QCD
problem set 11

1. Consider zero energy motion in the inverted double well potential $V(x)$

$$E = \frac{1}{2} m \dot{x}^2 - V(x)$$

and express the classical action for a motion between the two maxima in time interval $\left\{-\frac{T}{2}, \frac{T}{2}\right\}$ in terms of the integral over the potential. Calculate action explicitly for the following potential

$$V(x) = \frac{1}{8a^2}(a^2 - x^2)^2.$$

Note that the unity "1"in potnential $V(x)$ has dimension of energy/distance^2.

2. For the potential from problem 1 calculate the classical trajectory $\bar{x}(\tau)$ starting at $-T/2$ in $-a$ and ending at $T/2$ in a. This can done by using the fact that the instanton is a zero energy motion. From this condition you can calculate velocity in terms of a potential and then inegrating both sides over time and position you get the final answer. Compute classical velocity $d\bar{x}/d\tau$.

3. The operator responsible for the quantal part of the Euclidean propagator for any $V(x)$ reads as follows

$$D(\tau) = -m \frac{d^2}{d\tau^2} + V''[\bar{x}(\tau)].$$

Show explicitly that $d\bar{x}/d\tau$ computed in the previous problem is the eigenfunction of this operator to the eigenvalue equal zero. Normalize this solution explicitly and show that the normalization factor agrees with the general formula:

$$y_{\lambda=0}(\tau) = \sqrt{\frac{m}{S_E^0}} \frac{d\bar{x}}{d\tau}.$$

4. Show that in electrodynamics one can write

$$-\frac{1}{4} \int d^4x \, F^{\mu\nu} F_{\mu\nu} = \frac{1}{2} \int d^4x \, A^\mu (g_{\mu\nu} \Box - \partial_\mu \partial_\nu) A^\nu.$$

Transform this expression to the momentum space.