QCD
problem set 14

1. Flavor SU(3) left and right currents read

$$L_\mu^a = \bar{q}_L \gamma_\mu \frac{\lambda_a}{2} q_L, \;\; R_\mu^a = \bar{q}_R \gamma_\mu \frac{\lambda_a}{2} q_R$$

   where

$$q_{\substack{L\\R}} = \frac{1}{2}\left(1 \mp \gamma_5\right) q.$$

   Show that vector and axial currents take the following form

$$\begin{aligned} V_\mu^a &= R_\mu^a + L_\mu^a = \bar{q}\gamma_\mu \frac{\lambda_a}{2} q, \\ A_\mu^a &= R_\mu^a - L_\mu^a = \bar{q}\gamma_\mu \gamma_5 \frac{\lambda_a}{2} q. \end{aligned}$$

2. Effective Lagrangian describing Goldstone boson interactions reads

$$\mathcal{L}_{\rm eff} = \frac{F^2}{4}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{F_\pi^2 m_\pi^2}{4}\,\mathrm{Tr}(U + U^\dagger)$$

   where $U = \exp(i\phi/F)$ can be expressed in terms of the physical meson fields. For SU(2) we have:

$$\phi(x) = \sum_a \lambda_a \phi^a(x) = \left[\begin{array}{cc} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{array}\right]$$

   Expand $\mathcal{L}_{\rm eff}$ up to 4-field interactions and calculate flavor trace.

3. There exists another possible parametrization of U

$$U = \frac{1}{F_\pi}\left[\sigma(x) + i\,\vec{\tau}\cdot\vec{\pi}(x)\right] \text{ where } \sigma(x) = \sqrt{F_\pi^2 - \vec{\pi}^{\,2}(x)}.$$

   Calculate the effective lagrangian up to 4 fiels in this case.

4. In the SU(3) case the mass term in the effecive lagrangian is equal to

$$-\mathrm{const.}\,\mathrm{Tr}(UM^\dagger + MU^\dagger)$$

   where the quark mass matrix reads.

$$M = \mathrm{diag}[m_u, m_d, m_s].$$

Decomposition of the mass matrix in terms of λ_0, λ_3 and λ_8 can be found in lecture 14b. Matrix $U = \exp(i\phi/F)$ can be expressed in terms of the physical meson fields:

$$\phi(x) = \sum_a \lambda_a \phi^a(x) = \begin{pmatrix} \pi^0 + \eta/\sqrt{3} & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \eta/\sqrt{3} & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -2\eta/\sqrt{3} \end{pmatrix}.$$

Calculate the mass term (up to the quadratic terms in fields) and interpret all terms. Next, assume $m_u = m_d = m$ and calculate meson masses. There will be three masses for pions, kaons and eta expressed in terms of two parameters const.$\times m$ and const.$\times m_s$. Therefore there will be one, parameter independent, relation between these masses. Find this relation and check whether it is fullfiled experimentally.