QCD Lecture 1

October 4

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http://th-www.if.uj.edu.pl/~michal/QCD_2022/

A: Perturbative QCD

1. Introduction, Deep Inelastic Scattering, why QCD.
2. Field theory, Feynman rules (reminder).
3. Nonabelian SU(N) field theory, free case, interaction, color factors.
4. Renormalization, example: self-energy.
5. Running coupling constant.
6. Perturbative calculation of axial anomaly.

B: Path integral formulation of QCD

1. Introduction, reminder on the Dirac notation, path integral in QM.
2. Path integral for a classical scalar field, 2-point Green function, propagator in momentum space.
3. Fermions, functional determinants, Grassmann variables, Berezin integral.
4. Chiral transformation and its Jacobian.
5. Computing anomaly with the Fujikawa method, Atiyah-Singer theorem.
6. Theta term in QCD, topological current K_{mu}, fermion masses and theta term.
7. Quantization of the non-Abelian gauge theories: QCD vs. QED, Feynman rules, gauge fixing, Jacobian in the path integral.
8. Faddeev-Popov ghosts, Feynman rules for ghosts, on-shell Ward identities for QCD.
9. Lattice QCD.

C: Low energy QCD

1. Effective QCD: chiral symmetry, conserved currents and charges, chiral algebra, inclusion of quark masses.
2. Chiral Ward identities, QCD spectrum and chiral symmetry, quark condensate and Goldstone bosons, PCAC.
3. Nonlinear realization of chiral symmetry and Goldstone bosons, chiral lagrangian.
4. Heavy Quark Symmetry.

Francois Gelis,
"A Stroll Through Quantum Fields" (internet)

Richard Feynman,
"Feynman Lectures on Strong Interactions" (internet)

W. Greiner, S. Schramm, E. Stein
"Quantum Choromodynamics" (library)

J. Collins,
"Foundations of Perturbative QCD" (library)

S. Scherer,
"Introduction to Chiral Perturbation Theory" (internet)

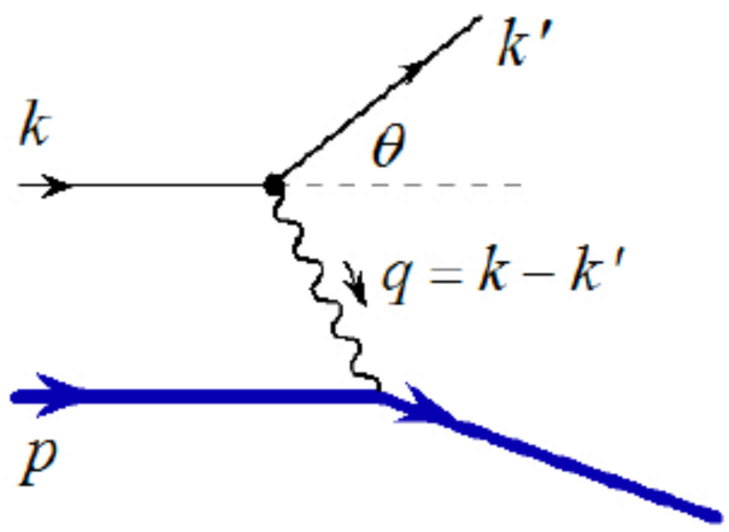
Assessment:

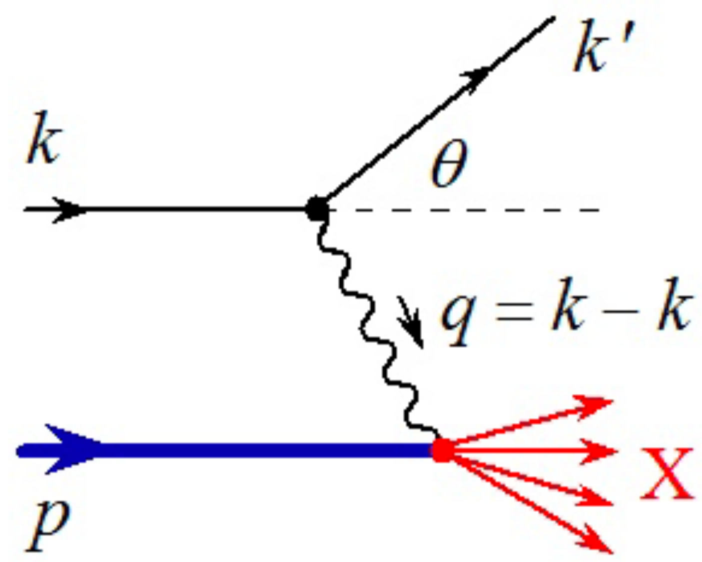
Active participation in tutorials (min. 3 full solutions)

Absence: max. 3 (no consequences),
up to 5 – written solutions of selected problems required,
over 5 – no assesment (in the case of illness individual assesment scheme might be possible)

Oral exam: list of ~ 10 broad topics (problems)
Each student can choose one problem she/he would like to discuss, and then she/he will be asked to discuss another problem chosen by the examiner.

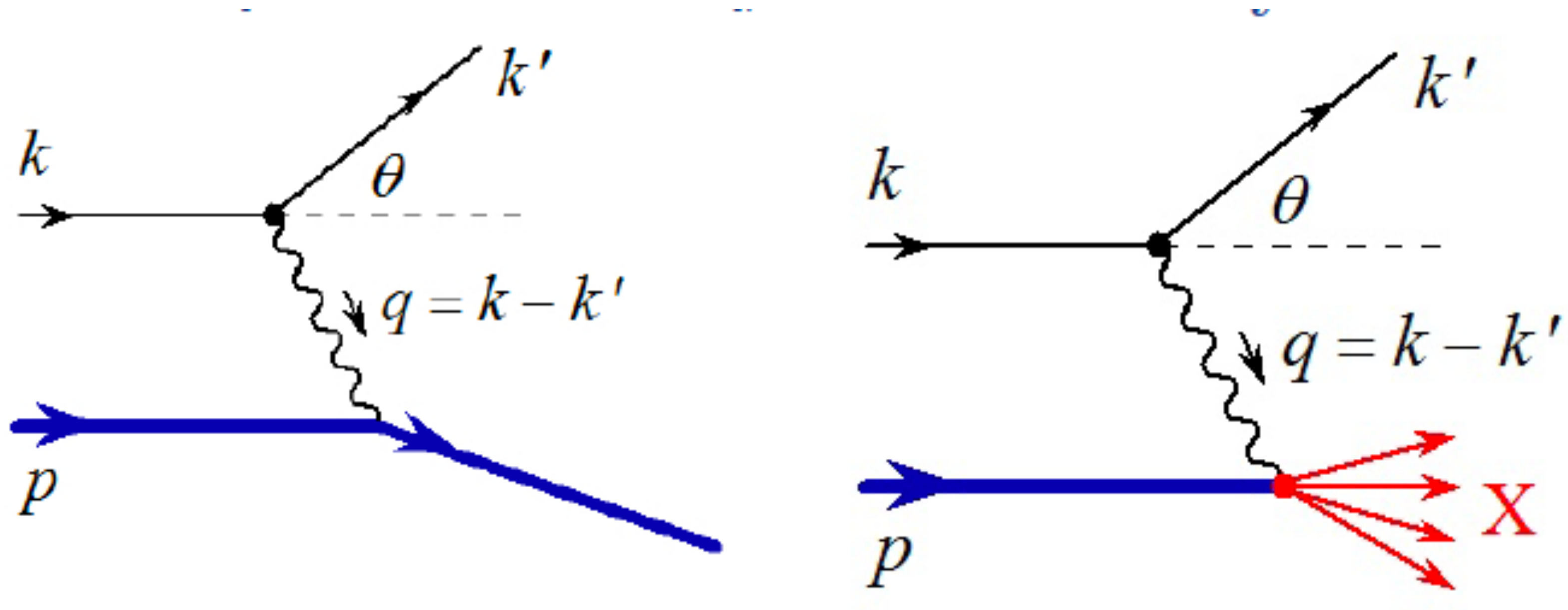
Deep Inelastic Scattering (DIS)





$$
\begin{aligned}
p &= M(1,0,0,0),\\
k &= \omega(1,0,0,1),\\
k' &= \omega'(1,\ \sin\theta\sin\varphi,\ \sin\theta\cos\varphi,\ \cos\theta)\\
q &= k - k' = p' - p.
\end{aligned}
$$

Deep Inelastic Scattering (DIS)



4-momentum transfer and energy transfer

$$q^2 = -2\omega\omega'(1-\cos\theta) = -4\omega\omega'\sin^2\frac{\theta}{2}, \qquad \nu = \omega - \omega'$$

on mass-shell condition for scattered proton (not present in the inelastic case):

$$Q^2 = -q^2 \qquad \delta((p+q)^2 - M^2) = \delta(2M\nu - Q^2) = \frac{1}{2M}\delta\left(\nu - \frac{Q^2}{2M}\right)$$

Elastic cross-section

on elementary fermion

$$\frac{d\sigma}{dQ^2} = \frac{\pi\alpha^2}{4\omega^2\sin^4\frac{\theta}{2}}\frac{e_p^2}{\omega\omega'}\left\{\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}\right\} \qquad \alpha = \frac{e^2}{4\pi}$$

Inelastic cross-section

on non-elementary fermion (proton)

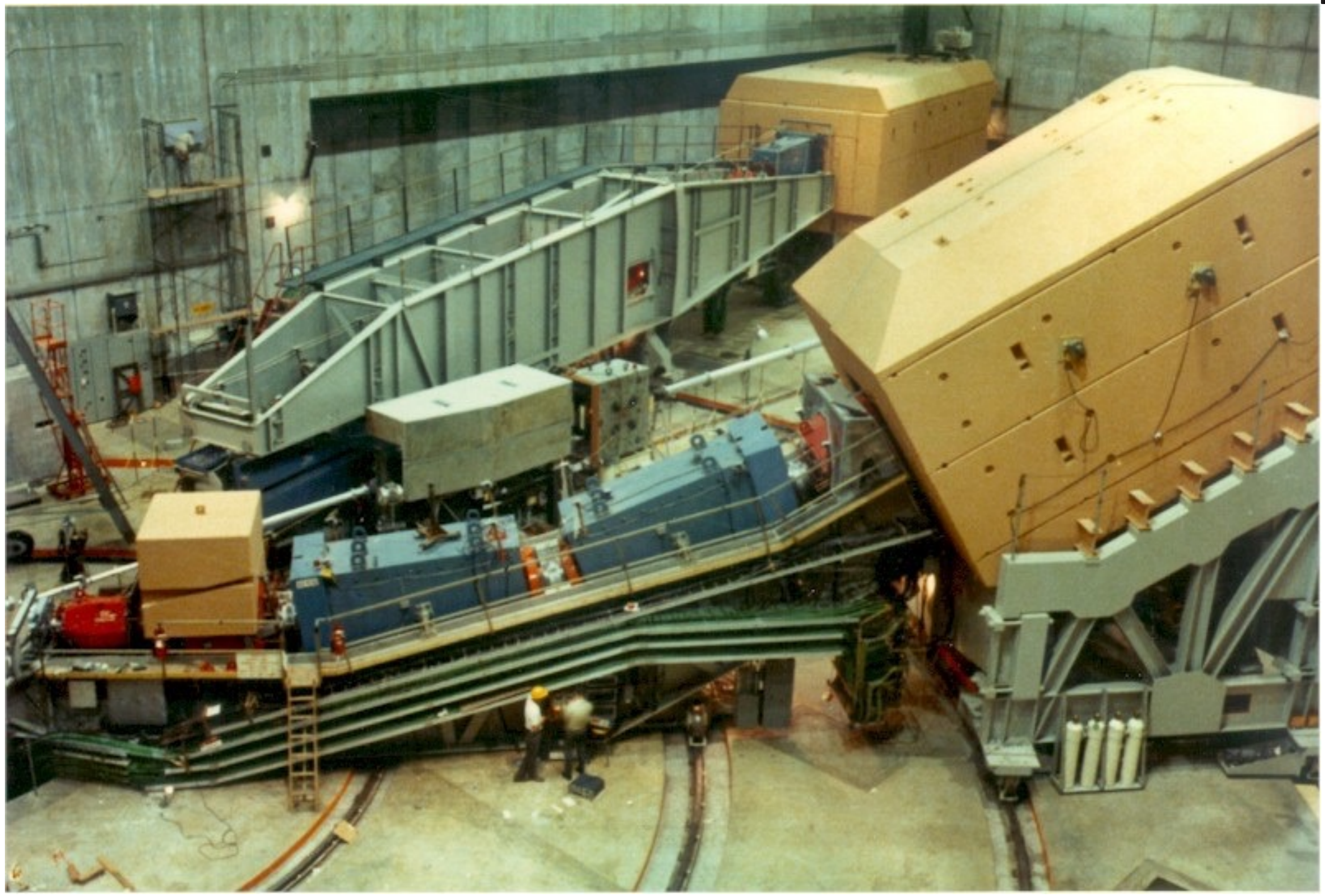
$$\frac{d\sigma}{dQ^2 d\nu} = \frac{\pi\alpha^2}{4\omega^3\omega'\sin^4\frac{\theta}{2}}\left\{W_2(Q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(Q^2,\nu)\sin^2\frac{\theta}{2}\right\}$$

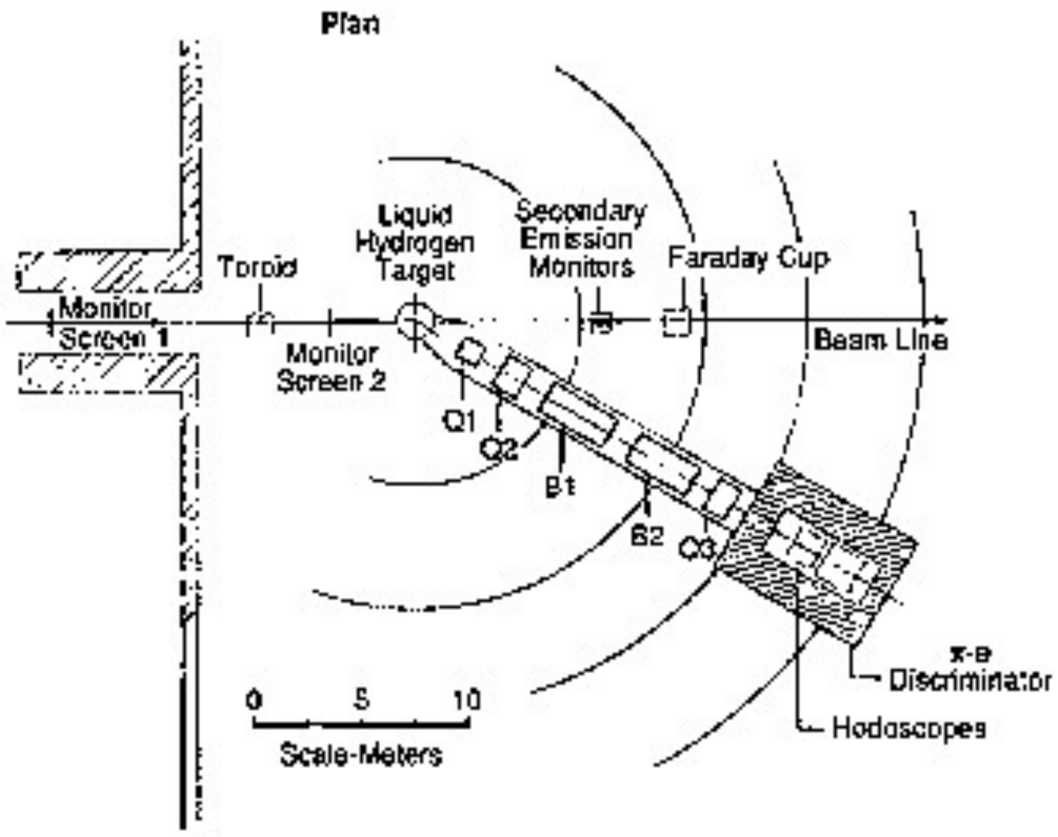
$$MW_1(Q^2,\nu) = F_1$$

$$\nu W_2(Q^2,\nu) = F_2$$

SLAC

SLAC built in1967
Length ~ 2 miles
Energy: 20 GeV





1968: convinced by James Bjorken analysis of DIS has been made

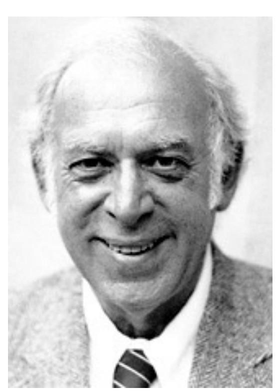
Interpretation was given by Richard Feynman

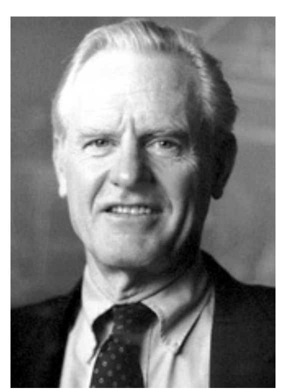
Nobel 1990:

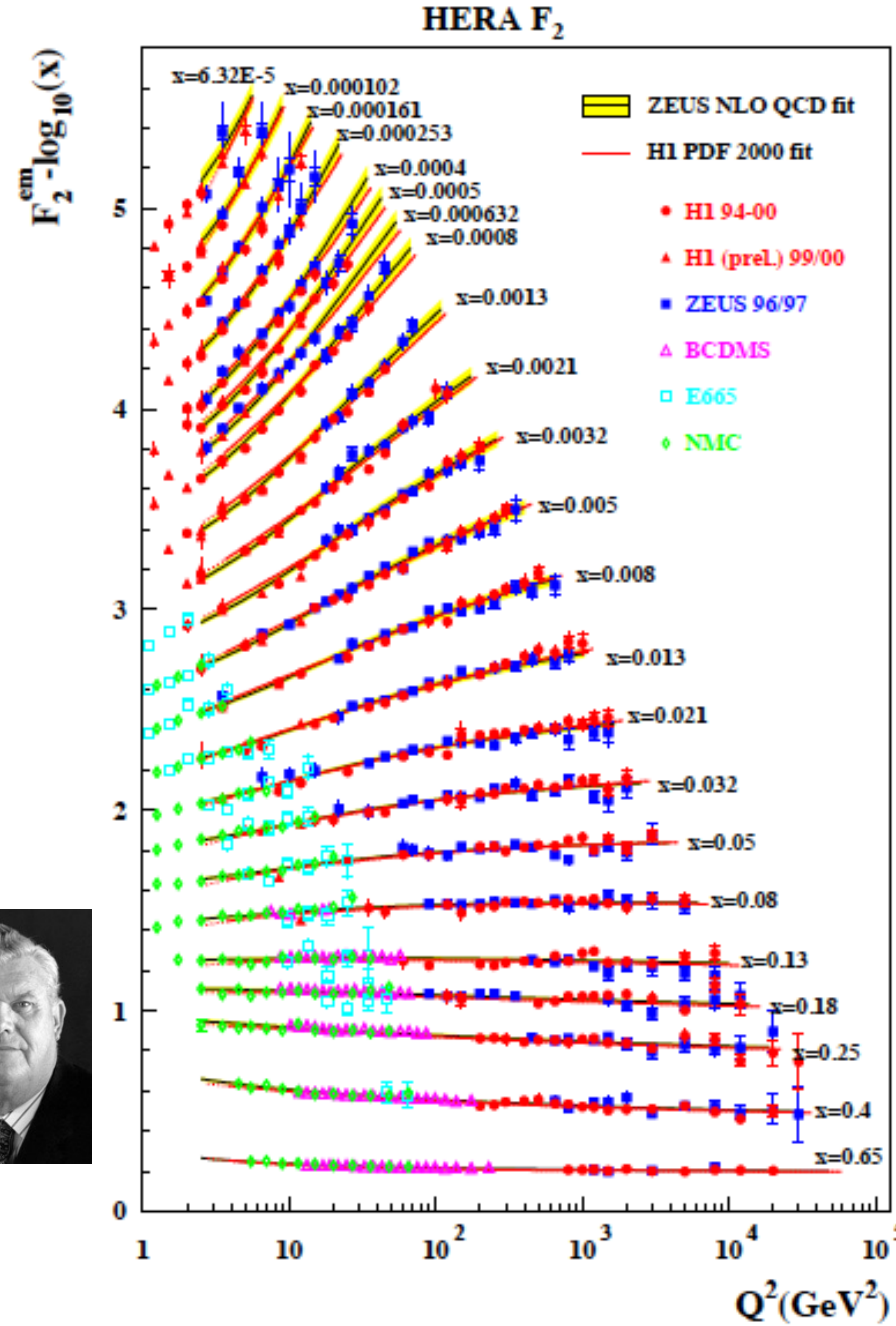
Jerome Friedman (MIT)

Henry Kendall (MIT)

Richard Taylor (SLAC)







Bjorken Scaling

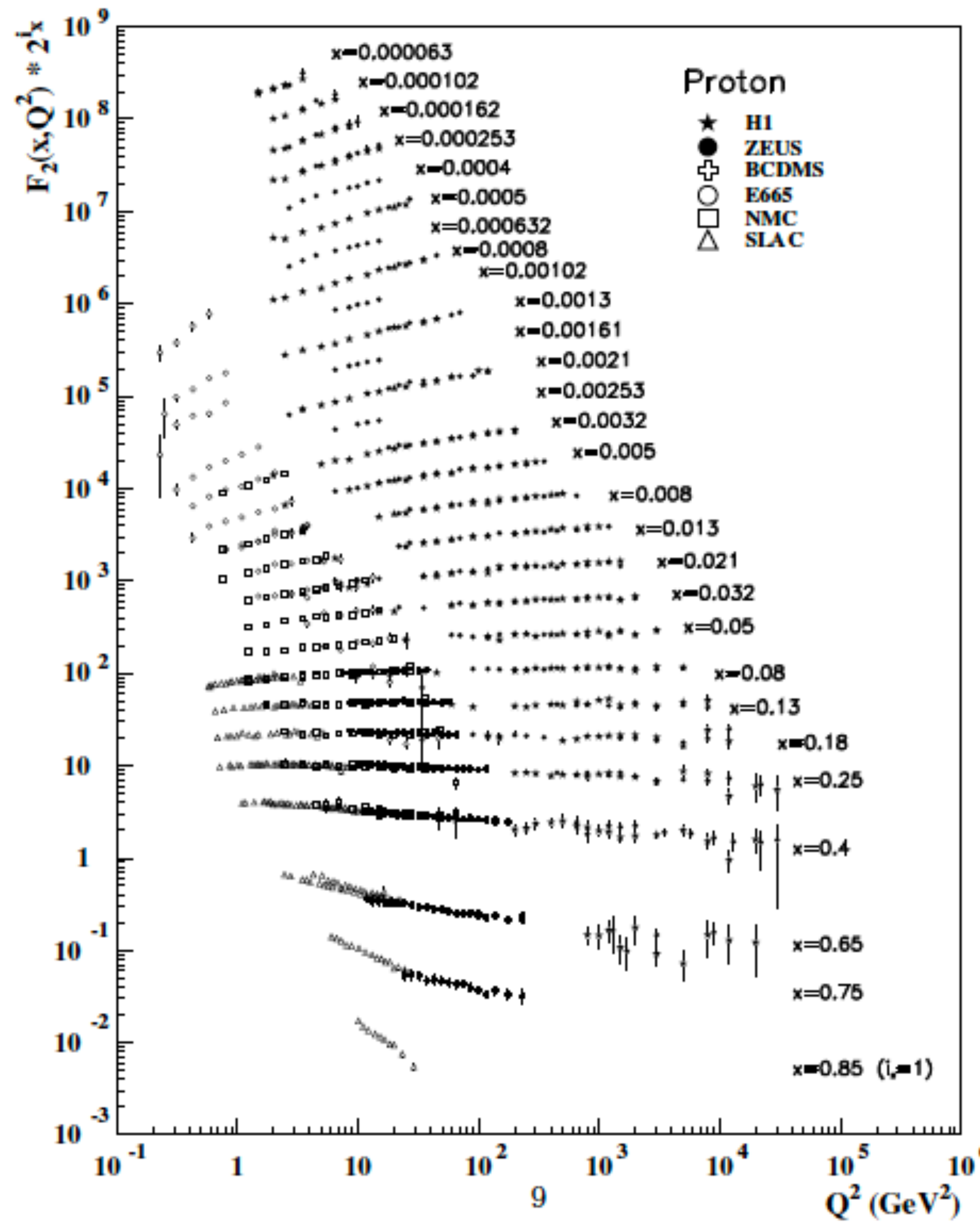
Bjorken limit:

$$Q^2, \nu \to \infty \qquad Q^2/\nu$$

$$MW_1(Q^2,\nu) = F_1(x)$$
$$\nu W_2(Q^2,\nu) = F_2(x)$$

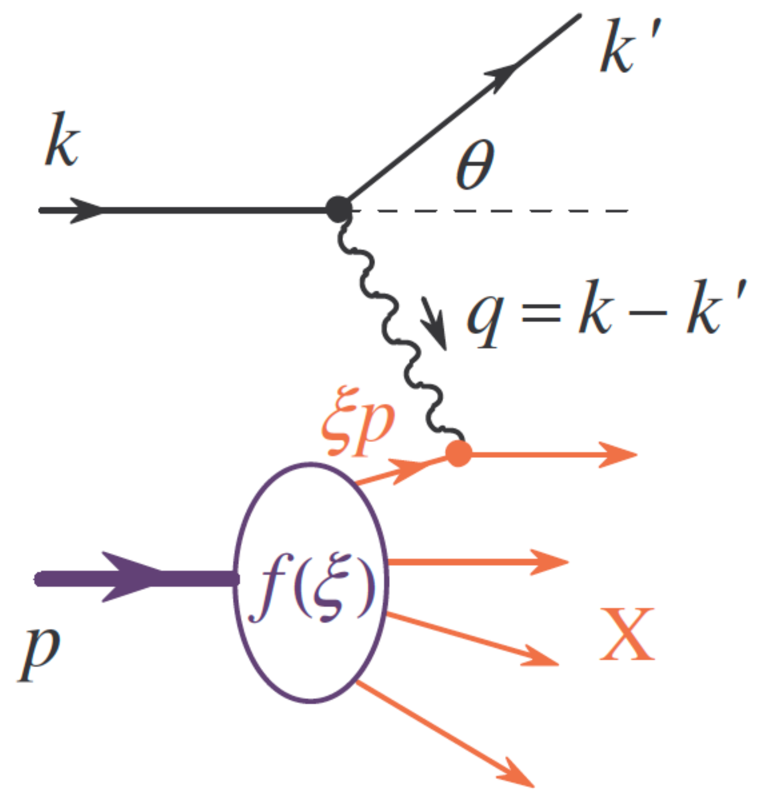
where:

$$x = \frac{Q^2}{2M\nu}$$



Feynman Parton Model

Inelastic scattering on proton
is a sum of elastic scattrings on partons
that are parallel to p
and carry momentum fraction ξ



In the proton rest frame we have to assume that parton mass is

$$m_\xi = \xi M$$

then the on-shell condition for the struck parton reads

$$(\xi p + q)^2 = m_\xi^2$$

$$\xi^2 M^2 + 2\xi M\nu - Q^2 = \xi^2 M^2 \;\rightarrow\; \xi = \frac{Q^2}{2M\nu} = x$$

ξ is the same as Bjorken x !