SPECTRAL ACTION: FROM NEUTRINO MASSES TO BIMETRIC GRAVITY

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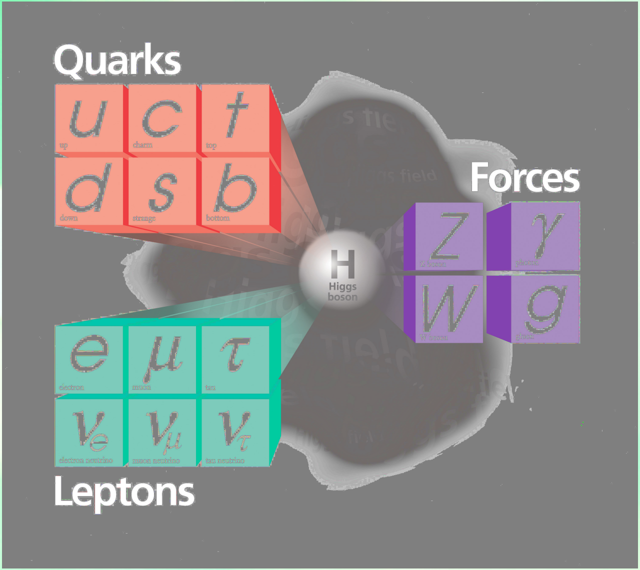
PARTICLE INTERACTIONS ARE ENCODED IN GEOMETRY



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GEOMETRY OF THE SPACETIME

1. the metric is encoded in the Dirac operator (or Laplace operator)
2. the spin connection is the lift to spinor bundle of the Levi-Civita connection,

Spectral triples provide a generalized setup for geometry

Define the geometry in an algebraic way.

Algebra $\mathcal{A}$, its (faithful) representation π on a Hilbert space $\mathcal{H}$, a selfadjoint operator D, satisfying several (reasonable) conditions.

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4. & ...+ make sure that you can compute something

Spectral action is an effective tool to get the action functional

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By parametrizing the degrees of freedom of the Dirac operator (in the classical case) through metric, torsion and connections (to allow for twisted Dirac operators) we obtain an esymptotic expansion of the action functional as:

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$$\mathcal{S}_{\Lambda}(D) = \Lambda^4 \alpha_4 \int_M \sqrt{g} + \Lambda^2 \alpha_2 \int_M \sqrt{(}g)(R(g) + \beta|T|^2) + \alpha_0 \int_M \sqrt{g}|F|^2 + \cdots$$

SPECTRAL ACTION IS IN MANY CASES COMPUTABLE

EXAMPLES

Spectral action is in many cases computable

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Physics *requires* that the full action is the action for bosonic fields (fields that are part of the Dirac operator) as well as the fermionic fields.

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WHY THE FERMIONIC ACTION IS NOT SPECTRAL ?

Physics *requires* that the full action is the action for bosonic fields (fields that are part of the Dirac operator) as well as the fermionic fields. Yet the action for fermions is of different type:

$$S_\Phi = \langle\Psi|\, D(g, A)\, |\Psi\rangle\,,$$

SPECTRAL ACTION CAN BE APPLIED TO FERMIONS

HOW TO MAKE FERMIONIC ACTION SPECTRAL ?

We propose (for the simplest Euclidean model) the cutoff fermionic action

$$S_{g,\Lambda} = \langle\Psi| \, |g_\Lambda(D_A)| \, |\Psi\rangle \,,$$

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Nonminimal fermion couplings

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THE YANG-MILLS SYSTEM

$$\Lambda^4 \int_M \sqrt{g}\, (\mathrm{tr} P_\Psi) = \Lambda^4 N \int_M \sqrt{g}\, \langle \Psi(x)|\,|\Psi(x)\rangle\,,$$
$$\Lambda^3 \int_M \sqrt{g}\, \langle \Psi(x)|\, D_A\, |\Psi(x)\rangle\,,$$
$$\Lambda^2 \int_M \sqrt{g}\, \langle \Psi(x)|\, \left(\tfrac{R}{12} + F\right) |\Psi(x)\rangle\,,$$

FERMIONIC SPECTRAL ACTION FOR NONCOMMUTATIVE SM

THE STANDARD MODEL AS A SPECTRAL GEOMETRY

The discrete Dirac operator written in the basis of fermions, taken in the order $f_R^u, f_R^d, f_L^u, f_L^d$ (each taken multiple times for N generations) is

$$D_F = \begin{pmatrix} & & \Upsilon_u^* & \\ & & & \Upsilon_d^* \\ \Upsilon_u & & & \\ & \Upsilon_d & & \end{pmatrix},$$

and the fluctuated discrete Dirac operator $D_F(H)$ is:

$$D_F = \begin{pmatrix} & & \Upsilon_u^* H^0 & \Upsilon_u^* H^- \\ & & -\Upsilon_d^* \overline{H^-} & \Upsilon_d^* \overline{H^0} \\ \Upsilon_u \overline{H^0} & -\Upsilon_d H^- & & \\ \Upsilon_u \overline{H^-} & \Upsilon_d H^0 & & \end{pmatrix},$$

where $H = H^0 + H^- j$ denotes a quaternionic field (Higgs doublet).

Fermionic spectral action for noncommutative SM

The Pauli-type terms for the SM geometry

Taking the square of the $D_F(H)$ at the Higgs vacuum expectation value we obtain:

$$(D_F(H_v))^2 = \begin{pmatrix} \Upsilon_e^* \Upsilon_e |H_v|^2 & \\ & \Upsilon_e^* \Upsilon_e |H_v|^2 \end{pmatrix},$$

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But for some nonscalar functions...

For any nonscalar function

$$f_\Lambda^\tau(x) = \tau f_\Lambda(x) \tau,$$

where $\tau = \sigma^2$ (Pauli matrix) the term $f_\Lambda^\tau(D^2)$ is certainly gauge covariant and, consequently, the terms in the fermionic spectral action remain gauge invariant.

FERMIONIC SPECTRAL ACTION FOR NONCOMMUTATIVE SM

EXPLICIT COMPUTATIONS LEADING TO WEINBERG TERM

Then the terms in the fermionic spectral action, that arise from $\mathrm{Tr}(P_\Psi\, f_\Lambda^\tau(\mathcal{D}^2))$ in the next-to-leading order, could be explicitly rewritten as:

$$\Lambda^2(\Upsilon_e \Upsilon_e{}^*) \left[(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} H^0 \\ -H^- \end{pmatrix} \right] \left[\left(\overline{H^0}, -\overline{H^-}\right) \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \right] + \text{h.c},$$

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Explicit computations leading to Weinberg term

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2. After the Higgs field gets its vacuum expectation value, a neutrino mass is generated, depending on the scale Λ - of the correct range.
3. Corrections for other leptons are negligible.

Look again at the bosonic action and gravity

The simplest noncommutative model is left-right:

Consider the algebra $\mathcal{A} = C^\infty(M) \otimes (\mathbb{C} \oplus \mathbb{C})$, represented on $L^2(S) \otimes \mathbb{C}^2$.

$$D_o = D \otimes \mathrm{id} + \gamma \otimes D_F, \qquad D_F = \begin{pmatrix} 0 & \phi \\ \phi^* & 0 \end{pmatrix}$$

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BUT THIS IS NOT THE MOST GENERAL DIRAC OPERATOR !

Let us consider instead:

$$D = \begin{pmatrix} D_1 & \gamma\phi \\ \gamma\phi^* & D_2 \end{pmatrix},$$

with two independent metrics.

$$D^2 = \begin{pmatrix} D_1^2 + \phi\phi^* & \gamma(D_1\phi - \phi D_2) \\ -\gamma(D_1\phi^* - \phi^* D_2) & D_2^2 + \phi\phi^* \end{pmatrix},$$

We compute the spectral action for the full two-sheeted spacetime model

Only in the FRLW Ansatz

$$D = \gamma^0 \partial_t + \frac{1}{a(t)} \left(\gamma^1 \partial_1 + \gamma^2 \partial_2 + \gamma^3 \partial_3 \right) + \gamma^0 \frac{3\dot{a}(t)}{2a(t)},$$

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We use pseudodifferential calculus and Wodzicki residue

Taking:

$$D = \gamma^0(\partial_t + H(t)) + A(t) D_3 + \gamma F(t),$$

where

$$H(t) = \begin{pmatrix} H_1(t) & 0 \\ 0 & H_2(t) \end{pmatrix}, \quad A(t) = \begin{pmatrix} \frac{1}{a_1(t)} & 0 \\ 0 & \frac{1}{a_2(t)} \end{pmatrix}, \quad F(t) = \begin{pmatrix} 0 & \phi(t) \\ \phi^*(t) & 0 \end{pmatrix}$$

The Robertson-Walker geometry for the two-sheeted model is exactly computable

The action (first two terms) is:

$$\mathcal{S} = 2\pi^2 \int dt \Big(\lambda^4 \left(a_1(t)^3 + a_2(t)^3\right)$$
$$-\lambda^2 c \left(\dot{a}_1(t)^2 a_1(t) + \dot{a}_2(t)^2 a_2(t)\right) - \lambda^2 c|\Phi|^2 \left(a_1(t)^3 + a_2(t)^3\right)$$
$$+ \lambda^2 c|\Phi|^2 \left(\frac{(a_1(t) - a_2(t))^2}{(a_1(t) + a_2(t))} \left(a_1(t)^2 + a_1(t)a_2(t) + a_2(t)^2\right)\right)\Big) + \mathcal{L}_m,$$

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The bimetric gravity models are closely related to Noncommutative Geometry

The additional term is an *interaction* between the two metrics that is similar to the terms in the bimetric theory of gravity.

Modified cosmology models (simplified)

The empty universe

$$r(t) \sim c_1 e^{-\sqrt{\frac{\Lambda}{24}+\frac{1}{4}\sqrt{6\Lambda+8\alpha}}} + c_2 e^{-\sqrt{\frac{\Lambda}{24}-\frac{1}{4}\sqrt{6\Lambda+8\alpha}}}.$$

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$$r(t) \sim t^{\frac{1}{4}} J_{\frac{\sqrt{5}}{4}}(\sqrt{-2(\Lambda+\alpha)}t),$$

Matter dominated universe

$$r(t) \sim t^{-\frac{1}{3}} \sin\left(\frac{1}{2}\sqrt{-2(\Lambda+\alpha)}t\right).$$

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7. **References for these results:**
   On almost commutative Friedmann-Lemaître-Robertson-Walker geometries, arXiv:1903.10158
   Fermionic spectral action and the origin of nonzero neutrino masses, (with Mairi Sakellariadou) arXiv:1903.09149