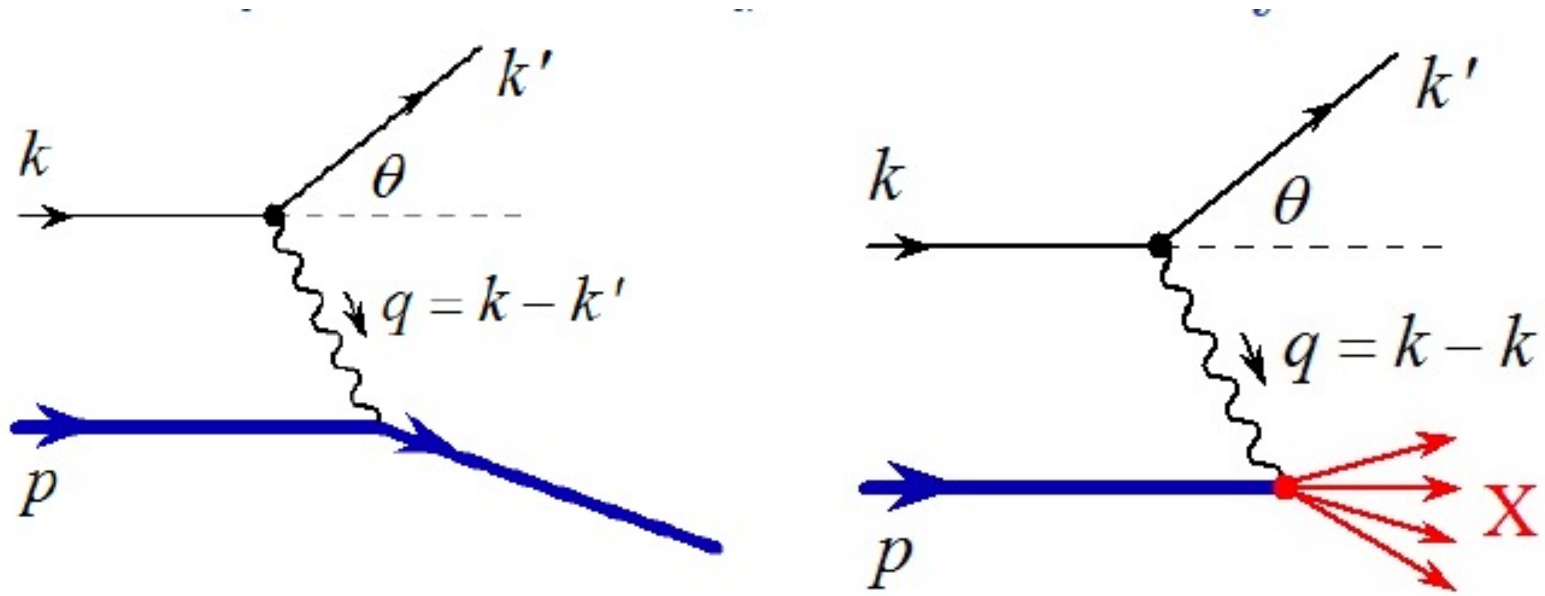
QCD Lecture 2

October 12

Deep Inelastic Scattering (DIS)



$$p = M(1, 0, 0, 0),$$
$$k = \omega(1, 0, 0, 1),$$
$$k' = \omega'(1,\ \sin\theta \sin\varphi,\ \sin\theta\cos\varphi,\ \cos\theta)$$
$$q = k - k' = p' - p.$$

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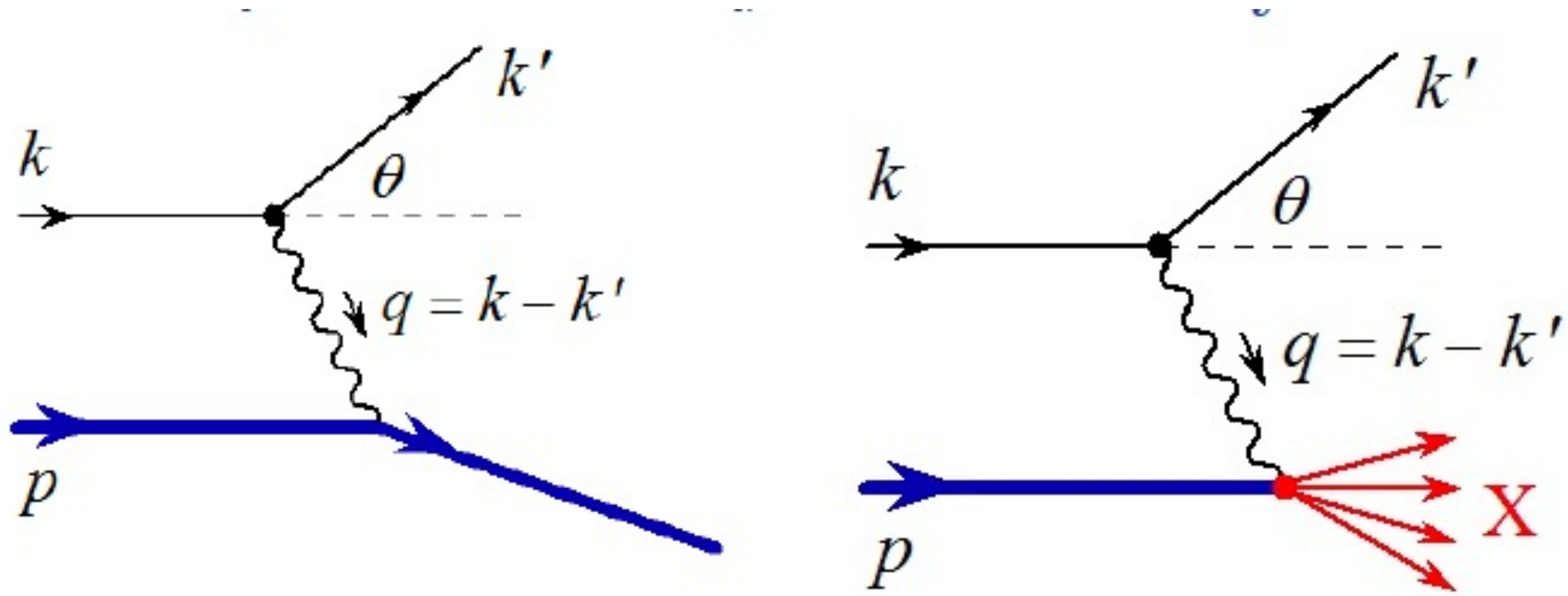

4-momentum transfer and energy transfer

$$q^2 = -2\omega\omega'(1-\cos\theta) = -4\omega\omega'\sin^2\frac{\theta}{2}, \qquad \nu = \omega - \omega'$$

on mass-shell condition for scattered proton (not present in the inelastic case):

$$\delta((p+q)^2 - M^2) = \delta(2M\nu - Q^2) = \frac{1}{2M}\delta\left(\nu - \frac{Q^2}{2M}\right)$$

Elastic cross-section

$$d\sigma = \frac{1}{4M\omega} \int \frac{1}{4} \sum_{\text{pol}} |\mathcal{M}_{fi}|^2 \, \frac{d^4k'}{(2\pi)^3} \delta(k'^2) \, \frac{d^4p'}{(2\pi)^3} \delta(p'^2 - M^2) \, (2\pi)^4 \delta(k + p - k' - p')$$

flux — matrix element — final state integration with on-shell cond. — mom.-energy conservation

perform dp' integration first, then

$$d\sigma = \frac{1}{4M\omega} \frac{1}{(2\pi)^2} \int \frac{1}{4} \sum_{\text{pol}} |\mathcal{M}_{fi}|^2 \, \underbrace{d\omega' d^3\boldsymbol{k}' \delta(\omega'^2 - k'^2)}_{=I} \, \delta((p+q)^2 - M^2)$$

compute I

Elastic cross-section

$$I = \int d\omega' d^3\boldsymbol{k}' \delta(\omega'^2 - \boldsymbol{k}'^2) = \int \boldsymbol{k}'^2 d|\boldsymbol{k}|' d\varphi \, d\cos\theta \, d\omega' \delta(\omega'^2 - \boldsymbol{k}'^2)$$

$$= 2\pi \int d\cos\theta \frac{\omega'^2 d\omega'}{2\omega'} = \pi \int \omega' d\omega' d\cos\theta.$$

We have assumed that the matrix element does not depend on φ

Change of variables:

$$Q^2 = -q^2 = 2\omega\omega'(1 - \cos\theta)$$
$$\nu = \omega - \omega',$$

Jacobian:

$$d\omega' d\cos\theta = \left| \frac{d(\omega', \cos\theta)}{d(\nu, Q^2)} \right| dQ^2 d\nu = \frac{1}{2\omega\omega'} dQ^2 d\nu$$

$$I = \frac{\pi}{2\omega} \int dQ^2 d\nu$$

Elastic cross-section

$$\delta((p+q)^2 - M^2) = \delta(2M\nu - Q^2) = \frac{1}{2M}\delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$\frac{d\sigma}{dQ^2} = \frac{1}{16M^2\omega^2}\frac{1}{4\pi}\int d\nu\,\frac{1}{4}\sum_{\text{pol}}|\mathcal{M}_{fi}|^2\,\delta\left(\nu - \frac{Q^2}{2M}\right)$$

Next: calculate amplitude squared averaged over initial polarizations and summed over final polarizations