QCD
problem set 11

1. Calculate the field tensor $F_{\mu\nu}$ for the pure gauge $A_\mu = -\frac{i}{g}\Omega\partial_\mu\Omega^\dagger$ where Ω is an SU(N) transformation matrix.

2. Prove that
$$\int\limits_{-T/2}^{T/2} d\tau_1 \int\limits_{\tau_1}^{T/2} d\tau_2 \ldots \int\limits_{\tau_{n-1}}^{T/2} d\tau_n = \frac{1}{n!}T^n.$$

3. Consider zero energy motion in the inverted double well potential $V(x)$
$$E = \frac{1}{2}m\dot{x}^2 - V(x)$$
and express the classical action for a motion between the two maxima in time interval $\left\{-\frac{T}{2}, \frac{T}{2}\right\}$ in terms of the integral over the potential. Calculate action explicitly for the following potential
$$V(x) = \frac{1}{8a^2}(a^2 - x^2)^2.$$
Note that the unity "1"in potnential $V(x)$ has dimension of energy/distance^2.

4. For the potential from problem 3 calculate the classical trajectory $\bar{x}(\tau)$ starting at $-T/2$ in $-a$ and ending at $T/2$ in a. This can done by using the fact that the instanton is a zero energy motion. From this condition you can calculate velocity in terms of a potential and then inegrating both sides over time and position you get the final answer. Compute classical velocity $d\bar{x}/d\tau$.

5. Show that the operator responsible for the quantal part of the Euclidean propagator for any $V(x)$ reads as follows
$$D(\tau) = -m\frac{d^2}{d\tau^2} + V''[\bar{x}(\tau)].$$
Show that $d\bar{x}/d\tau$ is the eigenfunction of this operator to the eigenvalue equal zero. Show that that this zero mode is normalized if
$$y_{\lambda=0}(\tau) = \sqrt{\frac{m}{S_E^0}}\frac{d\bar{x}}{d\tau}.$$