QCD
problem set 10

1. In QCD infinitensimal change of the gauge field under the gauge transformation
$$\Omega(x) = \exp(i\theta_a(x)T^a)$$
can be calculated from
$$\boldsymbol{A}_\mu^\Omega = \Omega^\dagger(x)\boldsymbol{A}_\mu\Omega(x) + \frac{i}{g}\Omega^\dagger(x)\partial_\mu\Omega(x)$$
and reads (show it):
$$g\,\delta A_\mu^a = g f^{abc}\theta_b(x)A_\mu^c - \partial_\mu\theta_a(x).$$
Calculate the change of the gauge condition
$$G^a(A_\mu) = n^\mu A_\mu^a$$
$g\,\delta G^a$ under this transformation.

2. Next calculate matrix
$$\mathcal{M}_{ab} = g\frac{\delta G^a}{\delta\theta_b}$$
and then rewrite the free part of the ghost action
$$\int d^4x \mathcal{L}_{FPG}^0 = \int d^4x \bar{\chi}_a \mathcal{M}_{ab}(g=0)\chi_b$$
in momentum space. The inverse of this term is the ghost propagator.

3. Applying gauge condition from problem 1 to the free part of the gluon action calculate the gluon propagator (inverse of gluonic operator derived from the free part of $F_{\mu\nu}F^{\mu\nu}$) in the axial gauge.

4. For a system of SU(3) scalar fields $\hat{\phi}_i(x)$ with $i = 1, 2, 3$ that satisfy the following commutation rules
$$\left[\hat{\phi}_i(t,\vec{x}), \hat{\pi}_j(t,\vec{x}\,')\right] = i\delta^{(3)}(\vec{x}-\vec{x}\,')\delta_{ij}$$
one defines charge operators
$$\hat{Q}^a(t) = -i\int d^3\vec{x}\,\hat{\pi}_i(t,\vec{x})T_{ij}^a\hat{\phi}_j(t,\vec{x})$$
where matrices T^a satisfy SU(3) commutation relations:
$$\left[T^a, T^b\right] = if^{abc}T^c.$$
Prove that
$$\left[\hat{Q}^a(t), \hat{Q}^b(t)\right] = if^{abc}\hat{Q}^c(t).$$