

Geometry through spectral functionals.

(popular abstract)

Since the beginning of modern mathematical approach to the description of the world, geometry has played a significant role in describing the universe at all scales (like particles, movement of bodies or the evolution of the Universe). This was seen already in classical mechanics and the laws of Newton, then became evident in electrodynamic and modern physical theories like general relativity and gauge theories.

Geometry is frequently seen as the study of spaces and the structures built on them yet there is another side to it, which has been best formulated in the famous question posed over 50 years ago by Mark Kac: "Can One Hear the Shape of a Drum?". Indeed, through listening to the vibrations of the drum (the frequencies of the sound) one can learn its shape, for example one can easily distinguish a circular drum from a square one or a triangular one.

Even though we know that the answer to this question is, in general, negative, the understanding that the "hearing" of the geometry (which, in mathematical language is related to the spectrum of special differential operators like the Laplace operator or Dirac operator) can provide substantial information about the shape and size of the objects. The typical example is the Weyl law, discovered over 100 year ago which links the volume of an object with the rate of the growth of its "frequencies".

The proposed project, which has roots in this simple question, proposes to study finer geometrical structures like Riemann and Ricci tensors by the methods of "hearing" (that is spectral analysis) of certain objects. The Riemann and Ricci tensors play a significant role in the description of geometries and, in particular, appear in the construction of gravity theories.

Further part of the project goes for beyond the classical geometries and looks for methods of "hearing" the geometrical properties of algebras that do not correspond classical spaces. For example, the classical analysis of geometric spaces allows us to use the spectral methods to "hear" the volume (as in the Weyl law) yet no such methods exist for noncommutative algebras.

We expect that the results of the project will allow to use the spectral methods to study far more objects and bring around the generalization of geometrical notions to a much bigger class objects than it is known now. This will allow to use it for the construction of mathematically sound and physically relevant models of space-time geometries to describe all fundamental interactions in nature.