Effective QCD - problem set 6
21.11.2017. Tuesday 14:00

room D-02-2

1. Find transformation properties of SU(3) source currents: vector, axial, vector-singlet, and source densities: scalar and pseudoscalar under parity transformation and charge conjugation.

2. Suppose we transform the quark fields by *local* transformations:

$$\begin{aligned} q_R &\to \exp\left(-i\theta(x)/3\right) V_R(x) q_R, \\ q_L &\to \exp\left(-i\theta(x)/3\right) V_L(x) q_L \end{aligned}$$

   where $V_{R,L}(x)$ are SU(3) matrices. Find transformation properties of above mentioned currents and densities that leave the QCD lagrangian invariant.

3. At the last problem class we have derived interaction lagrangian coming from $\mathcal{L}_2$

$$\mathcal{L}_2 = \frac{F_\pi^2}{4} \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)$$

   with

$$U = e^{\frac{i}{F_\pi}\vec{\tau}\cdot\vec{\phi}(x)}$$

   that took the following form:

$$\mathcal{L}_2^{4\phi} = \frac{1}{6F_\pi^2}\left(\partial_\mu\vec{\phi}\cdot\vec{\phi}\,\partial^\mu\vec{\phi}\cdot\vec{\phi} - \partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi}\,\vec{\phi}\cdot\vec{\phi}\right).$$

   Calculate the same interaction term for another parametrizaton of the U matrix

$$U = \frac{1}{F_\pi}\left[\sigma(x) + i\,\vec{\tau}\cdot\vec{\pi}(x)\right] \text{ where } \sigma(x) = \sqrt{F_\pi^2 - \vec{\pi}^{\,2}(x)}.$$

   In both cases calculate the mass term

$$\mathcal{L}_m = \frac{F_\pi^2 m_\pi^2}{4}\,\mathrm{Tr}(U + U^\dagger)$$

   up to 4 fields.

4. (*) Calculate Feynman vertices for both parametrizations.