QCD
problem set 7

1. General fermionic mass term reads (where M is complex):
$$M\,\bar{\psi}\frac{1+\gamma_5}{2}\psi + M^*\,\bar{\psi}\frac{1-\gamma_5}{2}\psi. \tag{1}$$
Prove that (1) is Hermitean. Show that chiral transformation
$$\psi \to e^{i\alpha\gamma_5}\psi$$
ammounts to
$$M \to e^{2i\alpha}M.$$

2. Prove that
$$\partial_\mu K^\mu = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$
where
$$K^\mu = \varepsilon^{\mu\nu\rho\sigma}\left(A^a_\nu F^a_{\rho\sigma} - \frac{g}{3}f^{abc}A^a_\nu A^b_\rho A^c_\sigma\right).$$
This calculation proves that anomaly is a total derivative.

3. Consider a complex scalar and/or fermion field theory coupled to the nonabelian gauge fields:
$$\begin{aligned}\mathcal{L}_\phi &= \left(D_\mu\phi(z)\right)^\dagger\left(D_\mu\phi(x)\right) - m^2\phi^\dagger(x)\phi(x) - V(\phi^\dagger(x)\phi(x)),\\ \mathcal{L}_\psi &= \bar{\psi}(x)\left(i\not{D}_x - m\right)\psi(x),\end{aligned}$$
with covariant derivative defined as
$$D_\mu = \partial_\mu - igA^a_\mu(x)\,T^a$$
where T^a are generators of SU(N) group. Lagrangian for the gauge fields reads
$$\mathcal{L}_A = -\frac{1}{4}F^a_{\mu\nu}(x)F^{a\,\mu\nu}(x).$$
Derive by means of the variational approach equations of motion for (analogs of Maxwell equations) the scalar and fermion theory. Prove the identity
$$[D_\mu, F^{\nu\rho}] + [D_\nu, F^{\rho\mu}] + [D_\rho, F^{\mu\nu}] = 0.$$