QCD
problem set 9

1. Prove that
$$\int\limits_{-T/2}^{T/2} d\tau_1 \int\limits_{\tau_1}^{T/2} d\tau_2 \ldots \int\limits_{\tau_{n-1}}^{T/2} d\tau_n = \frac{1}{n!}T^n.$$

2. Consider zero energy motion in the inverted double well potential $V(x)$
$$E = \frac{1}{2}m\dot{x}^2 - V(x)$$
and express the classical action for a motion between the two maxima in time interval $\left\{-\frac{T}{2}, \frac{T}{2}\right\}$ in terms of the integral over the potential. Calculate action explicitly for the following potential
$$V(x) = \frac{1}{8a^2}(a^2 - x^2)^2.$$
Note that the unity "1"in potnential $V(x)$ has dimension of energy/distance^2.

3. For the potential from problem 2 calculate the classical trajectory starting at $-T/2$ in $-a$ and ending at $T/2$ in a. This can done by using the fact that the instanton is a zero energy motion. From this condition you can calculate velocity in terms of potential and then inegrating both sides over time and position you get the final answer.

4. Going from Minkowski to Euclidean space may be sometimes cumbersome. Using prescription attached to this problem sheet calculate the argument of the exponent in the functional integral:
$$iS_M = -\frac{i}{4}\int d^4x_M\, F^{\mu\nu}F_{\mu\nu}$$
in Euclidean space. To this end calculate separately F_{ik} and F_{0k}.