Effective QCD - problem set 11
16.01.2018. Tuesday 14:00

room D-02-2

1. For G =SU(2)$_L\times$SU(2)$_R\times$U(1) the covariant derivative acting on a nucleon dublet
$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$$
takes the following form
$$D_\mu \Psi = \left(\partial_\mu + \Gamma_\mu - v_\mu^{(s)}\right)\Psi$$
where connection Γ_μ is given by:
$$\Gamma_\mu = \frac{1}{2}\left[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger\right]$$
and $u^2 = U$. In order to prove that $D_\mu\Psi$ transforms under G as
$$D'_\mu\Psi' = e^{-i\theta(x)}K(V_L(x), V_R(x), U(x))D_\mu\Psi$$
prove that
$$\partial_\mu K = K\Gamma_\mu - \Gamma'_\mu K.$$
Use
$$K = u'^\dagger V_R u = u' V_L u^\dagger$$
and
$$\begin{aligned} r'_\mu &= V_R r_\mu V_R^\dagger + iV_R\partial_\mu V_R^\dagger, \\ l'_\mu &= V_L l_\mu V_L^\dagger + iV_L\partial_\mu V_L^\dagger, \\ v_\mu^{(s)\prime} &= v_\mu^{(s)} - \partial_\mu\theta. \end{aligned}$$

2. Similarly, show that the so-called vielbein
$$u_\mu = u_\mu^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger$$
transforms as
$$u_\mu \to u'_\mu = K u_\mu K^\dagger.$$

3. Interaction lagragian of pions with the nucleon is given in tems of the nucleon mass m_N and so called axial coupling g_A:
$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\not{D} - m_N + \frac{1}{2}g_A\gamma^\mu\gamma_5 u_\mu\right)\Psi.$$
Using exponential parametrization of for the pion fields
$$U(x) = \exp\left(\frac{i}{F_0}\vec{\tau}\cdot\vec{\phi}(x)\right)$$
calculate one-pion and two-pion −nucleon interaction lagrangian (in this case external currents can be put to zero).