Effective QCD - problem set 9
12.12.2017. Tuesday 14:00

room D-02-2

1. Effective QCD lagrangian from problem set 4 can be supplemented by two 4 derivative terms, one of them taking a form of the commutator squared,
$$\mathcal{L}_{\rm eff} = \frac{F_\pi^2}{4}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\,\mathrm{Tr}\left(\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2\right),$$
where e is a free parameter. Calculate the lowest order interaction term by expanding U in terms of the Goldstone boson fields in SU(3)

2. Effective lagrangians discussed so far contained only even numbers of U fields. In order do describe physically possible transitions like $K^+K^- \to \pi^+\pi^-\pi^0$ we need anomalous term involving five fields U. Such term is called Wess-Zumino-Witten term and it cannot be written as a local action. Witten proposed to introduce the fifth time-like dimension characterized by the coordinate $0 \le \alpha \le 1$
$$x^\mu \to y^i = (x^\mu, \alpha)$$
and extending the U field
$$U = \exp\left(i\alpha\frac{\phi(x)}{F_0}\right),$$
where ϕ is the SU(3) meson field we have used before. Then the nonlocal action takes the following form
$$S_{WZW} = -\frac{i}{240\pi^2}\int_0^1 d\alpha \int d^4x\,\varepsilon^{ijklm}\,\mathrm{Tr}\left(L_i L_j L_k L_l L_m\right),$$
where
$$L_i = U^\dagger \frac{\partial U}{\partial y^i}$$
and ε^{ijklm} is totally antisymmetric. Expanding U in powers of ϕ allows to perform the integral over α. Calcualte the lowest order term in this expansion.

3. Prove that in 4 dimensional Minkowski space
$$\Delta(a) = a^\mu \int d^4r\,\partial_\mu f(r) = 2\pi i\,a^\mu \lim_{R\to\infty} R^2 R_\mu f(R).$$

4. Recall that the axial current amplitude to two photons is given by ($q = k_1 + k_2$)
$$\begin{aligned} T_{\mu\nu\lambda} &= -\int \frac{d^4p}{(2\pi)^4}\left\{\mathrm{Tr}\left[\frac{1}{\not p - m}\gamma_\lambda\gamma_5\frac{1}{\not p - \not q - m}\gamma_\nu\frac{1}{\not p - \not k_1 - m}\gamma_\mu\right]\right. \\ &\left. + \mathrm{Tr}\left[\frac{1}{\not p - m}\gamma_\lambda\gamma_5\frac{1}{\not p - \not q - m}\gamma_\mu\frac{1}{\not p - \not k_2 - m}\gamma_\nu\right]\right\}. \end{aligned}$$

Calculate $k_1^\mu T_{\mu\nu\lambda}$ using in the numerator the following trick

$$k_1 = (p - k_2 - m) - (p - q - m)$$

Then use the expansion from problem 3 for $a = k_1$ to show that

$$k_1^\mu T_{\mu\nu\lambda} = \frac{1}{8\pi} \varepsilon_{\nu\lambda\rho\sigma} k_1^\rho k_2^\sigma .$$