Effective QCD - problem set 2
24.10.2017. Tuesday 14:00
room D-02-2

1. Real scalar field lagrangian density reads as follows:

$$\mathcal{L}(\phi, \partial_\mu \phi) = \frac{1}{2}\partial_\mu \phi(x)\partial^\mu \phi(x) - \frac{1}{2}m^2\phi^2(x).$$

   Calculate Hamiltonian. Canonical equal-time quantization rules for real scalar field operators read:

$$\left[\hat{\phi}(t,\vec{x}), \vec{\pi}(t,\vec{x}\,')\right] = i\delta^{(3)}(\vec{x} - \vec{x}\,')$$

   and all other possible commutators are zero. Using decomposition

$$\hat{\phi}(t,\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^2\sqrt{2\omega_k}} \left[e^{-i\,kx}\hat{a}(\vec{k}) + e^{+i\,kx}\hat{a}^\dagger(\vec{k})\right]$$

   show that the canonical quatization rules are satisfied if

$$\left[\hat{a}(\vec{k}), \hat{a}^\dagger(\vec{k}\,')\right] = (2\pi)^3\delta^{(3)}(\vec{k} - \vec{k}\,')$$

   and the remaining two commutators vanish.

2. Show that $\psi_R^\dagger \psi_L$ and $\psi_L^\dagger \psi_R$ are invariant under proper Lorentz transformations, and $\psi_R^\dagger \sigma^\mu \psi_R$ and $\psi_L^\dagger \tilde{\sigma}^\mu \psi_L$are contravariant four-vectors.

3. Consider momentum in polar coordinates

$$\vec{p} = |\vec{p}|\left(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\right).$$

   Find two dimensional Weyl spinors $|\pm\rangle$ that are eigenstates of the helicity operator

$$\frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\left|\pm\right\rangle = \pm\left|\pm\right\rangle.$$

4. Lagrangian density for real scalar field ϕ takes the following form

$$\mathcal{L}(\phi, \partial_\mu \phi) = \frac{1}{2}\partial_\mu \phi(x)\partial^\mu \phi(x) - \frac{1}{2}m^2\phi^2(x) - \frac{\lambda}{4}\phi^4(x)$$

   Consider the case $\mu^2 < 0$. Irrespectively of the sign of μ^2 term the Hamiltonain has a symmetry $R : \phi \to -\phi$. Find minima of the potential. How do they transform under R? Check that $R^2 = 1$, $R = R^{-1} = R^\dagger$. Shift the fields by the field value at the minimum and find the resulting Lagrangian density.

5. Assume that the vacuum state is a linear superposition (with plus sign) of the two minima from the previous problem. Such vacuum state is invariant uner R. Show that this superposition is not stable against any perturbation H' that satisfies

$$RH'R^{\dagger} = -H'.$$

To this end calculate the first order correction to the ground state energy. Note that you have to use degenerate perturbation theory, because the second linear superposition with a minus sign has also the same energy. Use the above symmetry to eleminate some elements of the perturbation matrix. What are the eigenstates that diagonalize H'?