Effective QCD - problem set 7
28.11.2017. Tuesday 14:00
room D-02-2

1. Effective QCD lagrangian in the presence of sources reads

$$\mathcal{L} = \frac{F_0^2}{4} \operatorname{Tr}\left(D_\mu U (D^\mu U)^\dagger\right) + \frac{F_0^2}{4} \operatorname{Tr}\left(\chi U^\dagger + U \chi^\dagger\right)$$

where

$$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu,\ \chi = 2B_0(s + ip).$$

Variation of U takes the following form:

$$\delta U = i \sum_a \lambda_a \Delta_a.$$

Derive equations of motion for field U.

2. Interaction lagrangian of the weak current with the Goldstone boson field ϕ (set 4 problem 4) reads as follows:

$$\mathcal{L}_{W\phi} = -\frac{g}{\sqrt{2}} \frac{F_0}{2} \operatorname{Tr}\left[\left(W_\mu^+ T^+ + W_\mu^- T^-\right) \partial^\mu \phi\right]$$

where

$$T^+ = \begin{bmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ T^- = \left(T^+\right)^\dagger.$$

Express $\mathcal{L}_{W\phi}$ in terms of π and K fields.

3. Feynman amplitude for pion decay to $\mu^+ \nu_\mu$ takes the following form

$$\mathcal{M} = -G_F V_{ud} F_0 \left[\bar{u}_{\nu_\mu} \not{p} (1 - \gamma_5) v_{\mu^+}\right].$$

Calculate the decay width Γ from the following formula:

$$\Gamma = \frac{1}{2E_\pi} \int \frac{d^4 p_\mu}{(2\pi)^4} 2\pi \delta(p_\mu^2 - m_\mu^2) \frac{d^4 p_\nu}{(2\pi)^4} 2\pi \delta(p_\nu^2) \overline{|\mathcal{M}|^2} (2\pi)^4 \delta^{(4)}(p_\pi - p_\mu - p_{g\nu})$$

where *bar* over the amplitude squared denotes summation over final polarizations of μ and ν. What would be the result if the weak current would couple by just 1 instead of $1 - \gamma_5$?