QCD
problem set 6

1. Calculate the following traces:
$$\mathrm{Tr}\left[\not{P}\gamma_\lambda\gamma_5\not{P}\gamma_\nu\not{P}\gamma_\mu\right],\ \mathrm{Tr}\left[\not{P}\gamma_5\gamma_\nu\left(\not{P}-\not{k}\right)\gamma_\mu\right].$$

2. At the lecture we have computed
$$\Delta_{\mu\nu\lambda}(a)=T_{\mu\nu\lambda}(p\to p+a)-T_{\mu\nu\lambda}$$
(shorthand notation: $T_{\mu\nu\lambda}(p\to p+a)=T_{\mu\nu\lambda}(a)$) with the result
$$\Delta_{\mu\nu\lambda}(a)=\frac{1}{8\pi^2}\varepsilon_{\alpha\mu\nu\lambda}a^\alpha+(\mu\longleftrightarrow\nu,k_1\leftrightarrow k_2).$$
For
$$a=\alpha k_1+(\alpha-\beta)k_2$$
calculate
$$k_1^\mu\Delta_{\mu\nu\lambda}(a)\text{ and }k_2^\nu\Delta_{\mu\nu\lambda}(a).$$

3. In order to check vector current conservation for arbitrary momentum routing we apply the trick analogous to the one used for the axial current:
$$\begin{aligned}k_1^\mu T_{\mu\nu\lambda}(a) &= k_1^\mu\left(T_{\mu\nu\lambda}(a)-T_{\mu\nu\lambda}(0)\right)+k_1^\mu T_{\mu\nu\lambda}(0)\\ &= k_1^\mu\Delta_{\mu\nu\lambda}(a)+k_1^\mu T_{\mu\nu\lambda}(0)\end{aligned}$$
where
$$\begin{aligned}T_{\mu\nu\lambda}(0) &= -i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\frac{i}{\not{p}-m}\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{q})-m}\gamma_\nu\frac{i}{(\not{p}-\not{k}_1)-m}\gamma_\mu\right]\\ &\quad -i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\frac{i}{\not{p}-m}\gamma_\lambda\gamma_5\frac{i}{(\not{p}-\not{q})-m}\gamma_\mu\frac{i}{(\not{p}-\not{k}_2)-m}\gamma_\nu\right]\end{aligned}$$
Use the following tricks
$$\not{k}_1=\left(\not{p}-m\right)-\left((\not{p}-\not{k}_1)-m\right)$$
and
$$\not{k}_1=\left(\not{p}-\not{k}_2-m\right)-\left((\not{p}-\not{k}_1-\not{k}_2)-m\right)=\left(\not{p}-\not{k}_2-m\right)-\left((\not{p}-\not{q})-m\right)$$
to compute $k_1^\mu T_{\mu\nu\lambda}(0)$. The resulting expression a is difference of two terms where the first term is obtained from the second one by a shift $p\to p-k_1$. For such difference we may use
$$\Delta(a)=2i\pi^2a^\mu\lim_{R\to\infty}P^2P_\mu\,f(P)$$
where $a=-k_1$ and $f(P)$ is the second trace computed in for large P.

Present the final result for $k_1^\mu T_{\mu\nu\lambda}(a)$.

4. Hopf integral.

   Calculate Hopf integral

$$I = \int_{-\infty}^{+\infty} dx\, e^{iax^2}, \;\; a > 0$$

   as a contour integral over the complex plane. Choose the contour in such a way that the integral $\int dt\, e^{-bt^2}$ with positive b and real t appears. When the contribution of the large circle can be neglected? Is the phase of the result unique?