MODERN MATHEMATICS II (2023/24)
LIST OF TOPICS

Topology: elements of algebraic topology (4-6 hours)

Basic notions. Homotopy groups. Chain and cochain complexes. Basic definitions for homology and cohomology. Čech cohomology. Basic computations and examples

Differential manifolds (8-10 hours)

Manifolds: definition, maps, atlases, examples. Differential structures. Diffeomorphisms. Smooth maps between manifolds. (2-3 hours)
Tangent space. Vector fields: definition through tangent space (sections of the tangent space) and operational (derivations on smooth functions). Cotangent bundle. Pullback and pushforward maps, operations on vector fields. (4-6 hours)
Differential forms. External derivative. De Rham cohomology complex. Pullback of differential forms. (2-3 hours)

Fibre budles: vector and principal fibre bundles (4 hours)

Basic definition of a fibre bundle. Vector budle. Basic examples (Möbius band). Triviality of vector bundles. (2 hours)
Applications to Lie group. Vector fields over Lie groups. Principal fibre bundles. Triviality and examples, the Hopf fibration $SU(2) \to S^2$ with local trivialisations. (2 hours)

Different Views on Connections (6 hours)

Definition of the connection as a bundle of horizontal vectors. Connection form, pullback of the connection form and its behavior with respect to local trivialisations. Curvature form and its pullback. (2 hours)
Associated bundles. Connection on associated bundles. General notion of a connection on sections of a vector bundle. Curvature of the connection. (2 hours)
Linear connections, metric compatibility of connections. Torsion of the metric connection, Levi-Civita connection and its uniqueness. (2 hours)

Advanced topics (4-8 hours)

Chern classes and other topological terms. t'Hoft-Polyakov monopoles. Spinors. Supermanifolds. Superalgebras. Conformal symmetries, Virasoro algebras and QFT in 2 dimensions. Topological field theory. Gerbes.

Basic literature

C. Nash, S. Sen, *Topology and Geometry for Physicists*, Dover Publications, 2011
M. Nakahara, *Geometry, Topology and Physics*, IOP Publishing, 2003
C.Isham, *Modern Differential Geometry for Physicists*, World Scientific, 1999
M. Göckeler, T. Schücker, *Differential Geometry, Gauge Theories, and Gravity*, CUP 1989