Effective QCD - problem set 4
14.11.2017. Tuesday 14:00

room D-02-2

1. For the SU(2) case calculate the chiral lagrangian up to 4-field interaction. Use
$$U = \exp\left(\frac{i}{F_\pi}\vec{\tau}\cdot\vec{\pi}(x)\right)$$
and
$$\mathcal{L}_{\text{eff}} = \frac{F_\pi^2}{4}\,\text{Tr}\left(\partial_\mu U\,\partial^\mu U^\dagger\right). \qquad (1)$$

2. Prove that (for SU(N))
$$\text{Tr}\left(\partial_\mu U\,U^\dagger\right) = 0.$$

3. $\text{SU}_\text{R}(N)\times\text{SU}_\text{L}(N)$symmetry is implemented by the following transformation:
$$U(x) \to LU(x)R^\dagger$$
where
$$L, R = e^{-i\theta_a^{L,R}\lambda^a/2}.$$
Use Nother construction to calculate currents associated with L and R transformations. Expand currents up to the lowest possible order in the number of fields $\phi^a(x)$.

4. Add to (1) (with $F_\pi \to F_0$) the symmetry breaking term
$$\mathcal{L}_M = -\frac{F_0^2 B_0}{2}\,\text{Tr}\left(M(U\; + U^\dagger)\right)$$
where M =diag(m, m, m_s). Calculate $\mathcal{L}_M$ up to the lowest number of fields ϕ.Using
$$\lambda_a\phi^a(x) = \begin{bmatrix} \pi^0 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta^0 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta^0 \end{bmatrix}$$
calculate
$$\mathcal{L}_{\text{eff}} + \mathcal{L}_M$$
in terms of the physical fields. Read out the masses of these fields. Since these three masses will be expressed in terms of two "free"parameters $B_0 m$ and $B_0 m_s$, there exits one relation between these masses. Deive this relation and check its numerical accuracy.

5. Using the "hedhehog" Ansatz
$$U(x) = \exp\left(i\,\vec{n}\cdot\vec{\tau}\,P(r)\right)$$
calculate $\mathcal{L}_{\text{eff}}$. Derive e.o.m. for the function $P(r)$.