From noncommutative field theory towards topological recurrence

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Introduction

This project started in 1998 as an attempt to understand quantum field theories on noncommutative geometries.

- No interacting and mathematically consistent QFT is known in 4 dimensions.
- The hope was that the situation could improve on noncommutative spaces. Renormalisation and improvement in β-function were established.

Since 2009 we accumulated hints that something special is behind our computations, but we were unable to locate it.

Topological recursion

... is this special structure. It governs a remarkable variety of research lines in mathematics and physics and establishes beautiful connections between different fields.

Prehistory: Scalar fields on Moyal space

Moyal product $(f \star g)(x) = \int_{\mathbb{R}^{2D}} \frac{dydk}{(2\pi)^D} f(x + \frac{1}{2}\Theta k) g(x+y) e^{\mathrm{i}\langle k,y\rangle}$ gives rise to a very simple noncommutative geometry on which we define an action functional

$$S(\phi) = \int_{\mathbb{R}^D} \frac{dx}{(8\pi)^{D/2}} \Big(\frac{1}{2} \phi \star (-\Delta + \Omega^2 \|2\Theta^{-1} x\|^2 + m^2)\phi + \frac{\lambda}{p} \underbrace{\phi \star \dots \star \phi}_{p} \Big)(x)$$

- $\exists$ matrix basis $(e_{kl}(x))_{k,l\in\mathbb{N}}$ with $(e_{kl} \star e_{mn})(x) = \delta_{lm} e_{kn}(x)$, $\int dx\, e_{kl}(x) = \sqrt{\det(2\pi\Theta)}\delta_{kl}$
- Expand $\phi(x) = \sum_{k,l=1}^{\infty} \Phi_{kl} e_{kl}(x) \;\Rightarrow\; \int_{\mathbb{R}^D} dx\, \phi^{\star p} = \sqrt{\det(2\pi\Theta)} \mathrm{Tr}(\Phi^p)$
- For $\Omega = 1$ also the kinetic term is matrix product:

$$S(\phi) = \sqrt{\det(\Theta/4)}\, \mathrm{Tr}\Big(E\Phi^2 + \frac{\lambda}{p}\Phi^p \Big)$$

$E = c_0 \mathrm{id} + c_1 \mathrm{diag}(0,1,1,2,2,2,3,3,3,3,\dots)$ in 4D

- Need renormalisation $c_0 \mapsto m^2_{bare}$, $\Phi \mapsto \sqrt{Z}\Phi$

Measures for Euclidean quantum fields

Approximate Euclidean quantum fields by $N \times N$-matrices:

3. Kontsevich model $d\mu(\Phi) = \frac{1}{\mathcal{Z}} \int_{H_N} d\Phi \; e^{-N\,\mathrm{Tr}(E\Phi^2 + \frac{\lambda}{3}\Phi^3)}$
   - Computes intersection numbers on moduli space $\overline{\mathcal{M}}_{g,n}$ of stable complex curves [Kontsevich 92].
   - It is integrable via a relation (suggested by [Witten 91]) to the KdV hierarchy. Its moments obey topological recursion.
   - As NCQFT considered by [Grosse-Steinacker 05/06]
4. A quartic analogue $d\mu(\Phi) = \frac{1}{\mathcal{Z}} \int_{H_N} d\Phi \; e^{-N\,\mathrm{Tr}(E\Phi^2 + \frac{\lambda}{4}\Phi^4)}$
   - Passing from $\mathrm{Tr}(\Phi^3)$ to $\mathrm{Tr}(\Phi^4)$ is a minor step for QFT, but it destroys connections to mathematics.
   - The cubic vertices $\mathrm{Tr}(\Phi^3)$ encode simple zeros of the Strebel differential. One cannot make them disappear.

We show: both NCQFT-models share similar structures.

(Connected) correlation functions

= derivatives of $\log \mathcal{Z}(M) := \log \int_{H_N} d\mu(\Phi)\, e^{\mathrm{i}\Phi(M)}$ with respect to M_{kl}, with $\Phi(M) = \sum_{k,l} \Phi_{kl} M_{kl}$; they are zero unless organised in cycles:

$$\sum_{g=0}^{\infty} N^{2-2g-b} G^{(g)}_{|k_1^1 \dots k_{n_1}^1|\dots|k_1^b \dots k_{n_b}^b|} = \frac{\partial^{n_1+\dots+n_n} \log \mathcal{Z}(M)}{\partial \mathbb{M}_{k_{n_b}^b \dots k_1^b} \cdots \partial \mathbb{M}_{k_{n_1}^1 \dots k_1^1}}\Big|_{M \equiv 0}$$

where $\frac{\partial^n}{\partial \mathbb{M}_{k_n \dots k_1}} = \frac{N^n (-\mathrm{i})^n \partial^n}{\partial M_{k_n k_{n-1}} \cdots \partial M_{k_2 k_1} \partial M_{k_1 k_n}}$ and k_i^j pairwise different

- $\frac{\partial}{\partial M_{kl}}$ produces $\mathrm{i}\Phi_{kl}$ which contracts to

$$\frac{\partial \mathcal{Z}(M)}{\partial M_{kl}} = \frac{\mathrm{i}}{E_k + E_l} \int_{H_N} d\mu(\Phi) \Big(\mathrm{i} M_{lk} - \lambda N \sum_{p,q=1}^{N} \Phi_{kp} \Phi_{pq} \Phi_{ql} \Big) e^{\mathrm{i}\Phi(M)}$$

- Leads to identities between multiple M-derivatives: Dyson-Schwinger equations

Equations of motion for quartic Kontsevich model

Fourier transform $\mathcal{Z}(M) := \int_{H_N} d\mu(\Phi)\, e^{\mathrm{i}\Phi(M)}$ satisfies

1. $$-N(E_p - E_q)\sum_{k=1}^{N} \frac{\partial^2 \mathcal{Z}(M)}{\partial M_{pk}\partial M_{kq}} = \sum_{k=1}^{N}\Big(M_{kp}\frac{\partial \mathcal{Z}(M)}{\partial M_{kq}} - M_{qk}\frac{\partial \mathcal{Z}(M)}{\partial M_{pk}}\Big)$$

2. $$\frac{1}{N}\frac{\partial \mathcal{Z}(M)}{\partial E_p} = \sum_{k=1}^{N}\frac{\partial^2 \mathcal{Z}(M)}{\partial M_{pk}\partial M_{kp}} + \mathcal{Z}(M)\int_{H_N} d\mu(\Phi)\frac{1}{N}\sum_{k=1}^{N}\Phi_{pk}\Phi_{kp}$$

- They allow to express $\sum_{k=1}^{N} \frac{\mathcal{Z}(M)}{\partial M_{pk}\partial M_{kq}}$ in Dyson-Schwinger equations by fewer derivatives, thus cutting the SD-tower.
- Dyson-Schwinger equations complexify to equations for meromorphic functions in several complex variables $G^{(g)}(E_{k_1^1},\dots,E_{k_{n_1}^1}|\dots|E_{k_1^b},\dots,E_{k_{n_b}^b}) := G^{(g)}_{|k_1^1\dots k_{n_1}^1|\dots|k_1^b\dots k_{n_b}^b|}$
- We admit multiplicities $(E_1,\dots,E_N) = (\underbrace{e_1,\dots,e_1}_{r_1},\dots,\underbrace{e_d,\dots,e_d}_{r_d})$

Dyson-Schwinger equations

- We *define* a (Euclidean) QFT by Dyson-Schwinger equations for correlation functions.
- This definition is non-perturbative in the coupling constant λ, but formal in the $\frac{1}{N}$-expansion (which will not converge).

Theorem [Grosse-W 09]

The complexified planar 2-point function of the NC $\lambda\Phi^4$-model satisfies the non-linear closed equation

$$\Big(\zeta + \eta + \mu_{bare}^2 + \lambda \int_0^\infty dt\ \varrho_0(t)\, ZG^{(0)}(\zeta, t)\Big) ZG^{(0)}(\zeta, \eta)$$
$$= 1 + \lambda \int_0^\infty dt\ \varrho_0(t)\, \frac{ZG^{(0)}(t, \eta) - ZG^{(0)}(\zeta, \eta)}{t - \zeta}$$

where $\varrho_0(t) = \sum_k \frac{r_k}{N} \delta(t - e_k)$ if E has eigenvalues $\{e_k\}$ of multiplicities $\{r_k\}$.

All other $\frac{1}{N}$-expanded corr. functions satisfy affine equations!

Solution

Theorem [Panzer-W 18 for $\varrho_0 = 1$, Grosse-Hock-W 19]

1. Ansatz $G^{(0)}(x,y) = \dfrac{e^{\mathcal{H}_x[\tau_y(\bullet)]}\sin\tau_y(x)}{Z\lambda\pi\varrho_0(x)}$

   Z=renormalisation

   $\mathcal{H}_x[f] = \frac{1}{\pi}\fint \frac{dp\, f(p)}{p-x}$

2. $\tau_y(x) = \operatorname{Im}\log\big(y + I(x+\mathrm{i}\epsilon)\big)$ with $I(\zeta) = -R(-\mu^2 - R^{-1}(\zeta))$

3. $$\boxed{R(z) = z - \lambda(-z)^{D/2}\int_0^\infty \frac{dt\ \varrho_\lambda(t)}{(\mu^2+t)^{D/2}(t+\mu^2+z)}}$$

   $D = 2[\frac{\delta}{2}]$ at spectral dimension $\delta = \inf\Big(p:\ \int \frac{dt\,\varrho_0(t)}{(1+t)^{p/2}} < \infty\Big)$

4. ϱ_λ is implicit solution of $\varrho_0(R(x)) = \varrho_\lambda(x)$.

- Proof: [Cauchy 1831] residue theorem, [Lagrange 1770] inversion theorem, [Bürmann 1799] formula
- $\varrho_0(t) \equiv 1$ (2D Moyal) in terms of Lambert-W, $W(z)e^{W(z)} = z$:
  $I(\zeta) := \lambda W_0\big(\frac{1}{\lambda}e^{\frac{1+\zeta}{\lambda}}\big) - \lambda\log\big(1 - \lambda W_0\big(\frac{1}{\lambda}e^{\frac{1+\zeta}{\lambda}}\big)\big)$

$D = 4$ Moyal space: $\varrho_0(t) = t$ [Grosse-Hock-W 19]

- $\varrho_\lambda(x) \equiv \varrho_0(R(x)) = R(x) = x - \lambda x^2 \int_0^\infty \frac{dt\, \varrho_\lambda(t)}{(\mu^2+t)^2(t+x)}$
- If $\varrho_\lambda(t) \sim \varrho_0(t) = t$, then $R(x)$ bounded above. Consequently, R^{-1} would not be globally defined: triviality!
- Fredholm equation perturbatively solved by iterated integrals: Hyperlogarithms and $\zeta(2n)$ which can be summed to

$$\varrho_\lambda(x) = x \cdot {}_2F_1\left(\begin{matrix} \alpha_\lambda, 1-\alpha_\lambda \\ 2 \end{matrix} \middle| -\frac{x}{\mu^2} \right)$$

$$\alpha_\lambda = \begin{cases} \frac{\arcsin(\lambda\pi)}{\pi} & \text{for } |\lambda| \leq \frac{1}{\pi} \\ \frac{1}{2} + \mathrm{i}\frac{\operatorname{arcosh}(\lambda\pi)}{\pi} & \text{for } \lambda \geq \frac{1}{\pi} \end{cases}$$

Corollary

The interaction alters the spectral dimension to $4 - 2\frac{\arcsin(\lambda\pi)}{\pi}$ and thus avoids the triviality problem.

Gives non-perturbative integral representation for $G^{(0)}(\xi, \eta)$.

Direct solution for finite N

Theorem ([Schürmann-W 19], inspired by [Hock-Grosse-W 19])

Let $(\varepsilon_k, \varrho_k)$ be implicitly defined by $e_k = R(\varepsilon_k)$, $r_k = R'(\varepsilon_k)\varrho_k$

for $R(z) = z - \frac{\lambda}{N}\sum_{k=1}^{d} \frac{\varrho_k}{z+\varepsilon_k}$.

Then $G^{(0)}(\zeta,\eta) = \mathcal{G}^{(0)}(z,w)$ for $R(z)=\zeta$, $R(w)=\eta$ and

$$\boxed{\mathcal{G}^{(0)}(z,w) = \frac{1 - \frac{\lambda}{N}\sum_{k=1}^{d} \frac{r_k \prod_{j=1}^{d} \frac{R(w)-R(-\widehat{\varepsilon}_k^{\,j})}{R(w)-R(\varepsilon_j)}}{(R(z)-R(\varepsilon_k))(R(\varepsilon_k)-R(-w))}}{R(w)-R(-z)}}$$

where $u \in \{z, \hat{z}^1, \ldots, \hat{z}^d\}$ are all solutions of $R(u) = R(z)$.

(The symmetry $\mathcal{G}^{(0)}(z,w) = \mathcal{G}^{(0)}(w,z)$ is automatic)

Thus, planar 2-point function solved by the composition of a rational function $\mathcal{G}^{(0)}$ with inverse of another rational function R.

Topological recursion [Eynard-Orantin 07]

The exact solution by complex geometry suggests a link to topological recursion. TR recursively constructs, starting from a spectral curve consisting of

- a ramified covering $x : \Sigma \to \Sigma_0$ of Riemann surfaces,
- meromorphic differentials $\omega_{0,1}{=}ydx$ on Σ and $\omega_{0,2}$ on $\Sigma\times\Sigma$,

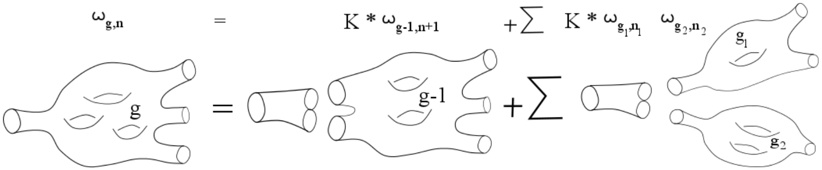
a family $\omega_{g,n}$ of meromorphic differentials on Σ^n, with poles at zeros of dx (ramification points), by

$$\omega_{g,n}(z_1,...,z_n) = \sum_i \operatorname*{Res}_{q\to\beta_i} K(z_1,q,\sigma_i(q))dz\Big(\omega_{g-1,n+1}(q,\sigma_i(q),z_2,...,z_n) + \sum_{\substack{g_1+g_2=g,\ (g_j,I_j)\neq(0,\emptyset)\\ I_1\uplus I_2=\{z_2,\dots,z_n\}}} \omega_{g_1,|I_1|+1}(q,I_1)\omega_{g_2,|I_2|+1}(\sigma_i(q),I_2)\Big)$$

- sum over ramification points β_i of x
- local involution $x(q)=x(\sigma_i(q))$ near β_i
- recursion kernel $K(z_1,z_2,z_3) = \frac{\frac{1}{2}\int_{z'=z_3}^{z_2}\omega_{0,2}(z_1,z')}{\omega_{0,1}(z_2)-\omega_{0,1}(z_3)}$]

Schematic structure of the topological recursion

Computes meromorphic differentials $\omega_{g,n}$ recursively in decreasing Euler characteristic $\chi = 2 - 2g - n$ by[1]



[1] https://upload.wikimedia.org/wikipedia/commons/7/74/Topological_recursion_illustration.png, B. Eynard, CC BY-SA 4.0 ⟨https://creativecommons.org/licenses/by-sa/4.0⟩, via Wikimedia Commons

Selected examples for topological recursion

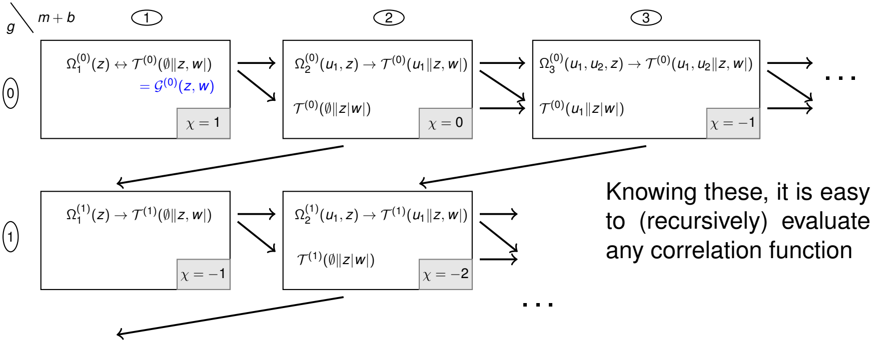
- $(\hat{\mathbb{C}} \to \hat{\mathbb{C}},\ x(z) = \alpha + \gamma(z+\frac{1}{z}),\ y(z) = \sum_j u_j(z^j - z^{-j}), \omega_{0,2} = B)$
  Hermitian one-matrix model, describes 2D quantum gravity, integrable [Gross-Migdal, Brézin-Kazakov, Douglas-Shenker 90]
- $(\hat{\mathbb{C}} \to \hat{\mathbb{C}},\ x(z) = z^2,\ y(z) = z,\ \omega_{0,2} = B)$
  [Kontsevich 92] model, equivalent formulation of 2D quantum gravity, generates intersection numbers on $\overline{\mathcal{M}}_{g,n}$ as conjectured by [Witten 91].
  The $\omega_{g,n}$ computed from TR are complexifications of $\Big\langle \int \frac{d\Phi}{\mathcal{Z}} \Phi_{k_1 k_1} \cdots \Phi_{k_n k_n} e^{-N\,\mathrm{Tr}(E^2\Phi^2 + \frac{\lambda}{3}\Phi^3)} \Big\rangle_c$ restriced to genus g.
- For $\mathfrak{X}$ a toric Calabi-Yau 3-fold (A-model), let $\mathcal{S}$ be the singular locus (B-model) of its mirror Calabi-Yau. Then $\omega_{g,n}$ of $\mathcal{S}$ generate Gromov-Witten invariants of $\mathfrak{X}$ (which classify stable genus-g maps into $\mathfrak{X}$) [Bouchard-Mariño-Klemm-Pasquetti 07]

Linear $\lambda\Phi^4$-equations [Branahl-Hock-W 20]

Recall that $d\mu(\Phi)$ depends on spectral values $e_1, \dots, e_d$. Define

$$\sum_{g=0}^{\infty} N^{2-2g-n} \Omega^{(g)}_{q_1,\dots,q_n} := \frac{\partial^{n-1}\Big(N \sum_{k=1}^{N} \int_{H_N} d\mu(\Phi) \Phi_{q_1 k} \Phi_{k q_1}\Big)}{\partial e_{q_2} \cdots \partial e_{q_n}} + \frac{\delta_{n,2}}{(e_{q_1} - e_{q_2})^2}$$

Their complexification forms with two families $\mathcal{T}(I\|z,w|)$, $\mathcal{T}(I\|z|w|)$ a system of equations to solve in decreasing χ:



Knowing these, it is easy to (recursively) evaluate any correlation function

Contact with topological recursion [BHW 20]

- Pass to meromorphic differentials
  $\omega_{g,m}(z_1, \dots, z_m) = \lambda^{2-2g-m}\Omega^{(g)}_m(z_1, \dots, z_m) \prod_{k=1}^m dR(z_i)$
- Intermediate steps of solution scheme extremely lengthy, but final result simple and structured:

$$\omega_{0,2}(u,z) = \frac{du\, dz}{(u-z)^2} + \frac{du\, dz}{(u+z)^2}$$

$$\omega_{0,3}(u_1,u_2,z) = -\sum_{i=1}^{2d} \frac{\Big(\frac{1}{(u_1-\beta_i)^2} + \frac{1}{(u_1+\beta_i)^2}\Big)\Big(\frac{1}{(u_2-\beta_i)^2} + \frac{1}{(u_2+\beta_i)^2}\Big) du_1\, du_2\, dz}{R'(-\beta_i)R''(\beta_i)(z-\beta_i)^2}$$
$$+ \Big[d_{u_1}\Big(\frac{\omega_{0,2}(u_2,u_1)}{(dR)(u_1)} \frac{dz}{R'(-u_1)(z+u_1)^2}\Big) + u_1 \leftrightarrow u_2 \Big]$$

where $\beta_1, \dots, \beta_{2d}$ are the ramification points of R, i.e. $dR(\beta_i) = 0$

Observation

The blue terms are exactly those of topological recursion, the red terms are a consistent extension called blobbed topological recursion [Borot-Shadrin 15].

Quartic Kontsevich model obeys BTR!

Proposition $(g,m) \in \{(0,3),(0,4),(0,5),(1,1)\}$ / Conjecture

Let $R : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$ be ramified cover identified in $\mathcal{G}^{(0)}(z,w)$ with ramification points $\beta_1,\dots,\beta_{2d}$. Define $\omega_{0,1}(z) = -R(-z)dR(z)$ and for $2-2g-m \leq 0$ the $\omega_{g,m}$ as before from $\Omega^{(g)}_m$.
Then parts $\mathcal{P}\omega_{g,m}$ containing the poles at ramification points:

$$\begin{aligned}
&\mathcal{P}_z\omega_{g,m}(u_1,\dots,u_{m-1},z) \\
&= \sum_{i=1}^{2d} \operatorname*{Res}_{q\to\beta_i} \frac{\frac{1}{2}\int_{q'=\sigma(q)}^{q'=q} B(z,q')}{\omega_{0,1}(q)-\omega_{0,1}(\sigma_i(q))} \Bigg(\omega_{g-1,m+1}(u_1,\dots,u_{m-1},q,\sigma_i(q)) \\
&\qquad + \sum_{\substack{I_1\uplus I_2=\{u_1,\dots,u_{m-1}\}\\ g_1+g_2=g\\ (I_1,g_1)\neq(\emptyset,0)\neq(I_2,g_2)}} \omega_{g_1,|I_1|+1}(I_1,q)\,\omega_{g_2,|I_2|+1}(I_2,\sigma_i(q)) \Bigg)
\end{aligned}$$

where σ_i = local Galois involution near β_i, i.e. $R(z)=R(\sigma_i(z))$, $\sigma_i(\beta_i)=\beta_i$, $\sigma_i \neq \mathrm{id}$ and $B(u,z) = \frac{du\,dz}{(u-z)^2}$ Bergman kernel.

Proof for genus $g = 0$ [Hock-W 21]

When trying to prove the conjecture for $g = 0$ we noticed surprising identities between $\omega_{0,m+1}(u_1, ..., u_m, -z)$ and $\omega_{0,k+1}(u_1, ..., u_k, z)$. They are of independent interest:

Definition

Let $x : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$ be a ramified covering with ramification points $\beta_1, ..., \beta_r$. For a global involution $\iota : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$, which neither fixes nor permutes the β_i, let $y(z) := -x(\iota z)$. Then a family $\{\omega_{0,n}\}_{n\geq 2}$ of meromorphic differentials is introduced by

$$\omega_{0,2}(w,z) = \frac{1}{2}\frac{dw\,dz}{(w-z)^2} + \frac{1}{2}\frac{d(\iota w)\,d(\iota z)}{(\iota w - \iota z)^2} - \frac{1}{2}\frac{dw\,d(\iota z)}{(w-\iota z)^2} - \frac{1}{2}\frac{d(\iota w)\,dz}{(\iota w - z)^2}$$

and for $m \geq 2$ by the involution identity

$$\omega_{0,m+1}(u_1, ..., u_m, z) + \omega_{0,m+1}(u_1, ..., u_m, \iota z) = \sum_{s=2}^{m} \sum_{I_1 \uplus \ldots \uplus I_s = \{u_1,\ldots,u_m\}} \frac{1}{s} \operatorname*{Res}_{w\to z} \left(\frac{dy(z)dx(w)}{(y(z)-y(w))^s} \prod_{i=1}^{s} \frac{\omega_{0,|I_i|+1}(I_i, w)}{dx(w)} \right) .$$

Theorem [Hock-W 21]

The involution identity has the (under mild assuptions unique) solution

$$\omega_{0,m+1}(u_1,\dots,u_m,z)$$
$$=\sum_{i=1}^{r}\operatorname*{Res}_{q\to\beta_i} K_i(z,q)\sum_{I_1\uplus I_2=\{u_1,\dots,u_m\}}\omega_{0,|I_1|+1}(I_1,q)\omega_{0,|I_2|+1}(I_2,\sigma_i(q))$$
$$-\sum_{k=1}^{m} d_{u_k}\Big[\operatorname*{Res}_{q\to\iota u_k}\sum_{I_1\uplus I_2=\{u_1,\dots,u_m\}}\tilde{K}(z,q,u_k)d_{u_k}^{-1}\big(\omega_{0,|I_1|+1}(I_1,q)\omega_{0,|I_2|+1}(I_2,q)\big)\Big]$$

where the recursion kernels are given by

$$K_i(z,q):=\frac{\frac{1}{2}\big(\frac{dz}{z-q}-\frac{dz}{z-\sigma_i(q)}\big)}{dx(\sigma_i(q))(y(q)-y(\sigma_i(q)))}\,,\tilde{K}(z,q,u):=\frac{\frac{1}{2}\big(\frac{d(\iota z)}{\iota z-\iota q}-\frac{d(\iota z)}{\iota z-u}\big)}{dx(q)(y(q)-y(\iota u))}.$$

The solution implies symmetry $z\mapsto\iota z$ of the rhs of the involution identity.

Back to quartic Kontsevich model

Theorem

For the choice

$$\iota z = -z\,, \qquad x(z) = R(z) := z - \frac{\lambda}{N}\sum_{k=1}^{d}\frac{\varrho_k}{\varepsilon_k + z}\,,$$

the solution of the involution identity coincides with the solution of the system for $(\Omega_n^{(0)}, \mathcal{T}^{(0)})$ found in [Branahl-Hock-W 20].

- Many more surprising combinatorial identites found on the way. Some conjectures were proved by Maciej Dołęga.
- There are a few examples for blobbed topological recursion (e.g. multitrace Hermitian matrix model, stuffed maps), but recursion kernel for blob is rather special.
- We have some ideas to extend the involution identity to higher g. The difficulty is more the $(\Omega_n^{(g)}, \mathcal{T}^{(g)})$-system.

Summary and outlook

- Quantum field theories on noncommutative geometries can be identified with matrix models.
- These are often exactly solvable or even integrable, which is best understood in terms of topological recursion.
- We have shown that these prospects also apply to the noncommutative $\lambda\phi^4$-model, which relates to blobbed TR.

Intersection numbers [Borot-Shadrin 15]

Forms $\omega_{g,m}$ which satisfy BTR encode intersection numbers on the moduli space $\overline{\mathcal{M}}_{g,m}$ of stable complex curves. We expect that they encode a geometric structure related to $z \mapsto -z$.

Integrability

Is not known in BTR. But this model with involution $z \mapsto -z$ and blob given by recursion kernel is special. We are optimistic.

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