Fermionic spectral action and the origin of neutrino mass.

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We propose that the fermionic action in the noncommutative description of the Standard Model is spectral in the same way as the bosonic action and discuss the terms that appear in the asymptotic expansion.

I. INTRODUCTION

The current experimental data provides evidence that at least two neutrinos do have small albeit nonzero masses [1]. There are several models that explain their origins both in the possible case where neutrinos are Dirac or Majorana particles [2]. It is worth noticing that the scale of neutrino masses when compared to other leptons is indeed much lower than the differences between the masses between other leptons or quarks. Therefore, one of the possible explanations that has been proposed [3] is that their origin is related to some effective terms in the action.

In the geometric interpretation of the Standard Model proposed within the framework of noncommutative geometry, neutrinos were originally proposed as massless Majorana particles [4]. Yet the experimental confirmation that beyond any doubt, neutrinos of different flavor oscillate has enforced the introduction of the neutrino masses into the model. Most of the explanations is based on the possibility of both Dirac and Majorana masses for the neutrinos and the see-saw

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mechanism [5].

In this note we propose that the consistent treatment of the fermionic action in the same way as the bosonic action [7] is constructed leads to some nontrivial corrections that introduce non-minimal fermion couplings. The idea behind the fermionic spectra action is, in principle, identical to the bosonic one. Assuming the existence of some energy cut-off Λ one proposes an effective action that depends on the spectrum of the Dirac operator but truncated at Λ, then computing the asymptotic expansion of the leading term in Λ. We propose a similar method for the fermionic action, where the action, usually taken as the expectation value of the Dirac operator for a fermionic field Ψ is, in effect, depending only on the truncated Dirac operator (where we consider only its part with eigenvalues less than Λ).

The idea of the paper is to study the qualitative consequences of the proposed spectral fermionic action leaving the detailed discussion with all aspects of the model (that will include the Lorentzian formulation [8], fermion doubling [9, 10], various forms of the fermionic action for different KO-dimensions etc) for future studies. Though usually, it is considered that bosonic spectral action is a window into very high energies [11], we believe that correction terms to fermionic spectral action can be observed even now. An early, much simpler toy model [12] suggested that fermionic spectral action might be responsible for additional mass terms, here we provide an extensive discussion which non-minimal interactions terms are possible.

II. THE FERMIONIC ACTION

The fermionic action for the spectral triple, which gives the dynamics and interactions between fermions and the bosonic field is usually formulated as the expectation value of the Dirac operator in the state given by a fermionic field Ψ:

$$S_f = \langle \Psi \,|\, D_A \Psi \rangle.$$

For the almost commutative geometries, with a specific KO dimension 2 (mod 8), which is used in the description of the Standard Model, one could consider another version of the action (on the total Hilbert space), in terms of the antisymmetric bilinear form:

$$S_{J,f} = \langle J\Psi \,|\, D_A \Psi \rangle,$$

which then could be restricted to the subspace of right-handed fermions, reducing the unnecessary doubling of the space of fermions [9].

In the noncommutative geometry approach there exists, however, a strange difference between the two parts of the action. While the bosonic part depends only on the reduced spectrum of the Dirac operator, the fermionic one uses explicitly the full spectrum. This has been first observed by Chamseddine [13], who postulated that also the fermionic action should have a similar form, yet the problem was not discussed until [12]

III. THE SPECTRAL FERMIONIC ACTION

We propose a natural method to write the above action functional, using similar cut-off regularization as in the bosonic case. Of course, using the spectral action generalization leads to the general problems with the interpretation of the Euclidean formulation of spinors and the reduction of the model due to the so-called fermion doubling (or, more correctly, double doubling).

We shall work completely in the Euclidean setup, leaving aside potential problems and looking only for potentially interesting terms that could lead to new physics. Our motivation comes directly from the bosonic spectral action, where the leading terms introduce one by one the geometry and interactions and the coupling between them. The fermionic term, on the other hand, by definition gives only the minimal coupling between the fermions and the gauge fields and the Higgs. We propose to investigate the type and the form of the expected non-minimal couplings that are motivated by the geometric structure of the interactions as described by the noncommutative geometry.

First of all, we propose (for the simplest Euclidean model)

$$S_{g,\Lambda} = \langle \Psi \,|\, g_\Lambda(D_A)\, \Psi \rangle,$$

for any suitable function g and taking $g_\Lambda(x) = g(\frac{x}{\Lambda})$. Observe, that we take a function of D_A and not D_A^2 on purpose, as the fermionic field Ψ is not necessarily symmetric with respect to the sign of the eigenvalues of D_A. Since any function could be split into the sum of an even and an odd function, and an even function could be taken as a function of D_A^2, using

$$g(x) = f(x^2) + x h(x^2),$$

we can study two contributions to the action:

$$S_{e,\Lambda} = \langle \Psi |\, f_\Lambda\big(D_A^2\big)\, \Psi \rangle, \qquad S_{o,\Lambda} = \frac{1}{\Lambda} \langle \Psi |\, D_A h_\Lambda\big(D_A^2\big)\, \Psi \rangle,$$

where in principle, f, h are two arbitrary function of the cut-off type. Using the fact that D_A is selfadjoint we can rewrite the latter expression as:

$$S_{o,\Lambda} = \frac{1}{\Lambda}\langle\Psi|\, h_\Lambda\big(D_A^2\big)\, D_A\Psi\rangle,$$

which would enable us to use the tools of heat trace expansion [14, 15] for the commutative and almost commutative geometries.

A. Commutative geometries

Let M be a Riemannian spin manifold and $L^2(S)$ its spinor bundle and let Ψ be a spinor field. We define P_Ψ is an endomorphism of the bundle of spinors that locally, at each point $x \in M$ projects on the spinor $\Psi(x)$, $\Phi(x) \mapsto \Psi(x)\langle\Psi(x)|\Phi(x)\rangle$, or, using the physics notation $P_\Psi = |\Psi(x)\rangle\langle\Psi(x)|$.

Let D be a Dirac operator that arises lifts the torsion-free Levi-Civita connection to the spinor bundle. Although one can consider Dirac operators that arise from connections with torsions, we concentrate on the generic case. The Dirac operators that are twisted by connections on a vector bundle . We shall interpret the above fermionic action terms as arising from the heat kernel expansion of the type:

$$S_{e,\Lambda} = \mathrm{Tr}\left(P_\Psi g_\Lambda\left(D^2\right)\right),$$

where. As P_Ψ is a local operator (endomorphism of the bundle on which D acts) we can, similarly as in the bosonic spectral action, use the formulae for the heat trace expansion:

$$\mathrm{Tr}\left(Te^{-tD^2}\right) = \sum_{n=0} t^{\frac{1}{2}(n-4)} \int_M a_n(x,T),$$

where

$$a_0(T,x) = (4\pi)^{-2}\mathrm{tr}\, T, \qquad a_1(T,x) = (4\pi)^{-2}\mathrm{tr}\left(T\left(-\frac{R}{6}+E\right)\right)$$

and using the moments of the functions f and h [15] as the coefficients. Here tr denotes the local trace operation in the endomorphisms of the spinor bundle taken at point x. Note that $\mathrm{tr}P_\Psi = \langle\Psi(x)\Psi(x)\rangle$ and therefore, for a 4-dimensional manifold we shall have the following leading terms, which arise from the fact that for the Dirac operator $E = \frac{R}{4}$:

$$\Lambda^4 \int_M \sqrt{g}\,(\mathrm{tr} P_\Psi) = \Lambda^4 \int_M \sqrt{g}\,\langle \Psi(x)|\Psi(x)\rangle,$$
$$\Lambda^2 \int_M \sqrt{g}\,\mathrm{tr}\left(P_\Psi \frac{R}{12}\right) = \Lambda^2 \int_M \sqrt{g}\,\frac{R}{12}\,\langle \Psi(x)|\Psi(x)\rangle.$$

The leading term, which resembles the cosmological constant term corresponds to the bare fermion mass term whereas the second term coming from the even part of spectral the action describes the non-minimal coupling of fermions to gravity through the scalar curvature [18, 19].

As for the odd part of the fermionic spectral action, we propose to rewrite it using similar arguments as:

$$S_{o,\Lambda} = \frac{1}{\Lambda}\mathrm{Tr}\left(P_{\Psi,D\Psi}\, h_\Lambda\left(D^2\right)\right),$$

where the local endomorphism of the spinor bundle $P_{\Psi,D\Psi}$ is of the form:

$$\Phi(x) \mapsto D\Psi(x)\langle\Psi(x)|\Phi(x)\rangle.$$

Similarly as in the even case, observe that $\mathrm{tr}\, P_{\Psi,D\Psi} = \langle\Psi(x)|\, D\Psi(x)\rangle$ with $(,)$ the local scalar product on the sections of the spinor bundle, which is valued in $C^\infty(M)$.

The odd component of the fermionic spectral action introduces as the leading term the fermion dynamics:

$$\Lambda^3 \int_M \sqrt{g}\,\langle\Psi(x)|D\Phi(x)\rangle,$$

while the next order terms would contribute (in the case of pure gravity) further higher coupling to scalar curvature:

$$\Lambda \int_M \sqrt{g}\,\frac{R}{12}\,\langle\Psi(x)|D\Phi(x)\rangle.$$

B. The Einstein-Yang-Mills system

The above discussed case extends to the situation of a simple noncommutative modification of geometry [7] where we take the algebra of $M_n(\mathbb{C})$ valued functions on the spin manifold M and use the family of Dirac operator constructed by gauge fluctuation of the Dirac operator D. Such family, which are obtained through the so-called internal fluctuations of the metric, is of the form:

$$D_A = D \otimes \mathrm{id} + A,$$

where A is a gauge potential ($A = \sum_i a_i[D, b_i]$) for $a_i, b_i \in C^\infty(M) \otimes M_n(\mathbb{C})$. This includes, in particular, the case of Dirac operators twisted by a connection on a vector bundle.

In case the spectral triple is real and satisfies order-one condition [] one should modify the above family by correcting A and adding (with a sign that comes from the KO-dimension) JAJ^{-1}. Since the square of the Dirac operator contains the gauge curvature F, the formulas for the heat trace expansion are modified, $E = \frac{R}{12} + F$, and then the first three leading fermionic action terms are modified as follows:

$$\begin{aligned}
&\Lambda^4 \int_M \sqrt{g}\,(\mathrm{tr} P_\Psi) = \Lambda^4 N \int_M \sqrt{g} \langle \Psi(x) | \Psi(x) \rangle, \\
&\Lambda^3 \int_M \sqrt{g}\, \langle \Psi(x) | D_A \Psi(x) \rangle, \\
&\Lambda^2 \int_M \sqrt{g}\, \langle \Psi(x) | \left(\frac{R}{12} + F \right) \Psi(x) \rangle,
\end{aligned}$$

The only difference with the previous case is, apart from the multiplicity of fermionic fields that comes from representation of the algebra $M_N(\mathbb{C})$, the minimal coupling of the fermions with the gauge field that comes in the second term as well as the Pauli interaction Lagrangian that appears in the third term, that locally looks like

$$\int_M \sqrt{g}\, \langle \Psi(x) | F \Psi(x) \rangle = \int_M \sqrt{g}\; \Psi(x)^\dagger \left(\gamma^\mu \gamma^\nu F_{\mu\nu}(x) \right) \Psi(x),$$

and which can be nontrivial even in the case of electrodynamics [].

C. The almost commutative geometries

In this section we consider product of two geometries, one, which is a smooth Riemannian manifold, and the second, finite-dimensional geometry, defined in the language of finite spectral triples. Since effectively the Dirac operator of the product is of the same type as in the case of Einstein-Yang-Mills system, there is no significant qualitative difference from the methods used while studying commutative geometries.

Let us recall the basic notions. We take as the underlying algebra of the model $\mathcal{A} = C^\infty(M) \otimes \mathcal{A}_F$, which is represented on the Hilbert space $L^2(S) \otimes \mathcal{H}_F$, and the Dirac operator is $\mathcal{D} = D_M \otimes \mathrm{id} + \gamma_5 \otimes D_F$, where again D denotes the standard Dirac operator on the spin manifold M and D_F is the Dirac operator of the finite spectral triple $(\mathcal{A}, \mathcal{H}, D_F)$. We refer for the details of construction of product geometries and related issues to [4].

The family of Dirac operators $\mathcal{D}_{\mathbb{A}}$ arises similarly as in the previous sections as fluctuations of the Dirac operator. They include both the classical gauge fields, with the unitary group of inner automorphisms of the algebra $\mathcal{A}_F$ as well as the gauge fields related to discrete geometry, which are interpreted as the Higgs field. More precisely,

$$\mathcal{D}_{\mathbb{A}} = D \otimes \mathrm{id} + \mathbb{A} + (\gamma^5 \otimes 1) D_F(H),$$

where $\mathbb{A}$ are the inner fluctuations of the Dirac operator D (containing, if we assume the reality of the spectral triple, also the real part of fluctuation) and $D_F(H)$ are inner fluctuations of the product geometry with respect to the discrete Dirac operator D_F. Note that both $\mathbb{A}$ and $D_F(H)$ are, from the technical point of view just matrix-valued functions on the manifold M, which are represented on $L^2(S) \otimes \mathcal{H}_F$.

As to obtain the leading terms in the spectral action we use the heat trace asymptotic expansion using the square of the Dirac operator, we use the result [] that

$$\mathcal{D}_{\mathbb{A}}^2 = \nabla^\star \nabla - E,$$

where ∇ is a connection on the spinor bundle tensored with $\mathcal{H}_F$ and E is the endomorphism of the latter bundle.

In local coordinates over the manifold, with γ^μ being the usual gamma matrices, we have:

$$E = -(D_F(\Phi))^2 - \sum_{\mu<\nu} \gamma^\mu \gamma^\nu \mathbb{F}_{\mu\nu} + i\gamma_5 \gamma^\mu \mathbb{M}(D_\mu(H)),$$

where $\mathbb{F}_{\mu\nu}$ is the curvature tensor of the gauge connections, $D_F(H)^2$ is the potential term for the fields H and the last term $\mathbb{M}(D_\mu(\Phi)$ is the endomorphism of the bundle that depends on the covariant derivative of fields H.

We shall analyze these terms in the next section, in the particular case of the almost commutative geometry underlying the Standard Model.

IV. THE APPLICATION TO THE STANDARD MODEL

Let us briefly recall here the basics of the Standard Model description as an almost commutative geometry. We take as an algebra $\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$, which is represented on a 16-dimensional Hilbert space that includes all fermions (assuming Dirac neutrinos) or 15-dimensional if we work with Majorana neutrinos only. For the details of the action in a convenient basis see [6] or [16, 17] for a most recent formulation and principles of constructing the Dirac operator.

The discrete Dirac operator written in the basis of fermions, taken in the following order $f_R^u, f_R^d, f_L^u, f_L^d$ (each taken multiple times for N generations),

$$D_F = \begin{pmatrix} & & \Upsilon_u^* & \\ & & & \Upsilon_d^* \\ \Upsilon_u & & & \\ & \Upsilon_d & & \end{pmatrix},$$

and the fluctuated discrete Dirac operator $D_F(H)$ is:

$$D_F = \begin{pmatrix} & & \Upsilon_u^* H^0 & \Upsilon_u^* H^- \\ & & -\Upsilon_d^* \overline{H^-} & \Upsilon_d^* \overline{H^0} \\ \Upsilon_u \overline{H^0} & -\Upsilon_d H^- & & \\ \Upsilon_u \overline{H^-} & \Upsilon_d H^0 & & \end{pmatrix},$$

where $H = H^0 + H^- j$ denotes a quaternionic field (Higgs doublet).

A. The fermionic spectral action for the SM

As we have seen in the previous section, the first term two terms of the fermionic spectral action are the bare mass term and the usual fermionic action. As the model is chiral, in the Lorentzian version the bare mass term is not possible. In the Euclidean version, however, it can appear in the model with the fermion doubling but will have to vanish if we require that the action terms are possible to restrict to the physical space of fermions (removing the fermion doubling [9]).

The next term yields the usual part of action, which includes the dynamical term for the fermions, minimal coupling to gauge fields and the coupling between the Higgs field and the fermions, which gives the mass terms in the broken symmetry phase.

The only possible corrections and new effects can be therefore visible in the third term, which is proportional to Λ^2. Of course, we shall have there similar terms as in the Einstein-Yang-Mills system, that is the non-minimal coupling of fermions to gravity (through the scalar curvature) and the Pauli-type interaction terms (coupling to curvature of connections) [20].

However, we shall additionally have the term of the fermionic spectral action that contains the square of the fluctuated discrete part of the Dirac operator, $D_F(H)^2$, that contains the Higgs field. We discuss now two interesting cases of Dirac and Majorana neutrinos, concentrating our analysis on the leptonic sector. Observe that the same could be, of course, used for the spatial part of the Dirac manifold leading to higher-derivative terms in the fermionic action that have been considered in many models [21, 22]

B. Dirac neutrinos

The square of the finite Dirac operator gives, in the case of the Standard Model the corrections to the fermion masses. using the notations of the first part of the section IV we compute $D_F(\Phi)^2$ when restricted to the lepton sector as:

$$D_F(H)_l^2 = \begin{pmatrix} \Upsilon_\nu^* \Upsilon_\nu |H|^2 + \Upsilon_R^* \Upsilon_R^* & 0 & 0 & \\ 0 & \Upsilon_e^* \Upsilon_e |H|^2 & 0 & 0 \\ 0 & 0 & \Upsilon_\nu^* \Upsilon_\nu |H|^2 & 0 \\ 0 & 0 & 0 & \Upsilon_e^* \Upsilon_e |H|^2 \end{pmatrix},$$

where Υ_m, Υ_ν are the mass and mixing matrices.

The order of the corrections, when compared to the main mass term are of the order $\frac{1}{\Lambda}$ and therefore will be negligible when compared to Dirac mass terms, when computed in the vacuum expectation value of the Higgs field $H = H_v$ (that is $H^0 = H_v, H^- = 0$).

C. The Majorana neutrinos model

In the noncommutative description of the Standard Model that uses only left-handed neutrinos there is no room for the neutrino mass terms. The natural interpretation of such model is in terms of Majorana neutrinos, as after restricting it to the physical subspace (reducing the fermion doubling) the natural restriction is that the neutrino spinor fields are their own antiparticles.

Of course, there are known mechanisms to generate possible mass terms, yet all of them involve quadratic coupling to the Higgs. In the view of the analysis of the fermionic spectral action from the previous section we see that it is possible to obtain terms of this type from the next leading term.

We shall analyze the situation in all details in this section.

Observe, that if there are no right-handed neutrinos, the fluctuations of the discrete Dirac operators on the leptonic sector are, in the basis (e_R, ν_L, e_L):

$$D_F(H) = \begin{pmatrix} & -\Upsilon_e^* \overline{H^-} & \Upsilon_e^* \overline{H^0} \\ -\Upsilon_e H^- & & \\ \Upsilon_e H^0 & & \end{pmatrix},$$

First of all, note that the first approach, by taking the term with the square of the finite part of the Dirac operator would not give anything new. Indeed, the resulting terms would only add small corrections to the usual mass terms at our energy limits. Taking the square of the $D_F(H)$ at the Higgs vacuum expectation value we obtain:

$$D_F(H_v)^2 = \begin{pmatrix} \Upsilon_e^* \Upsilon_e |H_v|^2 & & \\ & & \\ & & \Upsilon_e^* \Upsilon_e |H_v|^2 \end{pmatrix},$$

which similarly as in the Dirac masses case can only add small corrections to the already non-vanishing mass of charged leptons.

To see that some extra terms are possible we need to generalize the form of the spectral action to non-scalar functions. So far we have assumed that the function f_Λ, which we have taken to define the even part of the spectral action is scalar, that is for every operator T hat commutes with D_A we assume that $h_\Lambda(D_A)$ commutes with T as well. However, this is not necessary and we may consider other functions provided that the full gauge invariance will not be preserved.

Leaving aside the question about the classification of such functions, for the specific model, we observe the existence of a particular one. Let τ be the flip operator ($\tau = \sigma^2$ using the notation of Pauli matrices) acting on the Hilbert space of left-handed leptons (and identity elsewhere). As $\tau^2 = \mathrm{id}$ and, when restricted to the subspace of leptons, the conjugation by τ implements conjugation of the quaternions represented on the Hilbert space.

Therefore, for any scalar function $f_\Lambda^\tau(x) = \tau f_\Lambda(x) \tau$ is certanly gauge covariant and, consequently, the terms in the fermionic spectral action reimain gauge invariant. Observe that the leading term is not affected, whereas in the second term, the only the terms that include the parts that couple nondiagonally to left-handed leptons shall be changed.

That includes, in particular the terms arising from $D_F(H)^2$, which, after using τ become (again in the (e_R, ν_L, e_L) basis):

$$\tau D_F(H)\tau = \left(\begin{array}{ccc} & \Upsilon_e^* \overline{H^0} & -\Upsilon_e^* \overline{H^-} \\ \Upsilon_e H^0 & & \\ -\Upsilon_e H^- & & \end{array}\right)^2 .$$

Then the terms in the fermionic spectral action, that arise from $\mathrm{Tr}(P_\Psi f_\Lambda^\tau(\mathcal{D}^2)$ in the next-to-leading order, could be explicitly rewritten as:

$$\Lambda^2(\Upsilon_e {\Upsilon_e}^*) \left[(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} H^0 \\ -H^- \end{pmatrix} \right] \left[\left(\overline{H^0}, -\overline{H^-}\right) \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \right] + \text{h.c},$$

Note that the above term is gauge invariant. Clearly the product of the conjugate of the Higgs doublet (the latter in the fundamental representation of $SU(2)$) and the left-lepton doublet is $SU(2)$ invariant. Next, the hypercharges of the lepton doublet and the Higgs are, respectively $Y_L = -1 = Y_{H_u}$ and therefore the entire expression (as expected) is gauge invariant.

We identify this term as a Weinberg term (sometimes called Weinberg operator), which is used to describe effective mechanism of neutrino mass generation. As the operator is, in fact, non-renormalizable, the natural explanation used to be explain the physics behind the effective term as originating from yet unknown heavy intermediate particles.

After the Higgs field gets its vacuum expectation value, a neutrino mass is generated, depending on the scale Λ (recall that when compared to the usual mass term), Higgs vacuum expectation value and the charged lepton masses. If one assumes the Λ is much larger than the Higgs vacuum value (in many models this is around the scale of GUT) then the generated neutrino masses will necessarily be small.

V. CONCLUSIONS AND OUTLOOK BEYOND THE STANDARD MODEL

As we have shown, even the simplest model, which is based on the almost commutative geometry with the finite part described by a spectral triple, leads to the appearance of correction terms that give some non-minimal interactions between gravity and fermions, gauge theories and fermions as well as the Higgs field and fermions. It is interesting that no extra particles are required to explain the appearance of the neutrino masses and in the reasonable range.

Of course, we have demonstrated only that the spectral action for fermions in the Euclidean version could be constructed and computed leaving aside the problem whether any restriction could appear when considering the Lorentzian version, in particular with respect to the fermion doubling problem [9].

It is also interesting that the next order corrections lead to various terms that have been considered in various models, including Pauli interaction terms and non-minimal coupling to gravity. Connecting their origins to the same spectral action principle as in the case of neutrino masses could help to set possible limits on their observational evidence or set constraints on the models from cosmological observations in a similar way it can be done for the bosonic action [23, 24].

The neutrino mass corrections are possible in both Dirac and Majorana neutrino models, where a nonzero corrections generating possibly a small neutrino masses appear. The correction terms are non-renormalizable (which is similar to higher order terms from the bosonic action), yet could be treated as an effective description.

The model requires an extension of the assumed form of the cutoff function to include also a non-scalar part, which means that some internal permutations of the eigenspaces that are withing the range of the spectral projection P_Λ are allowed. This point certainly requires further studies, as it is necessary to understand the allowed freedom in the choice of the function. In particular, to introduce the neutrino mixing matrix one needs to generalize the cutoff function further, by adding a mixing to the function (that is modifying τ operator so that it is not diagonal for the three families).

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