QCD

lecture 14

1 Heavy quark lagrangian

We decompose the heavy quark field that enters the lagrangian for a heavy quark

$$\mathcal{L}_Q = \bar{Q}(x) \left(i\not{D} - M_Q \right) Q(x) \tag{1}$$

(where $D_\mu = \partial_\mu - ig\boldsymbol{A}_\mu$ is covariant QCD derivative) in the following way

$$Q(x) = e^{-iM_Q v\cdot x} \left[Q_v(x) + B_v(x) \right]$$

where

$$\begin{aligned} \frac{1+\not{v}}{2} Q_v(x) &= Q_v(x),\ \frac{1-\not{v}}{2} B_v(x) = B_v(x), \\ \frac{1-\not{v}}{2} Q_v(x) &= 0,\ \frac{1+\not{v}}{2} B_v(x) = 0. \end{aligned} \tag{2}$$

The second line of course follows from the first line due to the properties of the projection operators. Alternatively the same equations can written as

$$\not{v} Q_v(x) = Q_v(x),\ \not{v} B_v(x) = -B_v(x). \tag{3}$$

It is convenient to decompose the covariant derivative into two parts

$$i\not{D} = \not{v}\, iv \cdot D + i\not{D}_T. \tag{4}$$

Equation (4) can be in fact understood as a definition of D_T^μ

$$D_T^\mu = D^\mu - v^\mu v \cdot D, \tag{5}$$

which is indeed transverse to v: $v_\mu D_T^\mu = 0$.

We can write the heavy quark lagrangian

$$\begin{aligned} \mathcal{L}_Q &= \left[\bar{Q}_v(x) + \bar{B}_v(x) \right] e^{+iM_Q v\cdot x} e^{-iM_Q v\cdot x} \left\{ \not{v}\, i\, v \cdot D + M_Q \not{v} + i\not{D}_T - M_Q \right\} \left[Q_v(x) + B_v(x) \right] \\ &= \left[\bar{Q}_v(x) + \bar{B}_v(x) \right] \left\{ \not{v}\, i\, v \cdot D + M_Q \left(\not{v} - 1 \right) + i\not{D}_T \right\} \left[Q_v(x) + B_v(x) \right]. \end{aligned} \tag{6}$$

Now we shall apply to (6) relations (3)

$$\begin{aligned}\mathcal{L}_Q &= \bar{Q}_v(x)\, i\, v\cdot D\, Q_v(x) - \bar{B}_v(x)\, \left(i\, v\cdot D + 2M_Q\right)\, B_v(x) \\ &+\bar{Q}_v(x)\, i\not{D}_T B_v(x) + \bar{B}_v(x)\, i\not{D}_T Q_v(x). \end{aligned} \tag{7}$$

Note that there is no mass term for Q_v. Note also that there are no diagonal terms of $i\not{D}_T$

Let's summarize what has been achieved so far:

- We have performed field redefinition at tree level. We correctly describe couplings to $k^\mu \sim \Lambda_{\text{QCD}}$ gluons.
- Antiparticles are integrated out. They have $2M_Q$ mass gap.
- Possible mixing of B_v with *external* heavy quark Q_v is suppressed by $1/m_Q$.
- Number of heavy quarks is preserved (no $\mathrm{Q\bar{Q}}$ production).

2 Heavy quark symmetry (HQS)

First of all we have $U(N_h)$ symmetry, where N_h is number of heavy quarks, since $\mathcal{L}_Q$ is independent of M_Q.There is also spin symmetry. Consider spin operator

$$S^i_Q = \frac{1}{2}\begin{bmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{bmatrix} = \frac{1}{2}\gamma_5\gamma^0\gamma^i. \tag{8}$$

Consider an infinitesimal spin transformation

$$Q'_v = \left(1 + i\,\boldsymbol{\varepsilon}\cdot\hat{\boldsymbol{S}}_Q\right) Q_v. \tag{9}$$

This transformation changes lagrangian by

$$\delta\mathcal{L}_Q = \bar{Q}_v\left[i\, v\cdot D, i\,\boldsymbol{\varepsilon}\cdot\hat{\boldsymbol{S}}_Q\right] Q_v = 0. \tag{10}$$

The commutator is zero, because $v\cdot D$ does not contain γ matrices and $\boldsymbol{\varepsilon}\cdot\hat{\boldsymbol{S}}_Q$ is space-time independent. It follows that

$$\not{v}Q'_v = Q'_v, \tag{11}$$

which means that the spin symmetry acts within the two component subspace spanned by $\left(1+\not{v}\right)/2$. This has immidiate consequences for heavy quark interactions (via gluon exchange) with the "light stuff", which does not depend on the heavy quark spin. This not true in full QCD where $\hat{\boldsymbol{S}}_Q$ does not commute with $\not{D}$.

In heavy quark interactions at low energies v^μ is conserved, only k^μ changes, so that's why we can label fields by subscript v.

Lagrangian (6) allows for clear power counting in powers of $1/M_Q$. However there is still M_Q hidden in field normalization. Recall fermion filed quantization

$$Q(x) = \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2E_p}} \sum_s \left(e^{-ip\cdot x} u_Q(p,s)\hat{b}(\vec{p},s) + e^{+ip\cdot x} v_Q(p,s) d^\dagger(\vec{p},s) \right). \tag{12}$$

Remember that

$$p^\mu = M_Q v^\mu + k^\mu. \tag{13}$$

In heavy quark efective theory we are intersted only in $Q_v(x)$, which has been defined by factoring out explicitly $e^{-iM_Q v\cdot x}$. Since v is conserved

$$Q_v(x) \sim \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2E_p}} \sum_s e^{-ik\cdot x} u_Q(k,s)\hat{b}(\vec{k},s) + \ldots \tag{14}$$

and

$$i\partial_\mu Q_v(x) \sim k_\mu Q_v(x) \tag{15}$$

which means that coordinate x corresponds to the variations of quark momenta over scales $\sim \Lambda_{\rm QCD} \ll M_Q$. So $1/M_Q$ is explicit, except for state (*e.g.* heavy meson) normalization (see lecture 11)

$$\langle H(p')| \underbrace{H(p)\rangle}_{\text{dim } -1} = 2E_p(2\pi)^3\delta^{(3)}\left(\boldsymbol{p}' - \boldsymbol{p}\right) \tag{16}$$

with $E_p = \sqrt{M_H^2 - \boldsymbol{p}^2}$ where M_Q is hidden in M_H. It is therefore usefull to change state normalization

$$|H(p)\rangle = \sqrt{M_H}\left[|H(v,\boldsymbol{k})\rangle + \mathcal{O}\left(\frac{1}{M_Q}\right)\right] \tag{17}$$

with

$$\langle H(v',\boldsymbol{k}')| \underbrace{H(v,\boldsymbol{k})\rangle}_{\text{dim } -3/2} = 2v_0(2\pi)^3\delta_{v,v'}\delta^{(3)}\left(\boldsymbol{k}' - \boldsymbol{k}\right). \tag{18}$$

The same change is also introduced for spinors

$$u_Q(p,s) = \sqrt{M_Q}\, u_Q(v,\boldsymbol{k},s).$$

Note that $\sqrt{M_Q}$ cancels M_Q dependence of $1/\sqrt{2E_p}$ in Eq.(12) (in the leading order in $1/M_Q$).

3 Spectroscopy

An immediate consequence of heavy quark spin symmetry is degenreacy of spin 0 and spin 1 $\bar{Q}q$ (or $Q\bar{q}$) mesons. Since light quarks form SU(3)-flavor triplet we expect two triplets of almost degenerate (pseudo)scalar and vector mesons. The experemintal situation is summarized in Table below (antiparticles are not listed):

$\bar{Q}q$	name	charge	$s=0$ [Mev]	$s=1$ [Mev]	Δ_Q[MeV]
$\bar{c}d$	D^-	$-$	1879.65	2010.26	142.02
$\bar{c}u$	D^0	0	1864.83	2006.85	130.61
$\bar{c}s$	D_s^-	$-$	1968.34	2112.20	143.86
$\bar{b}d$	B^0	0	5279.65	5324.70	45.05
$\bar{b}u$	B^+	$+$	5279.34		45.36
$\bar{b}s$	B_s^0	0	5366.88	5415.40	48.52

We see that indeed vector-scalar splitting Δ is (almost) independent of the light quark content and is much smaller than meson masses. It is smaler for B mesons than for D mesons as expected, since degeneracy violation is of the order if $1/M_Q$. Particle Data Group estimates heavy quark masses to be

$$M_c \simeq 1.27 \text{ GeV}, \qquad M_b \simeq 4.18 \text{ GeV},$$

which gives

$$\frac{M_b}{M_c} \simeq 3.29$$

On the other hand on average

$$\frac{\Delta_c}{\Delta_b} \simeq 3.$$

Another phenomenological test of HQS for mesons consists in writing a mass formula for spin partners:

$$M_{\text{meson}} = \alpha M_Q + \beta + \begin{cases} \mu \frac{1}{M_Q} & \text{for} \quad s = 0 \\ \\ \mu^* \frac{1}{M_Q} & \text{for} \quad s = 1 \end{cases} \tag{19}$$

where * stands for vector mesons and constants α, β and $\mu^{(*)}$ are universal. Then

$$\delta = (M^*)^2 - M^2 = (M^* + M)(M^* - M) \underset{M_Q \to \infty}{\to} 2\alpha M_Q \frac{\mu^* - \mu}{M_Q} = \text{const.} \tag{20}$$

For nonstrange mesons we have:

$$\delta_D = 0.51 \div 0.55, \; \delta_B = 0.48$$

and for strange

$$\delta_{D_s} = 0.59, \; \delta_{B_s} = 0.52.$$

Situation is similar for baryons with one heavy quark. The other two light quarks (so called diquarks) may form antisymmetric and symmetric combinations corresponding to SU(3) flavor triplet and sextet. This can be seen in the following way. The product of two quark fields can be always decomposed into antisymmetric and symmetric parts that form SU(3) invariants:

$$q_1(x)q_2(y) = \frac{1}{2}\left(q_1(x)q_2(y) - q_2(x)q_1(y)\right) + \frac{1}{2}\left(q_1(x)q_2(y) + q_2(x)q_1(y)\right).$$

Antisymmetric part vanishes when $q_1 = q_2$, so it has only three components ud, us and ds, and symmetric part may have additionally uu, dd and ss components (six in total). Antisymmetric combination has spin 0, whereas symmetric one spin 1. This is illustrated in Fig. 1

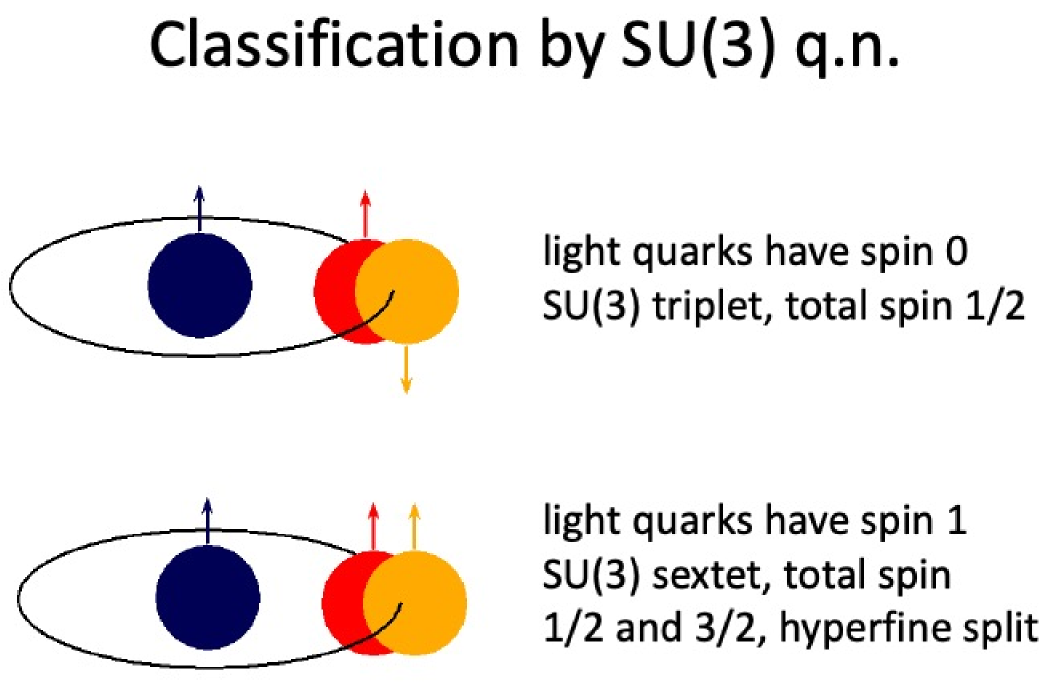

Figure 1: Heavy baryons as system composed from light diquark and heavy quark. Two invariant light quark configurations: triplet and sextet are possible.

This pattern is confirmed experimentally. Figs. 2 and 3 show masses and splittings in the c and b sector. Spin one sextet diquark can couple with heavy quark spin to spin 1/2 and 3/2, and these states should be – according to heavy quark spin symmetry – degenerate. In the charm sector this splitting is approximately 70 MeV and in the bottom sector 20 MeV. The ratio of this splittings is again approximately equal to 3 confirming $1/M_Q$ violation of heavy quark spin symmetry.

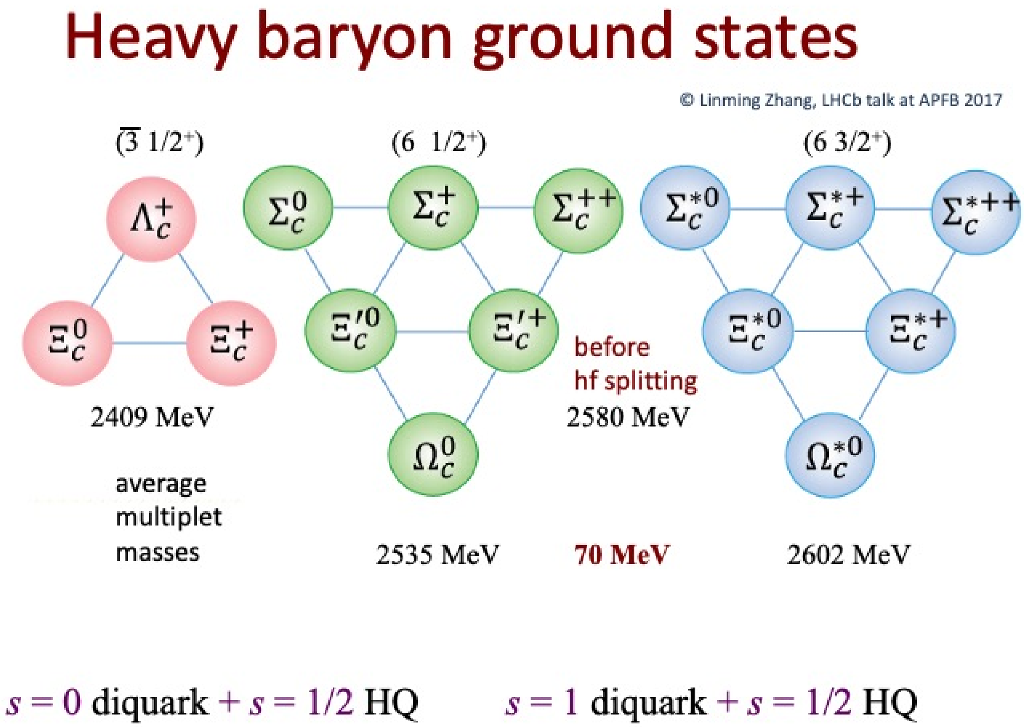

Figure 2: Heavy baryons with charm.

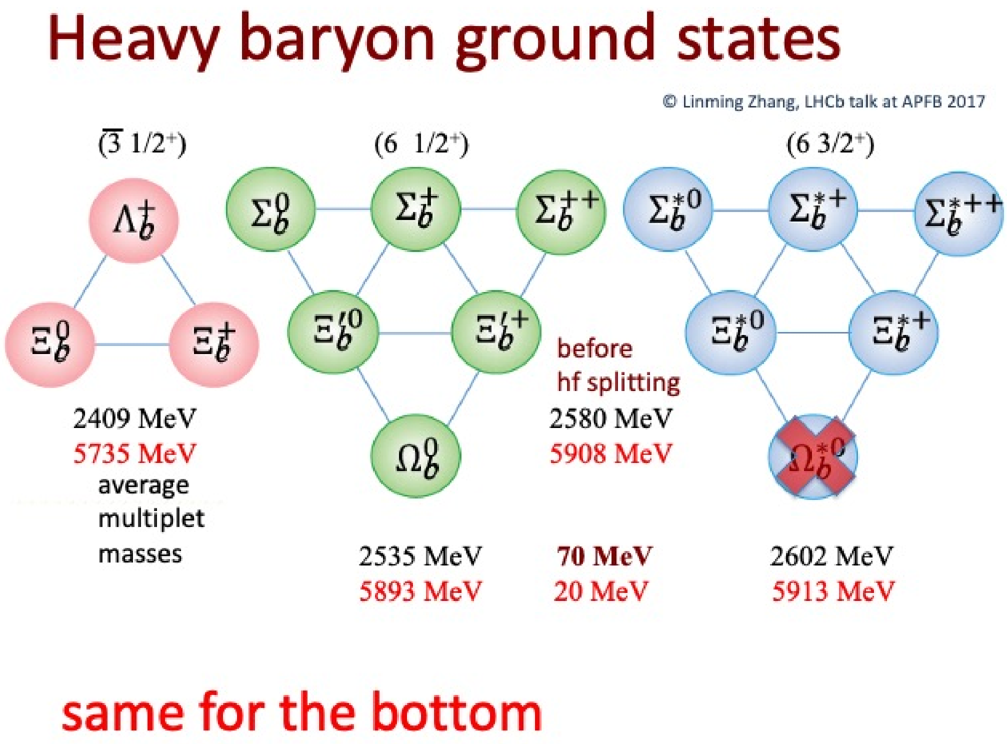

Figure 3: Heavy baryons with bottom quark. Ω_b^* has not been observed so far.

4 Covariant representation of fields[1]

HQS can relate matrix elements of weak currents such as for example

$$\langle D|\,\bar{c}\gamma^\mu(1-\gamma_5)b\,|B\rangle$$

describing decays of B mesons to D mesons or leptonic decays (like pion decay) through the current

$$\langle 0|\,\bar{q}\gamma^\mu\gamma_5 Q\,|H\rangle\,.$$

We want to construct *effective* fields $H_v^{(Q)}$ that desribe heavy mesons composed of $\bar{Q}_\alpha q_\beta$ (where α and β are Drac indices), that describe simultaneously (pseudo)scalar and vector mesons and have proper transformation properties with respect to the Lorentz transfor-

[1] A. Manohar and M. Wise *Heavy Quark Physics*, Cambridge University Press.

mation denoted by Λ:

$$x' = \Lambda x,\ v' = \Lambda v. \tag{21}$$

We require

$$H_{v'}^{(Q)\prime}(x') = D(\Lambda) H_v^{(Q)}(x) D(\Lambda)^{-1} \tag{22}$$

where $D(\Lambda)$ is spinor Lorentz transformation. This transformation can be alternatively written as

$$H_v^{(Q)}(x) \to H_v^{(Q)\prime}(x) = D(\Lambda) H_{\Lambda^{-1}v}^{(Q)}(\Lambda^{-1}x) D(\Lambda)^{-1}. \tag{23}$$

The field $H_v^{(Q)}(x)$ is a linear combination of the pseudoscalar field $P_v^{(Q)}(x)$ and the vector field $P_{v\mu}^{*(Q)}(x)$ that annihilate the $s_l = 1/2$ meson multiplet. Vector particles have a polarization vector ε_μ, with $\varepsilon^2 = -1$, and $v{\cdot}\varepsilon = 0$. The amplitude for $P_{v\mu}^{*(Q)}$ to annihilate a vector particle is ε_μ. A simple way to combine the two fields into a single field with the desired transformation properties is to define:

$$H_v^{(Q)} = \frac{1+\not{v}}{2}\left[\not{P}_v^{*(Q)}(\varepsilon) + iP_v^{(Q)}\gamma_5\right]. \tag{24}$$

Let's observe that

$$\not{v} H_v^{(Q)} = H_v^{(Q)},\ H_v^{(Q)}\not{v} = -H_v^{(Q)} \tag{25}$$

since $v \cdot P^{*(Q)} = 0$.

Recall Dirac matrices in the Dirac represenation:

$$\gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},\ \gamma^i = \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix},\ \gamma_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

In the rest frame $v_r^\mu = (1,0,0,0)$ component $P_{v_r 0}^{*(Q)} = 0$ because $v \cdot P^{*(Q)} = 0$ and

$$\begin{aligned} H_{v_r}^{(Q)} &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \left\{ -\begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} P_{v_r i}^{*(Q)} + i \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} P_{v_r}^{(Q)} \right\} \\ &= \begin{bmatrix} 0 & iP_{v_r}^{(Q)} - \boldsymbol{\sigma}\cdot\boldsymbol{P}_{v_r}^{*(Q)} \\ 0 & 0 \end{bmatrix}. \end{aligned} \tag{26}$$

The indices α and β of the field $\left[H_{v_r}^{(Q)}\right]_{\alpha\beta}$ label the spinor indices of the heavy quark Q_α and the light degrees of freedom, respectively. The field $H_{v_r}^{(Q)}$ transforms as a (1/2, 1/2) representation under $\hat{\boldsymbol{S}}_Q \otimes \hat{\boldsymbol{S}}_l$.

Recall that action of the symmetry operator $\mathcal{O}$ on some field ψ beloging to representation of this symmetry is

$$\mathcal{O}\psi\,\bullet + \psi\mathcal{O}\,\bullet$$

so that the field transformation is obtained by subtracting the last term (we are not nterested in action of $\mathcal{O}$ on some trial function denoted as $\bullet$), which gives a commutator

$$[\mathcal{O}, \psi]\,.$$

The spin operators $\boldsymbol{S}_Q$ and $\boldsymbol{S}_l$ for the heavy quark and light degrees of freedom acting on the $H_{v_r}^{(Q)}$ field are:

$$\begin{aligned} \left[\hat{\boldsymbol{S}}_Q, H_{v_r}^{(Q)}\right] &= \frac{1}{2}\boldsymbol{\Sigma}\, H_{v_r}^{(Q)}, \\ \left[\hat{\boldsymbol{S}}_l, H_{v_r}^{(Q)}\right] &= -\frac{1}{2}\, H_{v_r}^{(Q)}\boldsymbol{\Sigma} \end{aligned} \tag{27}$$

where

$$\Sigma^i = \frac{i}{4}\varepsilon_{ijk}\left[\gamma^j, \gamma^k\right] = \begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{bmatrix}. \tag{28}$$

Under infinitensimal rotations (neglecting angular momentum)

$$\begin{aligned} \delta H_{v_r}^{(Q)} &= i\boldsymbol{\varepsilon}\cdot\left[\left(\hat{\boldsymbol{S}}_Q + \hat{\boldsymbol{S}}_l\right), H_{v_r}^{(Q)}\right] \\ &= \frac{i}{2}\boldsymbol{\varepsilon}\cdot\left[\boldsymbol{\Sigma}\,, H_{v_r}^{(Q)}\right]. \end{aligned} \tag{29}$$

Let's compute this commutator

$$\begin{aligned} \frac{i}{2}\boldsymbol{\varepsilon}\cdot\left[\boldsymbol{\Sigma}\,, H_{v_r}^{(Q)}\right] &= \frac{i}{2}\varepsilon_i\left[\begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{bmatrix}, \begin{bmatrix} 0 & iP_{v_r}^{(Q)} - \sigma_j\left[P_{v_r}^{*(Q)}\right]_j \\ 0 & 0 \end{bmatrix}\right] \\ &= \frac{i}{2}\varepsilon_i\begin{bmatrix} 0 & -\left[\sigma_i, \sigma_j\right] \\ 0 & 0 \end{bmatrix}\left[P_{v_r}^{*(Q)}\right]_j \\ &= \varepsilon_i\varepsilon_{ijk}\begin{bmatrix} 0 & \sigma_k \\ 0 & 0 \end{bmatrix}\left[P_{v_r}^{*(Q)}\right]_j. \end{aligned} \tag{30}$$

This ammounts to

$$\begin{aligned} \delta P_{v_r}^{(Q)} &= 0, \\ \delta \boldsymbol{P}_{v_r}^{*(Q)} &= \boldsymbol{\varepsilon} \times \boldsymbol{P}_{v_r}^{*(Q)}, \end{aligned} \tag{31}$$

which are the transformations of spin zero and spin one fields, respectively.

Under transformations that correspond only to $\hat{\boldsymbol{S}}_Q$ or $\hat{\boldsymbol{S}}_l$ scalar and vector fileds mix (exercise). For example under $\hat{\boldsymbol{S}}_Q$

$$\begin{aligned} \delta P_{v_r}^{(Q)} &= -\frac{1}{2}\boldsymbol{\varepsilon} \cdot \boldsymbol{P}_{v_r}^{*(Q)}, \\ \delta \boldsymbol{P}_{v_r}^{*(Q)} &= \frac{1}{2}\boldsymbol{\varepsilon} \times \boldsymbol{P}_{v_r}^{*(Q)} - \frac{1}{2}\boldsymbol{\varepsilon}\, P_{v_r}^{(Q)}. \end{aligned} \tag{32}$$

5 Heavy meson decay constants

Decay constants (as in the case of Goldstone bosons) are defined as

$$\begin{aligned} \langle 0 |\, \bar{q}\gamma^\mu \gamma_5 Q(0)\, | P(p) \rangle &= -i \overset{\dim 1}{f_P}\, p^\mu = -i f_P\, M_P v^\mu, \\ \langle 0 |\, \bar{q}\gamma^\mu Q(0)\, | P^*(p,\varepsilon) \rangle &= \underset{\dim 2}{f_{P^*}}\, \varepsilon^\mu. \end{aligned} \tag{33}$$

If not for the HQS symmetry constants f_P and f_{P^*} would be independent. Note that the above relations are written in terms of fields normalized according to (16).

Currents can be written in terms of Q_v fields:

$$\bar{q}\Gamma^\mu Q = \bar{q}\Gamma^\mu Q_v + \ldots$$

In heavy quark effective theory we have one matrix element instead of two

$$\langle 0 |\, \bar{q}\Gamma^\mu Q_v(0)\, | H(v,p) \rangle \tag{34}$$

where $H(v,p)$ are heavy quark fields normalized according to (18). The current $\bar{q}\Gamma^\mu Q_v$ transforms under heavy quark rotation

$$\bar{q}\Gamma^\mu Q_v \to \bar{q}\Gamma^\mu\, D_Q(R) Q_v \tag{35}$$

where $D_Q(R)$ is the rotation matrix of a heavy quark filed. However, we need transformation of the current in terms of $H_v^{(Q)}$, which should transform in the same manner as (35). To find the proper representation we will use the following trick:

- Pretend that Γ^μ transforms as $\Gamma^\mu \to \Gamma^\mu D_Q^{-1}(R)$, so that the quark current is invariant.
- Write down operators that are invariant when

$$\begin{aligned} Q_v &\to D_Q(R) Q_v, \\ \Gamma^\mu &\to \Gamma^\mu D_Q^{-1}(R), \\ H_v^{(Q)} &\to D_Q(R) H_v^{(Q)}. \end{aligned}$$

- Set Γ^μ to $\gamma^\mu\gamma_5$ or γ^μ to obtain the operator with the correct transformation properties.

Remarks:

- The current must have single $H_v^{(Q)}$ field (matrix element (34) has only one meson).
- Field $H_v^{(Q)}$and Γ^μ must appear as a product $\Gamma^\mu H_v^{(Q)}$ to satisfy invariance property.
- For Lorentz covariance, the current must have the form

$$\mathrm{Tr}\left(X\,\Gamma^\mu H_v^{(Q)}\right)$$

where X is a Lorentz bispinor.

The only parameter that X can depend on is v, so

$$X = a_0(v^2 = 1) + a_1(v^2 = 1)\not{v}. \tag{36}$$

This form is compatible with Lorentz covarince and parity (not discussed). Recall (25)

$$H_v^{(Q)}\not{v} = -H_v^{(Q)}.$$

Hence

$$\begin{aligned} \mathrm{Tr}\left(X\,\Gamma^\mu H_v^{(Q)}\right) &= a_0\,\mathrm{Tr}\left(\Gamma^\mu H_v^{(Q)}\right) + a_1\,\mathrm{Tr}\left(\not{v}\,\Gamma^\mu H_v^{(Q)}\right) \\ &= \underbrace{(a_0 - a_1)}_{=a/2}\,\mathrm{Tr}\left(\Gamma^\mu H_v^{(Q)}\right). \end{aligned} \tag{37}$$

Therefore only one constant enters, rather than two:

$$\bar{q}\Gamma^\mu Q_v = \frac{a}{2}\,\mathrm{Tr}\left(\Gamma^\mu H_v^{(Q)}\right). \tag{38}$$

It is now a question of Dirac algebra to calculate the traces (exercise):

$$\frac{a}{2}\,\mathrm{Tr}\left(\Gamma^\mu H_v^{(Q)}\right) = a \times \begin{cases} -iv^\mu P_v & \text{for} \quad \Gamma^\mu = \gamma^\mu\gamma_5 \\ \\ P_v^{*\mu} & \text{for} \quad \Gamma^\mu = \gamma^\mu \end{cases} \tag{39}$$

and we get

$$\begin{aligned} \langle 0|\,\bar{q}\gamma^\mu\gamma_5 Q(0)\,|P(p)\rangle &= -i\sqrt{M_P}av^\mu = -if_P\,M_P v^\mu, \\ \langle 0|\,\bar{q}\gamma^\mu Q(0)\,|P^*(p,\varepsilon)\rangle &= \sqrt{M_{P^*}}a\varepsilon^\mu = f_{P^*}\varepsilon^\mu \end{aligned} \tag{40}$$

where we have "undone" change of normalization (17). Note that in fact up to $1/M_Q$ $M_P = M_{P^*}$. Looking at the definitions of decay constants we have

$$f_P = \frac{a}{\sqrt{M_P}},\ f_{P^*} = a\sqrt{M_{P^*}}. \tag{41}$$

Eliminating a we get

$$\frac{f_B}{f_D} = \sqrt{\frac{M_D}{M_B}}. \tag{42}$$

Experimentally f_P are not known and we take them from lattice QCD

$$\frac{f_B}{f_D} = \frac{173}{197} = 0.88,\ \sqrt{\frac{M_D}{M_B}} = \sqrt{\frac{1880}{5280}} = 0.60.$$

If true, there are large corrections to HQS symmetry.

This method can be applied to more complicated matrix elements reducing the number of independent parameters. For example weak decays

$$\begin{aligned} \bar{B} &\to D + l + \nu \\ \bar{B} &\to D^* + l + \nu \end{aligned}$$

can be parametrized in QCD by 6 independent formfactors:

$$
\begin{aligned}
\langle D(p')|\, V^\mu \left|\bar{B}(p)\right\rangle &= f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu, \\
\langle D^*(p',\varepsilon)|\, V^\mu \left|\bar{B}(p)\right\rangle &= g(q^2)\, \varepsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu\, (p+p')_\alpha (p-p')_\beta, \\
\langle D^*(p',\varepsilon)|\, A^\mu \left|\bar{B}(p)\right\rangle &= -if(q^2)\varepsilon^{*\mu} - i\varepsilon^*\cdot p\, \left[a_+(q^2)(p+p')^\mu + a_-(q^2)(p-p')^\mu\right].
\end{aligned}
$$

In heavy quark theory there is only one form-factor known as Isgur-Wise function $\xi(v\cdot v')$.