1. Consider scattering off an infinite "hard ball":

\[ V(r) = \begin{cases} 
0 & \text{dla } R < r \\
\infty & \text{dla } r < R 
\end{cases} \]

Calculate phase shifts from the condition \( A_l(R) = 0 \) where:

\[ A_l(r) = e^{i\delta_l(k)} \left[ j_l(kr) \cos \delta_l(k) - y_l(kr) \sin \delta_l(k) \right]. \]

Find low energy behaviour of \( \delta_l \). Calculate the cross-section for the lowest partial wave \( l = 0 \). As you will see the cross-section is not geometrical i.e. \( \sigma \neq \pi R^2 \).

2. Sum up higher partial waves

\[ \sigma_{\text{tot}} = \frac{4\pi}{k^2} \sum_l (2l + 1) \sin^2 \delta_l(k) \]

up to a maximal classically allowed \( l_{\text{max}} \sim k R \). To this end use

\[ \sin^2 \delta_l(k) = \frac{\tan^2 \delta_l(k)}{1 + \tan^2 \delta_l(k)} \]

and the formula for \( \tan \delta_l(k) \) in terms of spherical Bessel functions. Then use asymptotic form of Bessel functions. The resulting cross-section is still not geometrical \( (\sigma_{\text{tot}} = 2\pi R^2) \). Try to interpret this result (Sakurai, Advanced Quantum Mechanics, Chapt.7.6).

3. Using expansion for the wave function from the previous problem set

\[ \psi = \sum_l (2l + 1)i^l A_l(r) P_l(\cos \theta) \]

and conditions

\[ \beta_l = \frac{r}{A_l} \left. \frac{dA_l}{dr} \right|_{r=R}, \]

\[ \tan \delta_l = \frac{kR j'_l(kR) - \beta_l j_l(kR)}{kR y'_l(kR) - \beta_l y_l(kR)} \]

derive general formula (in terms of spherical Bessel functions) for the scattering length for the finite spherical well \( (V_0 > 0) \):

\[ V(r) = \begin{cases} 
0 & \text{dla } a < R \\
-V_0 & \text{dla } r < R 
\end{cases} \]

In particular calculate \( \tan \delta_0 \). Discuss two limits \( k \to 0 \) and \( k \to \infty \). How \( \delta_0 \) depends on \( V_0 \)?