QCD
problem set 9

1. $f(\boldsymbol{\psi})$ is a function of N independent Grassmann variables ψ_i. Prove the property used during the lecture that if $\psi_i = J_{ij}\theta_j$ then
$$\int d^N\boldsymbol{\psi} f(\boldsymbol{\psi}) = \det{}^{-1}(J) \int d^N\boldsymbol{\theta} f(\boldsymbol{\theta}). \tag{1}$$

2. Consider Gaussian integral
$$J(\mathcal{M}) = \int d^N\xi \, d^N\psi \, \exp\left(\psi_i \mathcal{M}_{ij} \xi_j\right)$$
where ψ_i and ξ_i $(i = 1, 2, \ldots N)$ are independent Grassmann variables. Expanding in a power series and commuting ξ's and ψ's show that
$$J(\mathcal{M}) = \det(\mathcal{M}).$$

3. Anomaly is proportional to the integral
$$\int d^4\boldsymbol{k} \, \mathrm{Tr}\left\{\gamma^5 t \mathcal{F}\left(-\left[i\not{k} + \frac{\not{D}_x}{M}\right]^2\right)\right\}.$$
Expand $\mathcal{F}$ for large M and show that the only term contributing to the above integral is the term proportional to $\mathcal{F}''(k^2)$.

4. Prove that
$$\partial_\mu K^\mu = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}$$
where
$$K^\mu = \varepsilon^{\mu\nu\rho\sigma}\left(A^a_\nu F^a_{\rho\sigma} - \frac{g}{3} f^{abc} A^a_\nu A^b_\rho A^c_\sigma\right).$$
This calculation proves that anomaly is a total derivative.

5. General fermionic mass term reads (where M is complex):
$$M \, \bar{\psi}\frac{1+\gamma_5}{2}\psi + M^* \, \bar{\psi}\frac{1-\gamma_5}{2}\psi. \tag{2}$$
Prove that (2) is Hermitean. Show that chiral transformation
$$\psi \to e^{i\alpha\gamma_5}\psi$$
ammounts to
$$M \to e^{2i\alpha} M.$$