Anomalies: list of problems

Problem 1: ABJ computation

Please check what are the minimal assumptions on f so that the expression:

$$\Delta(a) = \int dx^n \, \left(f(x+a) - f(x) \right),$$

gives the answer as in the „naive" ABJ computations:

$$\Delta(a) \sim \sum_{i=1}^{n} a^i \lim_{|x|\to\infty} x_i |x|^{n-2} f(x).$$

Problem 2: Fujikawa method in 2D

Please use the Fujikawa method (eigenvalues) to compute the anomaly (like in the lecture) but in 2 dimensions.

Problem 3: BRST

Let:

$$\mathbf{A}_\mu = A^a_\mu t^a, \qquad \mathbf{c} = c^a t^a, \qquad \mathbf{\Phi} = \Phi^a t^a,$$

where t^a are matrices generating a Lie algebra that satisfy $[t^a, t^b] = f^{abr} t^r$. All c^a are Grassman fields. Take s to be the following operator:

$$s\mathbf{A}_\mu = D_\mu \mathbf{c}, \;\; s\mathbf{c} = \alpha \mathbf{c}\mathbf{c}, \;\; s\bar{\mathbf{c}} = \beta \mathbf{\Phi}, \;\; s\mathbf{\Phi} = 0,$$

where α, β are real numbers and $c, \bar{c}$ are Grassman.
Please check for which α, β we have $s^2 = 0$ and what is the graded Leibniz rule so that both terms

$$\mathcal{L}_{YM} = -\frac{1}{4} F^{\mu\nu\,a} F^a_{\mu\nu}, \qquad \mathcal{L}_1 = \partial^\mu \Phi^a A^a_\mu + \frac{\xi}{2} \Phi^a \Phi^a + \partial_\mu \bar{c}^a (D_\mu c)^a,$$

are invariant .
Note:

$$\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu - i[\mathbf{A}_\mu, \mathbf{A}_\nu],$$
$$D_\mu(x) = \partial_\mu(x) - i[\mathbf{A}_\mu, x].$$