QCD
problem set 6

1. What is the pole structure of the scalar propgator
$$\frac{i}{(1+i0^+)k_0^2-(1-i0^+)(\boldsymbol{k}^2+m^2)}$$
compared to
$$\frac{1}{k^2-m^2+i0^+}.$$

2. Consider Gaussian integral
$$J(\mathcal{M})=\int d^N\xi\, d^N\psi\ \exp\left(\psi_i\mathcal{M}_{ij}\xi_j\right)$$
where ψ_i and ξ_i $(i=1,2,\ldots N)$ are independent Grassmann variables. Expanding in a power series and commuting ξ's and ψ's show that
$$J(\mathcal{M})=\det(\mathcal{M}).$$

3. Show that in electrodynamics one can write
$$-\frac{1}{4}\int d^4x\, F^{\mu\nu}F_{\mu\nu}=\frac{1}{2}\int d^4x\, A^\mu(g_{\mu\nu}\Box-\partial_\mu\partial_\nu)A^\nu.$$
Transform this expression to the momentum space.

4. Generating functional for electrodynamics in a covariant gauge is defined as:
$$\begin{aligned}Z_0[j^\mu] &= \int[D\omega(x)]\int[DA^\mu(x)]\exp\left(-i\frac{\xi}{2}\int d^4x\,\omega^2\right)\\ &\quad\times\delta(\partial_\mu A^\mu-\omega)\exp\left(i\int d^4x\left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+j_\mu A^\mu\right)\right),\end{aligned}$$
where ω, A^μ and $F^{\mu\nu}$ are functions of x. Show that it is equal to
$$Z_0[j^\mu]=\int[DA^\mu(x)]\ \exp\left(i\int d^4x\left(\frac{1}{2}A^\mu(g_{\mu\nu}\Box-(1-\xi)\partial_\mu\partial_\nu\right)A^\nu+j_\mu A^\mu\right).$$

5. Invert matrix
$$g_{\mu\nu}k^2-(1-\xi)k_\mu k_\nu$$