

# QCD

## problem set 12

1. In QCD infinitesimal change of the gauge field under the gauge transformation

$$\Omega(x) = \exp(i\theta_a(x)T^a)$$

can be calculated from

$$\mathbf{A}_\mu^\Omega = \Omega^\dagger(x)\mathbf{A}_\mu\Omega(x) + \frac{i}{g}\Omega^\dagger(x)\partial_\mu\Omega(x)$$

and reads (show it):

$$g\delta A_\mu^a = gf^{abc}\theta_b(x)A_\mu^c - \partial_\mu\theta_a(x).$$

Calculate the change of the gauge condition

$$G^a(A_\mu) = n^\mu A_\mu^a$$

$g\delta G^a$  under this transformation.

2. Next calculate matrix

$$\mathcal{M}_{ab} = g\frac{\delta G^a}{\delta\theta_b}$$

and then rewrite the free part of the ghost action

$$\int d^4x\mathcal{L}_{FG}^0 = \int d^4x\bar{\chi}_a\mathcal{M}_{ab}(g=0)\chi_b$$

in momentum space. The inverse of this term is the ghost propagator.

3. Applying gauge condition from problem 1 to the free part of the gluon action calculate the gluon propagator (inverse of gluonic operator derived from the free part of  $F_{\mu\nu}F^{\mu\nu}$ ) in the axial gauge.
4. For a system of SU(3) scalar fields  $\hat{\phi}_i(x)$  with  $i = 1, 2, 3$  that satisfy the following commutation rules

$$\left[\hat{\phi}_i(t, \vec{x}), \hat{\pi}_j(t, \vec{x}')\right] = i\delta^{(3)}(\vec{x} - \vec{x}')\delta_{ij}$$

one defines charge operators

$$\hat{Q}^a(t) = -i\int d^3\vec{x}\hat{\pi}_i(t, \vec{x})T_{ij}^a\hat{\phi}_j(t, \vec{x})$$

where matrices  $T^a$  satisfy SU(3) commutation relations:

$$[T^a, T^b] = if^{abc}T^c.$$

Prove that

$$\left[\hat{Q}^a(t), \hat{Q}^b(t)\right] = if^{abc}\hat{Q}^c(t).$$

5. Flavor SU(3) left and right currents read

$$L_\mu^a = \bar{q}_L \gamma_\mu \frac{\lambda_a}{2} q_L, \quad R_\mu^a = \bar{q}_R \gamma_\mu \frac{\lambda_a}{2} q_R$$

where

$$q_{L,R} = \frac{1}{2} (1 \mp \gamma_5) q.$$

Show that vector and axial currents take the following form

$$\begin{aligned} V_\mu^a &= R_\mu^a + L_\mu^a = \bar{q} \gamma_\mu \frac{\lambda_a}{2} q, \\ A_\mu^a &= R_\mu^a - L_\mu^a = \bar{q} \gamma_\mu \gamma_5 \frac{\lambda_a}{2} q. \end{aligned}$$