QCD
problem set 13

1. Express heavy quark lagrangian

$$\mathcal{L}_Q = \bar{Q}(x)\left(i\not{D} - M_Q\right)Q(x)$$

in terms of fields Q_v and B_v defined as follows:

$$Q(x) = e^{-iM_Q v\cdot x}\left[Q_v(x) + B_v(x)\right]$$

where

$$\not{v}Q_v(x) = Q_v(x),\ \not{v}B_v(x) = -B_v(x).$$

To this end decompose covariant derivative

$$i\not{D} = \not{v}\, iv\cdot D + i\not{D}_T.$$

- Show that

$$\bar{Q}_v = \bar{Q}_v\frac{1+\not{v}}{2},\ \bar{B}_v = \bar{B}_v\frac{1-\not{v}}{2}.$$

- Next show that

$$i\not{D}e^{-iM_Q v\cdot x}\left[Q_v + B_v\right] = e^{-iM_Q v\cdot x}\left\{M_Q\not{v} + \not{v}\, i\, v\cdot D + i\not{D}_T\right\}\left[Q_v + B_v\right].$$

- Prove that

$$\left(1+\not{v}\right)\not{D}_T = \not{D}_T\left(1-\not{v}\right).$$

- Using this identity show that

$$\bar{Q}_v(x)\not{D}_T Q_v(x) = 0,\ \bar{B}_v(x)\not{D}_T B_v(x) = 0.$$

- Show that

$$\bar{Q}_v\not{v}B_v = \bar{B}_v\not{v}Q_v = 0.$$

2. Spin operator can be defined as

$$S^i = \frac{1}{2}\gamma_5\gamma^0\gamma^i$$

or

$$S^i = \frac{i}{8}\varepsilon_{ijk}\left[\gamma^j, \gamma^k\right].$$

Calculate explicitly S^i given by these two expressions in the Dirac representations of γ matrices.

3. Heavy mesons consisting from a heavy antiquark and a light quark (or viceversa) can have spin 0 or 1 (why?). For the charm quark these mesons are called $D^{0,\pm}$ (spin 0) or $(D^*)^{0,\pm}$ (spin 1) and similarly for bottom quark: $B^{0,\pm}$ and $(B^*)^{0,\pm}$. Superscripts $0, \pm$ denote charge. Mesons with strange quark carry subscript s. Masses of these particles can be found at the web page of Particle Data Group (https://pdg.lbl.gov/2020/listings/contents_listings.html). For masses of the lightests spin 0 mesons ($M_{D,B}$) and spin 1 (M_{D^*,B^*}) of the same charge (both non-strange and strande) calculate
$$M_{X^*}^2 - M_X^2 = ?$$
where $X = D$ or B. You will find that these differences are almost the same both for D and B mesons.

4. Assume that heavy meson mass of spin $s = 0, 1$ is given by
$$M(s) = \alpha M_Q + \frac{\mu}{M_Q} \boldsymbol{S}_Q \cdot \boldsymbol{S}_l$$
where $\boldsymbol{S}_Q$ and $\boldsymbol{S}_l$ denote spin of a heavy antiquark (quark) and spin of a light quark (antiquark) respectively. Meson spin is equal $\boldsymbol{S} = \boldsymbol{S}_Q + \boldsymbol{S}_l$ and $\boldsymbol{S}^2 = s(s+1)\,\mathbf{1}$. Calculate masses of spin 0 and spin 1 mesons and then calculate the difference from the previous problem.