $\otimes$ DGLAP (RAPGAP)

**$\sigma(e^+ p \rightarrow e^+ \bar{b}b X \rightarrow e^+ \mu^\pm \text{Jet X})$**

$Q^2 > 2 \text{ GeV}^2$, $0.05 < y < 0.7$

$P_\mu > 2 \text{ GeV}$, $30^\circ < \theta_\mu < 160^\circ$

$E_{\text{Breit}} > 6 \text{ GeV}$, $-2 < \eta_{\text{Jet}} < 2.5$

$P_T^\mu > 2 \text{ GeV}$, $30^\circ < \theta_\mu < 160^\circ$

$E_{\text{Breit}} > 6 \text{ GeV}$, $-2 < \eta_{\text{Jet}} < 2.5$
Resolved - $\gamma$, NLO and $k_t$ - factorization

$k_t$ factorization

- NLO corrections
- anomalous $\gamma$
- even in NLO
- includes NNLO
- includes NNNLO
- includes NNNNLO

$k_t$ factorization has no problem with:

- negative weights....
- matching to PS
- matching to hadronization
$$k_t$$ - and collinear factorization

off - shell matrix element

➤ 1-loop correction to
Born approximation

➤ high energy limit of NLO !!!

$$\sigma = \int \frac{dz}{z} d^2 k_t \hat{\sigma}(\frac{t}{z}, k_t) \mathcal{F}(z, k_t)$$

$$\mathcal{F}(z, k_t) = \int \frac{dz}{z} \tilde{\mathcal{F}}(x/z, k_t; Q_0) \tilde{f}_0(z, Q_0)$$

factorize $$k_t$$ dependence in $$\mathcal{F}$$ and insert in $$\sigma$$

➤ improved coefficient and splitting functions to all order