Effective QCD - problem set 12
23.01.2018. Tuesday 14:00
room D-02-2

1. Nonrelativistic reduction of the Dirac hamiltonian.

   Consider Dirac equation for a point-like, relativistic, charged particle

   $$i\partial_t\Psi = \left[\vec{\alpha}\cdot\left(\vec{p}-q\vec{A}\right)+\beta m+qA_0\right]\Psi \equiv H\Psi$$

   in the canonical representation for matrices $\vec{\alpha}$ and $\beta = \mathrm{diag}[1_{2\times 2}, -1_{2\times 2}]$. For simplicity let's assume $\vec{A}=0$. Define

   $$\Psi(\vec{x},t) = e^{-im}\Psi'(\vec{x},t)$$

   and derive equation for Ψ'. Decompose Ψ' into a large and a small component

   $$\Psi' = \frac{1}{2}(1+\beta)\Psi_L + \frac{1}{2}(1-\beta)\Psi_S$$

   and write a system of two first order (in time and ∂_x) differencial equations that mix both components.

2. Show that the system from the previous problem can formally solved for Ψ_S

   $$\Psi_S = \frac{1}{2m+i\partial_t - qA_0}\vec{\sigma}\cdot\vec{p}\,\Psi_L \equiv \mathcal{D}\Psi_L.$$

   Expand operator $\mathcal{D}$ in powers of $1/m$ up to the second order. Use Maxwell equation to eliminate derivatives of A_0. Using this expansion derive equation for the large component Ψ_L.

   [Answer: $(i\partial_t - qA_0)\Psi_L = \left(p^2/2m - q/4m^2\left\{\vec{\nabla}\cdot\vec{E}+\vec{\sigma}\cdot\vec{E}\times\vec{p}+i\vec{E}\cdot\vec{p}\right\}\right)\Psi_L$]

   Compare the result with the Foldy-Wouthuysen transformation discussed *e.g.* in the book of Bjorken and Drell.