QCD
problem set 5

1. For the Altarelli-Parisi probabilities defined in lecture notes calculate Mellin moments:
$$\int_0^1 dz\, z^{n-1} P_{ab}(z) = \gamma_{ab}^{(n)}.$$
The calculation of $\gamma_{qq}^{(n)}$and $\gamma_{GG}^{(n)}$ requrie certain trick. We have
$$\gamma_{qq}^{(n)} = C_F \int_0^1 dz\, z^{n-1} \left(\frac{1+z^2}{1-z}\right)_+ = C_F \int_0^1 dz\, (z^{n-1}-1) \left(\frac{1+z^2}{1-z}\right). \tag{1}$$
To this end use (and prove) the following identity
$$\frac{z^{n-1}}{1-z} = -\sum_{k=0}^{n-2} z^k + \frac{1}{1-z}. \tag{2}$$

2. You should get that $\gamma_{qq}^{(1)} = 0$. Interpret this result.

3. Write explicitly DGLAP equation for the singlet and gluon contributions for $n = 2$. You should get that $q_{n=2}^S(t) + G_{n=2}(t) = const.$ Interpret this result and present arguments that $const. = 1$.

4. Find DGLAP equation for
$$f(t) = \frac{4C_F}{3} q_{n=2}^S(t) - \frac{n_f}{3} G_{n=2}(t)$$
and solve it. You should find that $f(t) \underset{t\to\infty}{\longrightarrow} 0$.

5. At $t \to \infty$ we can therefore, using the result of the previous problem, express $q_{n=2}^S(\infty)$ in terms $G_{n=2}(\infty)$,and then – plugging this into the constraint of problem 3 – compute $G_{n=2}(\infty)$. Find numerical value of $G_{n=2}(\infty)$ for $n_f = 3, 4, 5, 6$. Find out whether for finite t the value of $G_{n=2}(t)$ is larger or smaller than $G_{n=2}(\infty)$.