QCD
problem set 4

1. Calculate using *Mathematica* package FeynCalc the $L_{\mu\nu}$ tensor

$$L^{\mu\nu}(p,p') = \frac{1}{2}\,\mathrm{Tr}\left[\gamma^{\mu}\,(p' + M)\,\gamma^{\nu}(p + M)\right]$$

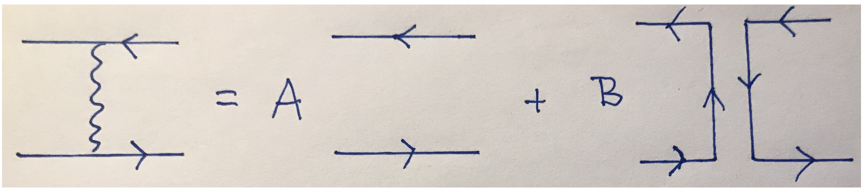
   where $p^2 = p'^2 = M^2$. Check that $L^{\mu\nu}$ can be decomposed into a sum of two gauge invariant tensors given at the lecture.

2. Using kinematics defined at the lecture calculate

$$p_\mu p_\nu L^{\mu\nu}(k,k'),\ g_{\mu\nu} L^{\mu\nu}(k,k').$$

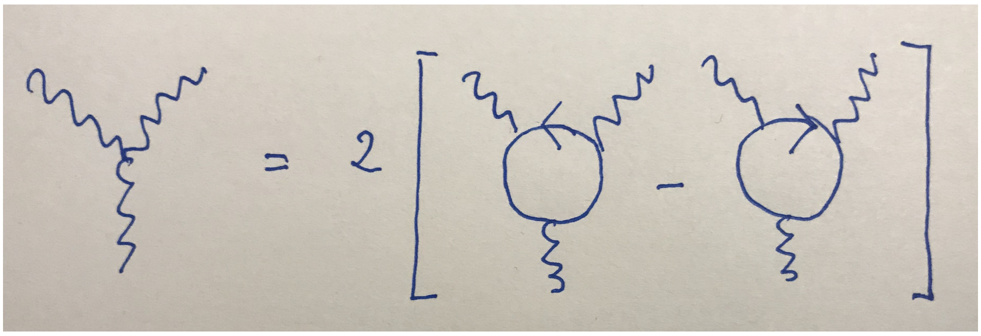
3. Color factors.

   - With the help of the graphical methods for the SU(N) group described at the lecture, find coefficients A and B for the following decomposition of the one gluon exchange:



     and interpret the result for $N \to \infty$.
     HINT: contract fermion lines in two possible ways (remember that contractions must preserve the direction of the arrow).

   - Prove the following identity:



     HINT: start from the commutation relation for the generators in the fundamental representation and contract it with an extra generator keeping the arrows direction. Then use the normalization condition for the generators. In these color diagrams what really matters is topology, so we can turn them around or reflect without changing the result (except for possible change of sign if we flip legs of an antisymmetric tensor).

- Using relations from previous problems calculate C_A the Casimir operator for the adjoint representation: