Effective QCD - problem set 10
9.01.2018. Tuesday 14:00
room D-02-2

1. Effective QCD lagrangian from problem set 4 can be supplemented by two 4 derivative terms, one of them taking a form of the commutator squared,

$$\mathcal{L}_{\text{eff}} = \frac{F_\pi^2}{4}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\,\mathrm{Tr}\left(\left[U^\dagger\partial_\mu U, U^\dagger \partial_\nu U\right]^2\right),$$

where e is a free parameter. Calculate the lowest order interaction term by expanding U in terms of the Goldstone boson fields in SU(3). Use explicit form of the SU(3) structure constnats $d_{ijk}$and f_{ijk}.

2. Loop corrections to the masses of Goldstone bosons (self energies) calculated from the lowest order lagrangian $\mathcal{L}_2$

$$\mathcal{L}_2 = \frac{F_0^2}{4}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{F_0^2}{2}\,\mathrm{Tr}\left(M(U^\dagger + U)\right)$$

with $M = \mathrm{diag}(m, m, m_s)$ are of the same order as the p^4 effective lagrangian. Here the relevant terms are the ones correspnding to the Gasser-Leutwyler coefficients $L_4 - L_8$:

$$\begin{aligned}
\mathcal{L}_4 \;=\; & \ldots + L_4\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right) \\
& + L_5\,\mathrm{Tr}\left[\left(\partial_\mu U \partial^\mu U^\dagger\right)\left(\chi U^\dagger + U\chi^\dagger\right)\right] \\
& + L_6\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 \\
& + L_7\left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 \\
& + L_8\,\mathrm{Tr}\left(\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger\right) + \ldots
\end{aligned}$$

with $\chi = 2B_0 M$. Expand $\mathcal{L}_4$ up to two fields Using exponential parametrization of U) and show that

$$\begin{aligned}
\mathcal{L}_4^{2\phi} \;=\; & \frac{1}{2}(a_\eta \partial_\mu \eta \partial^\mu \eta - b_\eta \eta^2) \\
+ \; & \frac{1}{2}(a_\pi \partial_\mu \pi^0 \partial^\mu \pi^0 - b_\pi \pi^0 \pi^0) \\
+ \; & a_\pi \partial_\mu \pi^+ \partial^\mu \pi^- - b_\pi \pi^+ \pi^- \\
+ \; & a_K \partial_\mu K^+ \partial^\mu K^- - b_K K^+ K^- \\
+ \; & a_K \partial_\mu K^0 \partial^\mu \bar{K}^0 - b_K K^0 \bar{K}^0
\end{aligned}$$

and calculate coefficients a and b in terms of the quark masses and constants $L_4 - L_8$.